

Reliability Based Comparison Between ACI 318-05 and NBR 6118

Comparação Entre as Normas ACI 318-05 e NBR 6118 Com Base na Teoria da Confiabilidade



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Abstract

This paper presents reliability analyses for reinforced concrete structural members subjected to bending and shear, such as beams and slabs. The analyses are developed for members designed according to American Standard ACI 318-05 and to Brazilian Standard NBR 6118. The strength limit state functions are developed for reinforced rectangular sections, for different values of cross section sizes and reinforcement ratios. The considered reliability models use typical values for average values, standard deviations and bias factors for both resistance and loads variables. The analyses consider all the possibilities regarding the relationship between applied dead and live loads. The results obtained in the analyses are the reliability indexes for structures designed according to the two considered standards. It is shown that, although the design philosophy of the two standards is quite different, the obtained reliability levels can be compared and are generally very similar.

Keywords: reliability analysis; concrete structures; flexural design; shear design.

Resumo

Este artigo apresenta Análises de Confiabilidade para elementos de concreto estrutural submetidos a flexão simples e cisalhamento, tais como vigas e lajes. As análises são desenvolvidas para elementos projetados de acordo com a Norma Americana ACI 318-05 e com a Norma Brasilieira NBR 6118. As funções de estado limite de resistência são desenvolvidas para seções retangulares de concreto armado, para diferentes dimensões das seções transversais e diferentes porcentagens de armadura. Os modelos considerados nas Análises de Confiabilidade usam valores numéricos típicos para valores médios, valores característicos e desvios padrão das variáveis relativas a resistências e cargas. As análises consideram todas as possibilidades relativas às relações entre cargas perma nentes e variáveis aplicadas. Os resultados obtidos nas análises são os índices de confiabilidade para estruturas projetadas de acordo com as duas normas consideradas. É mostrado que, embora as filosofias que norteiam as duas normas sejam muito diferentes, os índices de confiabilidade obtidos são comparáveis e geralmente bastante similares.

Palavras-chave: análise de confiabilidade; estruturas de concreto; dimensionamento à flexão; dimensionamento ao cisalhamento

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Introduction

The crescent economical integration among the countries around the world, which includes the market of services in civil engineering, lead to the necessity of a mutual deeper knowledge of the technical standards and of the specific design requirements established in each of these countries. In this way, this paper proposes to perform a comparison between the design of reinforced concrete structures, according to ACI 318-05¹ and to NBR 6118². It is to be noticed that, being the Brazil the biggest country of Latin America, is among the few ones (with Paraguay and Uruguay) that does not adopt formally the Spanish version of ACI 318-05 for regulating the design of concrete structures.

Both analyzed standards, ACI 318-05 and the NBR 6118, consider the structural safety verifications through the LRFD (Load Resistance Factor Design), following a semiprobabilistic approach, which can be represented by this symbolic requirement:

Design strength \geq Required strength	(1)
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Uncertainties are inherently present in both sides of the inequality, i.e., in the evaluation of the actions and of the resistance of the structural members. In this semi-probabilistic approach, these uncertainties are accounted for by the use of the load and resistance factors defined in the design standards. These factors are applied for reducing the nominal values of the structural strengths as well as for increasing the nominal values of the actions, accounting for possible under-strength or over-loading of the structural members. The nominal values of resistances and actions are defined as values with a given probability of being attained or exceeded, respectively, in the service life of the structure.

It is nowadays recognized that a rational basis for evaluating the actual risks of failure in a structure can only be achieved by a full probabilistic approach. However, it should be recognized that this approach can give only a "nominal" evaluation for the probability of the actual failure, as long as several relevant variables are not included in the reliability models, such as human errors, deterioration of the structure, etc. In this "nominal" reliability evaluation of the safety in a given concrete structure, the several variables involved, such as the concrete and reinforced steel strengths, geometric dimensions and acting loads shall be treated as random variables.

It has been considered herein that for the comparison between ACI 318-05 and NBR 6118, a mere analysis of the load and resistance factors defined by the standards, or even the direct comparison between examples of structures designed according their requirements, would be useless in a quantitative point of view, as long as their design philosophies are quite different. One of the main points of difference to be noticed is related to the resistance safety factors, different for the concrete and the steel in NBR 6118, and global (factors ϕ) in ACI 318-05.

It is to be pointed out that the Brazilian Standard is strongly influenced by the European technical tradition and also by the characteristics of low seismicity of Brazil. The proposed comparison is then done herein from the results of reliability analyses for structural members designed according the two standards. It is considered that the obtained reliability indexes express a quantitative measure of the reliability of the structures with respect to actual risks of failure. For obtaining a representative set of numerical values that would permit the comparisons, several reliability analyses have been done, for reinforced rectangular sections with different values of cross section sizes and reinforcement ratios, and considering all the possibilities regarding the relationship between applied dead and live loads. In this way, a consistent comparison between the standards is possible, based on actual quantitative results.

The considered reliability models use typical values found in the literature for the average values, standard deviations and bias factors for both resistance and loads variables. Reliability based studies for defining load factors for the ASCE-7³ have been performed by Ellingwood et al.⁴. Reliability analyses of reinforced concrete structures designed according to ACI 318-05 have been performed for combinations of dead, live, wind and snow loads (for instance, by Nowak and Collins⁵; Nowak and Szerszen⁶; Szerszen and Nowak⁷; and Szerszen et al.⁸).

The present study is focused in structural members, such as beams and slabs, subjected only to bending and shear, typical of building structures, designed considering the code provisions of the ACI 318-05 and of the NBR 6118. The reliability indexes for the possible loading combinations of dead and live loads are then evaluated and compared.

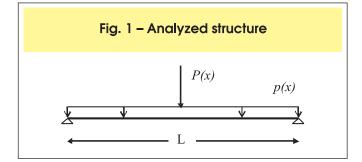
Research Significance

The evaluation of the safety levels implied in the adoption of a given technical standard can be a significant parameter in the establishment of the confidence of the standard for their users. The use of the reliability analysis provides a rational tool for this evaluation. The results presented herein, of reliability indexes obtained with structural elements designed according to ACI 318-05 and NBR 6118, for bending and shear, provide some quantitative elements for the discussions related to the comparisons and use of these two standards.

Analyzed Examples

Without loss of generality regarding any other cases of reinforced concrete structural members subjected to bending or shear, a simply supported element (beam or slab) is considered, as shown in Figure 1.

The element is subjected to a general loading case, composed by concentrated (P(x)) and distributed (p(x)) loads



acting along the member length L. These loads are, in general, composed by a combination of dead and live loads, producing a maximum bending moment equal to M and a maximum shear force equal to V.

Two structural sections have been considered: a beam of dimensions 250 x 1000 mm and a slab of 150 mm of thickness. Brazilian steel CA-50 (with nominal strength $f_y = 500$ MPa), and concrete Brazilian Class C25 (which corresponds to ACI 318-05 nominal concrete strength $f'_c = 25.94$ MPa) are considered.

The adopted steel areas for the flexural and shear reinforcements of the beam and of the slab are defined in Table 1. Two conditions have been considered for the flexural reinforcement of the two elements: A_{s.max} and A_{s.min} (respectively, maximum and minimum amounts of reinforcement, according to the most rigorous criteria between both considered standards). It is to be noticed that the minimum flexural reinforcement according to ACI 318-05 is almost twice the one defined by NBR 6118. For the shear reinforcement, three conditions have been considered: $a_{_{\text{sw, max}}},~a_{_{\text{sw, ave}}}$ and $a_{_{\text{sw,min}}}$ (maximum, average and minimum amounts of reinforcement; for the slab, minimum reinforcement corresponds to zero shear reinforcement). It should be noted that differently from the ACI 318-05, NBR 6118 don't allow beams without shear reinforcement.

Resistance Models

The actual resistance R of a structural element can be generically expressed by the product of the following factors (Szerszen et al.⁸):

$$R = R_n \times M_F \times F_F \times P_F$$
 (2)

In this formula, R_n stands for the nominal values of the resistances of concrete and steel; M_F stands for the material factor, which reflects the statistical variations in the strength properties of the materials; F_F stands for the fabrication factor, which reflects the uncertainties in the reinforced concrete fabrication, regarding the deviation of geometric dimensions from the design values; P_F stands for the professional factor, which reflects the uncertainties in the adopted method of structural analysis, i.e., the deviation between the analytically predicted capacity and the actual "in-situ" performance of the structural member.

A study of these factors, to be used in the reliability analyses, was presented by Szerszen et al.⁸; the values adopted herein are based on this study, and also in the experience gathered from the Brazilian construction industry.

The statistical properties of the materials are defined in Table 2 and the fabrication and professional factors in Table 3. Subscripts b and s are used in the Tables, corresponding to the adopted numerical values, when different, respectively for beams and slabs. It is to be noticed the difference between the coefficient of variation 0.10 considered for the concrete compression resistance in the specimens for the compression tests and the coefficient 0.15 considered for the actual resistance of the concrete in the structures.

Table 1 – Nominal values of steel areas (flexure and shear)			
Analysis	Steel area	Beam	Slab
Flexure: mm² or mm² / m	$A_{s,max}$	3150	1680
	$A_{s,min}$	630	336
Shear: mm ² / m or mm ² / m ²	A _{s,max}	1680	6720
	$A_{\mathrm{s,ave}}$	969	3360
	$A_{s,min}$	257	0

Table 2 – Statistical properties of the materials				
Variable	Description	Average values (µ)	Coefficients of variation (σ/μ)	Nominal values
f _y	Steel yielding stress	545 MPa	0.05	500 MPa
f _c	Concrete compression resistance	29.95 MPa	0.15	25 MPa
f_{ct}	Concrete tension resistance	2.565 MPa	0.20	1.796 MPa

Strength Capacity of the Structural Elements in Bending

The Figure 2 shows the equilibrium for the design condition, in the critical section of a structural member, where the maximum bending moment M occurs. The relevant member geometric dimensions are b, d and h, respectively, width, effective and total heights; the steel reinforcement area is A_s. The steel resisting force R_s is expressed as a function of the steel area A_s and the steel strength f_y; the concrete resisting force R_c is expressed as a function of the section width b, the depth of the neutral axis c and the compressive concrete strength f_c:

 $R_{s} = A_{s} \cdot f_{v}$

(3a)	
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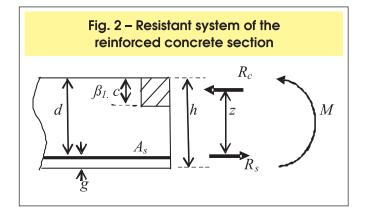
$R_{c} = 0.85 . b . \beta_{1} . c . f_{c} $ (3)

where:

 β_1 = factor defined as the ratio of the depth of equivalent rectangular stress block to the distance from the extreme compression fiber to the neutral axis; for f'_c = 25.94 MPa, β_1 = 0.85.

It should be noted that in the probabilistic equilibrium equations, the strength values f_y and f_c can assume values different from the nominal ones, representing then the actual stresses present in the steel and in the concrete in a given equilibrium situation.

Table 3 – Fabrication and professional factors					
Variable	Description	Average values (μ)	Coefficients of Nominal values variation (σ/μ)		
рΦ	Reinforcement area variation	1.00	0.015	1.00	
b	Width of the section, beam	0.25 m	0,08	0.25 m	
p*	Width of the section, slab	1.00 m	0	1.00 m	
p _z	Level arm variation	1.00	0.04	1.00	
h _b	Height of the section, beam	1.00 m	0.04	1.00 m	
h	Height of the section, slab	0.15 m	0.133	0.15 m	
g⊳	"Concrete cover" (g=h-d), beam	0.10 m	0.15	0.10 m	
g,	"Concrete cover" (g=h-d), slab	0.03 m	0,20	0.03 m	
P _{fb}	Professional factor, bending	1.02	0.06	1.00	
P _{fs}	Professional factor, shear	1.075	0.10	1.00	



As in bending R_s is equal to R_c , the value of c results equal to:

$$c = \frac{A_s \cdot f_y}{0.85 \cdot b \cdot \beta_1 \cdot f_c}$$
(4)

The equilibrium between the acting moment M in the section and the internal resisting forces, R_s of the steel reinforcement and R_c of the concrete, leads to the expression:

$$M = A_{s} \cdot f_{y} \left(d - \frac{\beta_{1}}{2} \cdot \frac{A_{s} \cdot f_{y}}{0.85 \cdot b \cdot \beta_{1} \cdot f_{c}} \right)$$
(5)

In the semi-probabilistic design according to ACI 318-05, nominal values are taken for the design variables, and a global strength reduction factor $\varphi=0.9$ is considered. In the semi-probabilistic design according to NBR 6118, the nominal resistances of the steel and of the concrete are reduced by resistance factors respectively equal to $\gamma_{\rm s}=1.15$ and $\gamma_{\rm c}=1.4.$

The equilibrium expressed in terms of the probabilistic variables is written as:

$$M_{P} = A_{s}.p_{\Phi}.f_{y}.p_{f}.p_{z}\left((h-g) - \frac{1}{2}.\frac{A_{s}.p_{\Phi}.f_{y}}{0.85.b.f_{c}}\right)$$
 (6)

The definition of the probabilistic variables and their respective considered numerical values is given in Tables 2 and 3. The subscript "P" means that the bending moment is evaluated probabilistically.

Strength Capacity of the Structural Elements in Shear

For the semi-probabilistic shear design according to ACI 318-05 (Eqs. (11-3) and (11-15)), the following usual design equations are considered, valid for elements with or without shear reinforcement:

$$V_{d} = \phi \left(0.17 \cdot \sqrt{f'_{c}} \cdot b \cdot d + a_{sw} \cdot f_{y} \cdot d \right)$$
(MPa units) (7)

In these equations, V_d is the design shear force, $a_{sw} = \frac{A_v}{2}$

is the transversal reinforcement by unit length (A_v – stirrups area and s – their spacing). As shown by MacGregor and Wight⁹, and also as summarized in the NCHRP-Report 549¹⁰, the limitation of the maximum shear reinforcement in ACI 318-05 is equivalent to the verification of the maximum compression stresses in the concrete diagonal struts. In this paper, the ACI 318-05 value $\varphi = 0.75$ and also the value $\varphi = 0.9$ have been analyzed.

This last value has been considered since the ACI Committee 318, permanently in charge of the revision of this standard, is presently studying a possible revisi on for this coefficient.

For the semi-probabilistic design according to NBR 6118 (Design Model I, struts angle $\theta = 45^{\circ}$), the following design equation is considered:

$$V_d = 0.6.f_{ctd}.b.d + a_{sw}.0.9.d.f_{yd}$$
 (8)

In this equation, $f_{ctd} = \frac{f_{ct}}{\gamma_c}$ and $f_{yd} = \frac{f_y}{\gamma_s}$ are the design values for the concrete and the steel resistance in tension,

values for the concrete and the steel resistance in tension, obtained by dividing their nominal resistance values by resistance factors equal to $\gamma_s = 1.15$ and $\gamma_c = 1.4$.

According to NBR 6118, it is only possible to have slabs without stirrups when the design shear force $\rm V_d$ is not superior to:

$$V_d = 0.25. f_{ctd}.k.(1.2 + 40\rho_1).b.d$$
 (9)

where:

k = 1.6 – d; ρ_1 = longitudinal reinforcement ratio. Assuming d = 0.12m and ρ_1 = 0.0015:

$$V_d = 0.25.f_{ctd} \cdot 1.48.(1,2+40.0.0015)b.d = 0.4662.f_{ctd} \cdot b.d$$
 (10)

Two equilibrium equations are defined in the following for the probabilistic analysis. The first equation is defined for the probabilistic check of the reinforcement:

$$V_P = p_f \cdot (0,33.f_{ct} \cdot b + a_{sw} \cdot p_{\phi} \cdot f_{yt} \cdot 0,90)(h-g)$$
(11)

This equation is defined with basis in the corresponding Equation (7) for the semi-probabilistic design according to the ACI 318-05. The equation is also valid for members without shear reinforcement. The subscript "P" means that the shear force is evaluated probabilistically. The parameter f_{ct} corresponds to the average splitting tensile strength of the concrete, according to the ACI 318-05.

It is assumed that $f_{ct} = \frac{\sqrt{f'_c}}{2}$ (MPa); the term $0.17.\sqrt{f'_c}$ in Equation (7) is then accordingly replaced

by $0.17.2f_{ct} = 0.33.f_{ct}$ in Equation (11).

In the second equation, for checking the compression stresses in the concrete diagonal struts, the maximum allowable design shear force defined below is considered, according to NBR 6118:

$$V_{P} = 0.27.p_{f}.(1 - \frac{f_{ck}}{250})f_{c}.b.(h-g)$$
(MPa units)
(12)

In this equation, $\rm p_{f}$ is the professional factor and $\rm f_{ck}$ (considered herein as 25 MPa), is the nominal resistance of the concrete.

Load Models

The following loading combinations are to be considered when defining the design loads:

ACI 318-05:
$$U = max \begin{cases} 1.4D \\ 1.2D + 1.6L \end{cases}$$
; NBR 6118: $U = 1.4D + 1.4L$ (13)

According to Szerszen et al.⁸, for dead loads and castin-place concrete, the bias factor (ratio average/nominal values) and the coefficient of variation in a Normal distribution can be taken respectively as $\lambda_{\rm D}$ = 1.05 and $V_{\rm D}$ = 0.10. For the maximum 50-year live load, a Gumbel distribution is adopted, with as $\lambda_{\rm L}$ = 0.934 and $V_{\rm L}$ = 0.20. These values correspond to a non-exceedance probability of 70% in 50 years, which is the criterion of NBR 6118 for defining the nominal live loads. From the nominal values of the loads, as defined in the pertinent standards, the average values of the loads are obtained considering the defined bias factors.

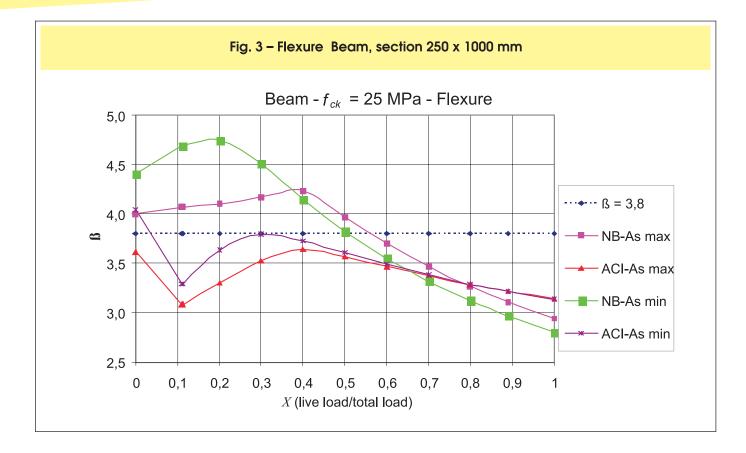
Results

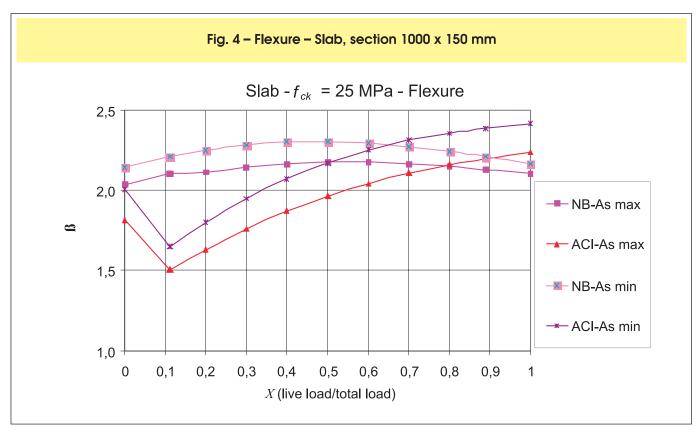
Considering the described reliability models, the analyses are performed using the computer program COMREL¹². The reliability indexes are obtained with one of the standard methods (FORM, SORM and Monte Carlo) available in the program, whichever is the most adequate in each situation. Main results of the analyses are presented in Figures 3 to 8.

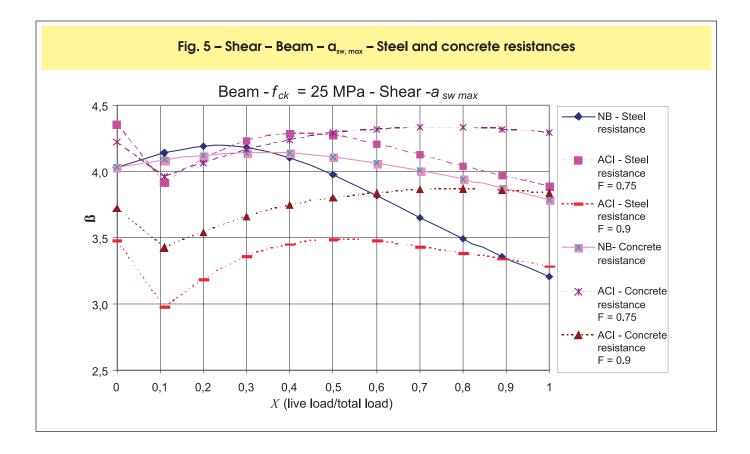
Figure 3 presents the results for the flexural analyses of the beam. The reliability indexes obtained with the NBR 6118 are greater for small values of χ (ratio live loads/total loads) and equivalent to the ones of the ACI 318-05 for higher values of χ . It is also plotted in the figure the reference value β = 3.8. This reference reliability index is the one defined for the "Ultimate Design States" in Eurocode 1¹², for a reference period of 50 years and for its "Consequence Class CC2". This Class corresponds to "medium consequence for loss of human life, economic, social or environmental consequences considerable", and includes the structures of hotels, schools and residential buildings. It is to be noticed that the flexural design is reasonably covered, according this criterion, for usual values of χ , for both ratios of flexural reinforcement, $A_{s,\,max}$ and $A_{s_{,\,min}}$ for NBR 6118; for ACI 318-05, the obtained values of β are smaller than to 3.8.

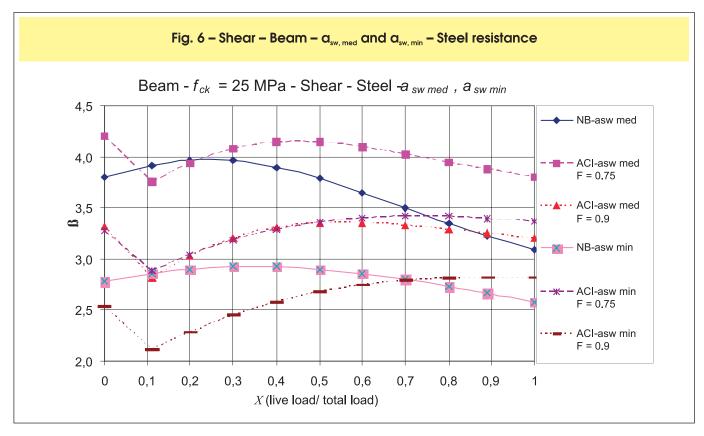
Figure 4 presents the results corresponding to the flexural analyses of the slab. The reliability indexes obtained with the NBR 6118 are greater for small values of χ (ratio live loads/total loads) and equivalent to the ones of the ACI318-05 for higher values of χ . It is to be noticed that obtained reliability indexes are much smaller than the ones obtained for the beam, for both ratios of reinforcement, A_{s, max} and A_{s, min}. This can be explained by the fact that the flexural resistance of the slab is much more sensitive to the variations of the concrete cover than the beam.

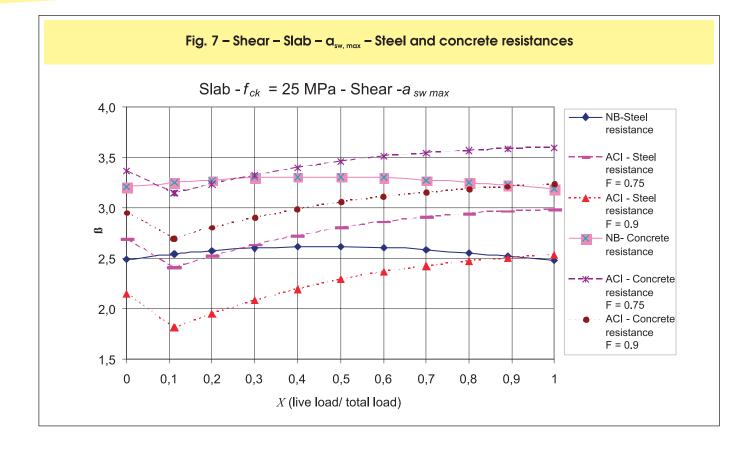
Figures 5 and 6 present the results corresponding to the shear analyses of the beam. The results of two verifications are shown: "reinforcement" and "concrete". The first one, the check of the reinforcement, is done according to Equation (11), for the three conditions a_{sw. max}, $a_{sw, ave}$ and $a_{sw, min}$. The second one is related to check of the maximum possible compression stresses in the concrete diagonal struts according to Equation (12). This check is done only for the critical situation, the shear force corresponding to the maximum shear reinforcement, $a_{_{\text{sw, max}}}$. The analyses are done for NBR 6118 and ACI 318-05, with $\phi = 0.75$ and $\phi = 0.9$. It can be seen that the obtained reliability indexes for the concrete are not the critical ones, regarding the corresponding for the reinforcement. For the three reinforcement conditions, the reliability indexes obtained with the NBR 6118 are equivalent to the ones of ACI 318-05 (with $\phi = 0.75$) for small values of χ (ratio live loads/total loads) and are smaller for greater values of χ . Reliability indexes obtained with ACI 318-05, $\phi =$ 0.9, are much smaller than the ones obtained with the two previously analyzed criteria.

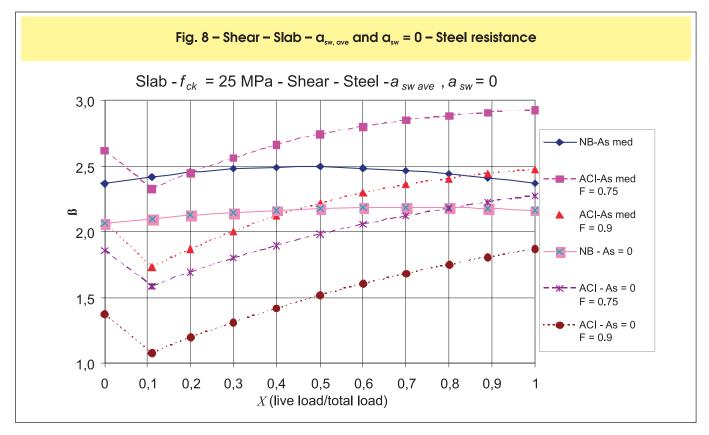












Figures 7 and 8 present the results corresponding to the shear analyses of the slab. The results of the "reinforcement" and "concrete" verifications are shown. The check of the reinforcement is done according to Equation (11), for the three conditions of reinforcement: $a_{sw, max'} a_{sw, ave}$ and a_{sw} = 0. The concrete check is done according to Equation (12), only for the critical situation, corresponding to $a_{_{\mbox{sw},\,\,\mbox{max}}}$. The analyses are done for NBR 6118 and ACI 318-05 (with $\phi =$ 0.75 and $\varphi=0.9$). It is to be first noticed that the obtained reliability indexes are always smaller than the ones obtained for the beam. The obtained reliability indexes for the concrete are again not the critical ones, regarding the corresponding for the reinforcement. For $a_{_{SW,max}}$ and $a_{_{SW,ave}}$, the reliability indexes obtained with the NBR 6118 are equivalent to the ones of ACI 318-05, with $\phi = 0.75$, for small values of χ (ratio live loads/ total loads) and are smaller for greater values of χ . For $a_{sw} = 0$, the results of NBR 6118 are the most conservative ones. Reliability indexes obtained with ACI 318-05, $\phi = 0.9$, are again much smaller than the ones obtained with the two other criteria.

Findings

Reliability indexes obtained for slabs are always smaller than the corresponding ones obtained for beams. Reliability indexes obtained with NBR 6118 are generally adequate compared with the corresponding ones of ACI 318-05, with $\varphi=0.75$; the exception to be noted is the shear design, for the greater values of the parameter χ (live loads/total loads). Considering the actual code limits for maximum shear reinforcement, the check of the compression stresses in the concrete diagonal struts proved to be not critical in the analyzed cases. The NBR 6118 criterion for slabs without stirrups is conservative, with respect to the ones of ACI 318-05, with $\varphi=0.75$. Reliability indexes obtained with ACI 318-05, $\varphi=0.9$, have shown to be excessively low, mostly for the case without shear reinforcement.

Conclusions

Based on the results presented in this paper, the following conclusions are drawn:

1. For flexural and shear design, in the analyzed cases, the criteria defined by the Brazilian Standard NBR 6118 lead generally to the same level of safety obtained using ACI 318-05 ($\phi = 0.75$).

2. The multiple criteria for the loading combinations defined in ACI 318-05, lead to a more uniform value for the reliability indexes, as a function of the parameter χ . Future revisions in Brazilian Standard could eventually consider this design approach.

3. In NBR 6118, the reliability indexes decrease with the increase of χ . A point that was not considered in this study is that, in some situations, where live loads are higher than 5kN/m², the load coefficients of this Standard are changed to $\gamma_f = 1.35$ and $\gamma_{fq} = 1.5$ (respectively, dead and live loads coefficients). This criterion can eventually mitigate the appointed tendency of the reliability indexes.

Anyway, this issue could be eventually taken in consideration in a future revision of NBR 6118.

4. Clearly, the adoption of the factor $\phi = 0.9$ for the shear design in ACI 318-05 leads to low reliability indexes, mostly for elements without shear reinforcement. In the point of view of the Reliability Analysis, this consideration would lead to a relatively unsafe design, and shall be carefully analyzed before a possible future adoption in ACI 318.

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