Volume 3, Number 2 (June 2007) p. 143-150 ISSN 1809-1121
Revista IBRACON de Estruturas
IBRACON Structural Journal

Global Safety Format for Design based on Nonlinear Analysis

Formato de Segurança Global Baseado em Análise Não-Linear

V. CERVENKA [1]
vladimir.cervenka@cervenka.cz

J. CERVENKA [1]
jan.cervenka@cervenka.cz

Z. JANDA [1]
zdenek.janda@cervenka.cz

Abstract

The safety format suitable for design of reinforced concrete structures using non-linear analysis requires a global approach. The performance of various safety formats is compared on four examples ranging from statically determinate structures with a bending mode of failure up to indeterminate structures with a shear failure.

Keywords: nonlinear analysis, reinforced concrete, finite element method, structural analysis, probabilistic analysis

Resumo

O formato de segurança adequado para o projeto de estruturas de concreto armado usando análise não linear requer um enfoque global. O desempenho de vários formatos de segurança é comparado em quatro exemplos variando de estruturas estaticamente determinadas com um modo de falha por flexão até estruturas indeterminadas com falha por cortante.

Keywords: análise não-linear, concreto armado, método dos elementos finitos, análise estrutural, análise probabilística

[1] *Cervenka Consulting, www.cervenka.cz, Predvoje 22, 162 00, Prague, Czech Republic*

1 Introduction

Non-linear analysis is becoming a frequent tool for design of new and assessment of existing structures. This development is supported by the rapid increase of computational power as well as by new capabilities of the available software tools for numerical simulation of structural performance.
On the other hand the code provisions provide very little guidance how to use the results of non-linear analysis for structural assessment or design. The safety formats and rules that are usually employed in the codes are tailored for classical assessment procedures based on beam models, hand calculation, linear analysis and local section checks. The non-linear analysis is by its nature always a global type of assessment, in which all-structural parts, or sections interact. Until recently the codes did not allow applying the method of partial safety factors for non-linear analysis, and therefore, a new safety format was expected to be formulated. Certain national or international codes have already introduced new safety formats based on overall/global safety factors to address this issue. Such codes are, for instance, German standard DIN 1045-1 [6] or Eurocode 2 EN 1992-2 [7]. This paper will try to compare several possible safety formats suitable for non-linear analysis: partial factor method, global format based on EN 1992-2, and fully probabilistic method. A new alternative safety format was proposed by CERVENKA [5], which is based on a probabilistic estimate of the coefficient of variation of resistance.
A standard assessment procedure based on partial safety factors usually involves the following steps:
(1) Linear elastic analysis of the structure considering all possible load combinations. Results are actions in some critical sections, which could be referred as design actions and can be written as:

$$E_d = \gamma_{S1} S_{n1} + \gamma_{S2} S_{n2} + \ldots\ldots \gamma_{Si} S_{ni} \quad (1)$$

They include safety provisions, in which the nominal loads S_{ni} are amplified by appropriate partial safety factors for loading γ_{Si}, where index i stands for load type, and their combinations.
(2) Design resistance of a section is calculated using design values of material parameters as:

$$R_d = r(f_d, \ldots),\ f_d = f_k / \gamma_m \quad (2)$$

The safety provision for resistance is used on the material level. The design value of material property $f_d = f_k / \gamma_m$ is obtained from the characteristic value f_k by its reduction with an appropriate partial safety factor γ_m. The random variability of material properties is covered by the partial safety factors individually for each material.
(3) Safety check of limit state is performed by design condition, which requires, that design resistance is greater then design action:

$$E_d < R_d \quad (3)$$

Note, that in the partial factor method the safety check is ensured in local material points. However, the probability of structural failure, i.e. the probability of violation of the design criteria is not known.
In the above outlined procedure, the non-linear analysis should be applied in step 1) to replace the linear analysis. Following the current practice an engineer will continue to steps 2), 3) and perform the section check using the internal forces calculated by the non-linear analysis. This is a questionable way since in non-linear analysis material criteria are always satisfied implicitly by the constitutive laws. Instead, a global check of safety should be performed on a higher level and not in local sections. This is the motivation for the introduction of new safety formats for non-linear analysis.
The final step of the verification process often involve assessment of serviceability conditions, i.e. deflections, crack width, fatigue, etc. In certain cases, these serviceability conditions might be the most important factors affecting the assessment conclusions.
Another important point is that a non-linear analysis becomes useful when it is difficult to clearly identify the sections to be checked. This occurs in structures with complicated geometrical forms, with openings, special reinforcement detailing, etc. In such cases, usual models for beams and columns are not appropriate, and non-linear analysis is a powerful alternative.
The above discussion shows that it would be advantageous to check a global structural resistance to prescribed actions rather than checking each individual section. This approach can bring the following advantages:
(a) The nonlinear analysis checks automatically all locations and not just those selected as critical sections.
(b) The global safety format gives information about the structural safety, redundancy and robustness. This information is not available in the classical approach of section verification.
(c) The safety assessment on global level can bring more economic solution by exploiting reserves due to more comprehensive design model and a risk of unsafe design is reduced.
However, the above enthusiastic statements should be accepted with caution. There are many aspects of the assessment process, which require engineering judgment. Also many empirical criteria must be met as required by codes. Therefore, the global safety assessment based on non-linear analysis should be considered as an additional advanced tool, which should be used, when standard simple models are not sufficient.

The non-linear analysis offers an additional insight into the structural behaviour, and allows engineers to better understand their structures. It is often referred as a virtual testing. On the other hand, non-linear analysis is usually more demanding then a linear one, therefore an engineer should be aware of its limits as well as benefits. Other disadvantage is that the force super-position is not valid anymore. The consequence is that a separate non-linear analysis is necessary for each combination of actions.
Finally, a note to terminology will be made. The term for global resistance (global safety) is used here for assessment of structural response on higher structural level then a cross section. In technical literature, the same meaning is sometimes denoted by the term overall. The term global is introduced in order to distinguish the newly introduced check of safety on global level, as compared to local safety check in the partial safety factor method. This terminology has its probabilistic consequences as will be shown further in the paper. The proposed global approach makes possible a reliability assessment of resistance, which is based on more rational probabilistic approach as compared to partial safety factors. The presented study is based on the paper by CERVENKA [5].

2 Safety formats for non-linear analysis

2.1 Design variable of resistance

Our aim is to extend the existing safety format of partial factors and make it compatible with nonlinear analysis. First we introduce a new design variable of resistance $R=r(f, a, \ldots, S)$. Resistance represents a limit state. In a simple case this can be a single variable, such as loading force, or intensity of a distributed load. In general this can represent a set of actions including their loading history. We want to evaluate the reliability of global resistance, which is effected by random variation of basic variables f - material parameters, a – dimensions, and possibly others.
Random variation of resistance is described by a statistical distribution which can be characterized by following parameters: R_m - mean value of resistance, R_k - characteristic value of resistance, (corresponding to the probability 5%), R_d - design value of resistance. For our further derivations it is important to realize, that the characteristic and design values reflect the random scatter of the resistance, which in probabilistic terms means that they reflect distribution function of resistance and its parameters, namely the standard deviation.
The resistance is determined for a certain loading pattern, which is here introduced by the symbol of actions S. It is understood that unlike material parameters and dimensions, which enter the limit state function r as basic variables, the loading is scalable, and includes load type, location, load combination and history. The objective of the resistance R is to determine the load magnitude for a given loading pattern of S.
In general, action E_d and resistance R_d, which appear in design equation (3), can include many components (for example vertical and horizontal forces, body forces, temperature, etc,) and can be described by a point in a multi-dimensional space. It is therefore useful to define a resistance scaling factor k_R, which describes safety factor with respect to the considered set of design actions. In the simplified form, considering one pair of corresponding components it can be described as:

$$k_R = \frac{R}{E_d} \qquad (4)$$

Then, the design condition can be rewritten as:

$$\gamma_R < k_R \qquad (5)$$

Where γ_R is required global safety factor for resistance. Factor k_R can be used to calculate the relative safety margin m_R for resistance:

$$m_R = k_R - 1 \qquad (6)$$

It remains to determine the design resistance R_d. The following methods will be investigated and compared:

(a) Proposed method ECOV, i.e. estimate of coefficient of variation for resistance.
(b) EN 1992-2 method, i.e estimate of R_d using the overall safety factor from Eurocode 2 EN 1992-2.
(c) PSF method, i.e. estimate of R_d using the partial factors of safety
(d) Full probabilistic approach. In this case R_d is calculated by a full probabilistic non-linear analysis.

Furthermore, the limit state function r can include some uncertainty in model formulation. However, this effect can be treated separately and shall not be included in the following considerations. It should be also made clear, that we have separated the uncertainties of loading and resistance (and their random behaviour). Our task is reduced to the calculation of design resistance R_d to be used in Eq. 3.

2.2 ECOV method – estimate of coefficient of variation

This method was inspired by the global safety analysis by HOLICKY [8]. It is based on the idea, that the random distribution of resistance, which is described by the coefficient of variation V_R, can be estimated from mean R_m and characteristic values R_k. The underlying assumption is that random distribution of resistance is according to lognormal distribution, which is typical for structural resistance. In this case, it is possible to express the coefficient of variation as:

$$V_R = \frac{1}{1.65} \ln\left(\frac{R_m}{R_k}\right) \quad (7)$$

Global safety factor γ_R of resistance is then estimated as:

$$\gamma_R = \exp(\alpha_R \beta V_R) \quad (8)$$

where α_R is the sensitivity (weight) factor for resistance reliability and β is the reliability index. The above procedure enables to estimate the safety of resistance in a rational way, based on the principles of reliability accepted by the codes. Appropriate code provisions can be used to identify these parameters. According to Eurocode 2 EN 1991-1, typical values are $\beta = 4.7$ (one year) and $\alpha_R = 0.8$. In this case, the global resistance factor is:

$$\gamma_R \cong \exp(-3.76) V_R \quad (9)$$

and the design resistance is calculated as:

$$R_d = R_m / \gamma_R \quad (10)$$

The key steps in the proposed method are to determine the mean and characteristic values R_m, R_k. It is proposed to estimate them using two separate nonlinear analyses using mean and characteristic values of input material parameters, respectively.

$$R_m = r(f_m, \ldots), \; R_k = r(f_k, \ldots) \quad (11)$$

It can be argued, why not to calculate R_d directly from Eq.(2) as we do in partial factor method. One of the reasons is the fact that design material values f_d are extremely low and do not represent a real material. A simulation of real behaviour should be based on mean material properties and safety provision should be referred to it. Analysis based on extremely low material properties may result in unrealistic redistribution of forces, which may not be on the conservative side. It may also change the failure mode. Therefore, the characteristic value f_k, which is not so far from a mean, but well reflects the scatter is preferred for analysis. Then a transformation within estimated distribution function is performed as described by Equations (7), (8), (9).

The method is general and reliability level β and distribution type can be changed if required. It reflects all types of failure. Its sensitivity to random variation of all material parameters is automatically included. Thus, there is no need of special modifications of concrete properties in order to compensate for greater random variation of certain properties. However, the method requires two separate non-linear analyses.

2.3 EN1992-2 method

Design resistance is calculated from

$$R_d = r(\tilde{f}_{ym}, \tilde{f}_{cm} \ldots, S) / \gamma_R \quad (12)$$

Material properties used for resistance function are considered by mean values. The mean steel yield strength $\tilde{f}_{ym} = 1.1 f_{yk}$. For concrete the mean strengths is reduced to account for greater random variation of concrete properties, $\tilde{f}_{cm} = 1.1 \frac{\gamma_s}{\gamma_c} f_{ck}$

where γ_s and γ_c are partial safety factors for steel and concrete, respectively. Typically this means that the concrete compressive strength should be calculated as $\tilde{f}_{cm} = 0.843 f_{ck}$. This method allows to treat the steel and concrete failure models in a unified way. The global factor of resistance shall be $\gamma_R = 1{,}27$. The evaluation of resistance function is done by nonlinear analysis assuming the material parameters according to the above rules.

2.4 PSF method – partial safety factor estimate

Design resistance R_d can be estimated using design material values as

$$R_d = r(f_d, \ldots, S) \quad (13)$$

In this case, the structural analysis is based on extremely low material parameters in all locations as was already mentioned in the end of Section 2.2. This may cause deviations in structural response, e.g. in failure mode. It may be used as an estimate in absence of a more refined solution.

2.5 Full probabilistic analysis

Probabilistic analysis is a general tool for safety assessment of reinforced concrete structures, and thus it can be applied also in case of non-linear analysis. A limit state function can be evaluated by means of numerical simulation. In this approach the resistance function r (r) is represented by non-linear structural analysis and loading function s(s) is represented by a action model. Safety can be evaluated with the help of reliability index β, or alternatively by failure probability P_f taking into account all uncertainties due to random variation of material properties, dimensions, loading, and other. More about the probabilistic analysis is presented in the paper by NOVAK et al [9] and here we shall only briefly outline this approach. The probabilistic analysis is more general, but can be used only for determination of design value of resistance function r (r) expressed as R_d. It involves random sampling, and includes the following steps:

(1) Numerical model based on non-linear finite element analysis. This model describes the resistance function r (r) and can perform deterministic analysis of resistance for a given set of input variables.

(2) Randomization of input variables (material properties, dimensions, boundary conditions, etc.). This can also include some effects of actions, which are not in the action function s (s) (for example pre-stressing, dead load etc.). Random properties are defined by random distribution type and its parameters (mean, standard deviation, etc.). They describe the uncertainties due to statistical variation of resistance properties.

(3) Probabilistic analysis of resistance and action. This can be performed by numerical method of Monte Carlo-type, such as LHS sampling method. In this an array of resistance values is generated, which represents a distribution function of global resistance by a set of points. Based on this the distribution function of resistance can be calculated including type, mean, standard deviation, etc. This fully described the random properties of the resistance, and it can be used as a rational basis fot the safety verification.

(4) Evaluation of safety using reliability index β or probability of failure.

Probabilistic analysis is so far an ultimate tool for safety assessment. It can reveal reserves, which can not be discovered by conventional methods. However, it is substantiated mainly in cases, where real random properties of material or other parameters can be exploited.

2.6 Nonlinear analysis

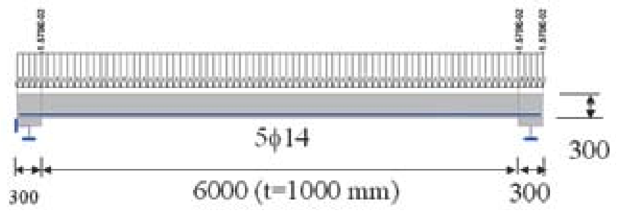
Examples in this paper are analysed with program ATENA for non-linear analysis of concrete structures. ATENA is capable of a realistic simulation of concrete behaviour in the entire loading range with ductile as well as brittle failure modes as shown in papers by CERVENKA [3], [4]. The numerical analysis is based on finite element method and non-linear material models for concrete, reinforcement and their interaction. Tensile behavior of concrete is described by smeared cracks, crack band and fracture energy, compressive behavior of concrete is described by damage model with hardening and softening. In the presented examples the reinforcement is modelled by truss elements embeded in two-dimensional isoparametric concrete elements. Nonlinear solution is performed incrementally with equlibrium iterations in each load step.

3 Examples of application

The performance of presented safety formats will be tested on several examples ranging from simple statically determinate structures with bending failure mode up to statically indeterminate structures with shear failure modes.

Example 1: Simply supported beam in bending.
Simply supported beam is uniformly loaded as shown in Fig.1. The beam has a span of 6m, rectangular cross/-section of h=0.3m, b=1m. It is reinforced with 5Ø14 along the bottom surface. The concrete type is C30/37 and reinforcement has a yield strength of 500 Mpa. The failure occurs due to bending with reinforcement yielding.

Figure 1 – Beam geometry with distributed design load for example 1



Example 2: Deep shear beam
Continuous deep beam with two spans is shown in Figures 2 and 3. It corresponds to one of the beams tested at Delft University of Technology by ASIN [1]. It is a statically indeterminate structure with a brittle shear failure.

Figure 2 – Deep beam in Example 2

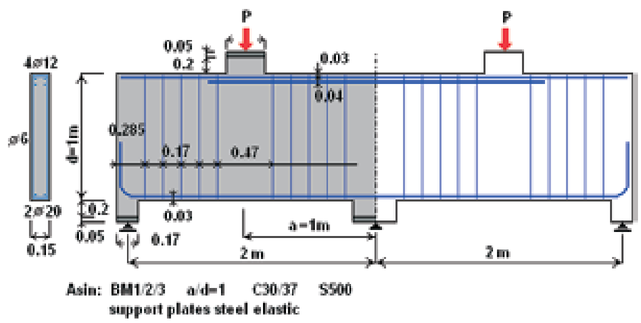


Figure 3 – Laboratory test of deep beam

Example 3: Bridge pier

This example is chosen in order to verify the behavior of the various safety formats in the case of a problem with second order effect (i.e. geometric nonlinearity). It is adopted from a practical bridge design in Italy that was published by Bertagnoli et. al. (2004). It is a bridge pier loaded by normal force and moment in the top, Figure 4.

Figure 4 – Bridge pier in Example 3

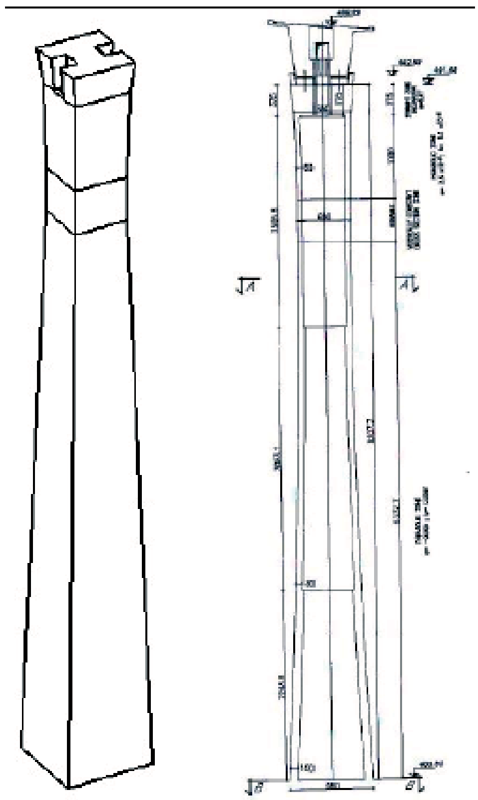
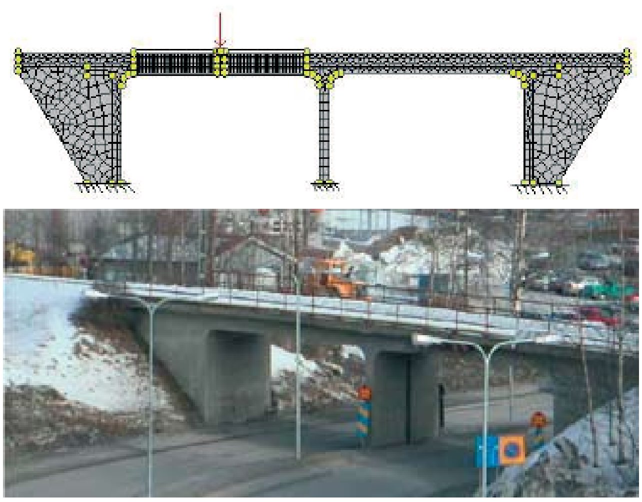


Figure 5 – Railway bridge in Example 4. Top – model, bottom real structure.



Example 4: railway bridge frame structure

The bridge frame structure in Sweden shown in Figure 5 fails by a combined action of bending and shear. It is an existing bridge that was subjected to a field test up to failure by a single load in the middle of the left span.

In the non-linear analysis, the load is gradually increased up to failure. Typical results are illustrated on the case of simple supported beam of Example 1. Figure 6 shows the beam response for increasing load using various safety methods described in Section 2. The straight dashed line denoted as ENV1992-1 represents the load-carrying capacity given by standard design formulas based on beam analysis by hand calculation and critical section check by partial factor method. The curve denoted as PSF, thus corresponds to the partial factor method from Section 2.4, in which the used material parameters are multiplied by the corresponding factors of safety. These two methods are based on the same safety format, PSF, and the differences are only due to different analysis models used: cross section analysis with zero tensile strength of concrete (hand calculation) and FE analysis utilizing real tensile strength. The other curves corresponds to the analyses with different material properties as specified by the safety format approaches that are presented in Section 2.

The response curve EN1992-2 is obtained from an analysis, where the material parameters are given by Section 2.3. For the ECOV method (Section 2.2), two separate analyses are needed: one using mean material properties, and one with characteristic values. The results from these two analyses are denoted by the labels "Mean" and "Char." respectively.

For each example, a full probabilistic analysis was also performed. Each probabilistic analysis consisted of several (at least 32 to 64) non-linear analyses with ran-

Figure 6 – Load-displacement diagrams for beam in Example 1

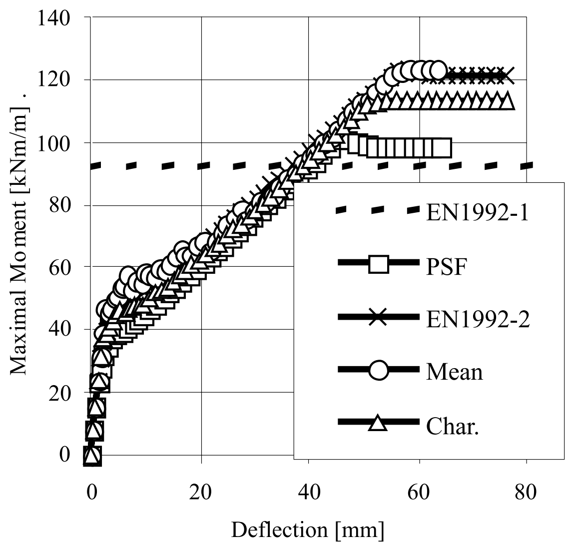


Table 1 – Comparison of various safety formats

	R_d/R_d^{PSF}			
	PSF	ECOV	EN 1992-2	Probabilistic
Example 1 Bending	1.0	1.0	0.95	0.96
Example 2 shear beam	1.0	1.02	0.98	0.98
Example 3 bridge pier	1.0	1.06	0.98	1.02
Example 4 bridge frame	1.0	0.97	0.93	1.01
average	1.0	1.01	0.96	0.99

domly chosen material properties. The design resistance is then obtained by a probabilistic analysis the calculated resistances.
Calculated design resistances for all examples and various methods are compared in Table 1. The design resistances are normalized with respect to the values obtained for PSF method to simplify the comparison. This means that the design method based on partial factors – PSF, which is the current design practice is taken as a reference.

4 Concluding remarks

The paper presents a comparison of several safety formats for the safety assessment based on non-linear analysis. A new method called ECOV (Estimate of Coefficient Of Variation) of ultimate state verification suitable for non-linear analysis of reinforced concrete structures is described. The proposed method can capture the resistance sensitivity to the random variation of input variables, and thus it can reflect the effect of failure mode on the safety. It requires two non-linear analyses with mean and characteristic values of input parameters, respectively. Other safety formats suitable for non-linear analysis that are based on global resistance are also presented. They are: the global approach proposed by EN 1992-2, fully probabilistic analysis and a simple approach based on design values of the input parameters, i.e. characteristic parameters reduced by the partial safety factors. The last approach is usually not recommended by design codes, but practicing engineers often overlook this fact, and use this approach if a non-linear analysis is available in their analysis tools. The consequences are investigated in this paper.
The discussed safety formats are tested on four examples. They include ductile as well as brittle modes of failure and second order effect (of large deformation). For the investigated range of problems, all the methods provide quite reliable and consistent results.

Based on the limited set of examples the following conclusions are drawn:
(a) The differences between all methods are not significant. None of the simple methods, PSF, EN 1992-2 and ECOV is superior to others.
(b) EN 1992-2 method using a fixed global factor $\gamma_R = 1{,}27$ gives more conservative results comparing to other methods.
(c) The proposed ECOV method gives results consistent with PSF and Probabilistic method.
(d) The PSF method, gives results consistent with Probabilistic analysis. It is a natural extension of the conventional PSF approach to the design based on non-linear analysis.

Fully probabilistic analysis is sensitive to the type of random distribution assumed for input variables. It offers a rational safety assessment in which real random properties of materials and other parameters can be utilized. The presented study was too limited to draw generally valid conclusions. However, it supports the authors experience that nonlinear analysis can be applied using any of the presented safety formats. The choice of the safety format depends on the specific situation (design of new structure, assessment of existing structure, knowledge of specific material data). The methods are currently subjected to further validation by authors for other types of structures and failure modes.
The research presented in this paper was in part resulting from the project supported by Grant no. 1ET409870411 of the Czech Academy of Sciences. The financial support is greatly appreciated.

5 References

[1] Asin, M. (1999) The Behaviour of Reinforced Concrete Continuous Deep Beams. Ph.D. Dissertation, Delft University Press, The Netherlands, 1999, ISBN 90-407-2012-6

[2] Bertagnoli, G., Giordano L., Mancini, G. (2004), Safety format for the nonlinear analysis of concrete structures. STUDIES AND RE SEARCHES –V.25, Politechnico di Milano, Italy.
[3] Cervenka V. (1998): Simulation of shear failure modes of R/C structures. In: Computational Modeling of Concrete Structures (Euro-C 98), eds. R. de Borst, N. Bicanic, H. Mang, G. Meschke, A.A.Balkema, Rotterdam, The Netherlands, 1998, 833-838.
[4] Cervenka V. (2002). Computer simulation of failure of concrete structures for practice. 1st fib Congress 2002 Concrete Structures in 21 Century, Osaka, Japan, Keynote lecture in Session 13, 289-304
[5] Cervenka J., Cervenka, V., Janda, Z., Safety Assessment in Fracture Analysis of Concrete structures, Proceedings of the 6th International Conference on Fracture Mechanics of Concrete Structures, FRAMCOS-6, Catania, Italy, June 2007, Taylor&Francis Group, ISBN 978-0-415-44616-7, pp.1043-1049.
[6] DIN 1045-1 (1998), Tragwerke aus Beton, Stahlbeton und Spannbeton, Teil 1: Bemessung und Konstruktion, German standard for concrete, reinforced concrete and pre-stressed concrete structures
[7] EN 1992-2, (2005), Eurocode 2 – Design of concrete structures – Concrete bridges – Design and detailing rules
[8] Holický, M.(2006) Global resistance factors for reinforced concrete members. ACTA POLYTECHNICA 2006, CTU in Prague.
[9] Novák, D., Vořechovský, M., Lehký, D. & Rusina, R., Pukl, R. & Červenka, V. Stochastic nonlinear fracture mechanics finite element analysis of concrete structures. Proceedings of 9th Int. conf. on Structural Safety and Reliability Icossar, Rome, Italy, 2005