

## Comparative Analysis of Design Code Criteria for Shear Strength of Reinforced Concrete Beams

## Análise Comparativa dos Critérios Normativos de Dimensionamento ao Cisalhamento em Vigas de Concreto Armado



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### Abstract

The NBR 6118:2003 introduces significant changes in the shear design criteria for reinforced concrete (RC) beams. Two design models are presented and in one of them allowance is given to the designer to vary the angle  $\theta$  of the concrete struts between  $30^\circ$  and  $45^\circ$ . The objective of this paper is to evaluate these shear design procedures in terms of safety, precision and economy with respect to test results of RC beams, with and without stirrups built with normal strength concrete ( $f_c \leq 50\text{MPa}$ ). EUROCODE 2:2003 and ACI 318:2005 shear design criteria are also analyzed. The comparative analysis strongly suggests a revision of NBR 6118 design equation for the concrete contribution in the shear strength of RC beams. For beams with web reinforcement and concrete with  $f_c \leq 50\text{MPa}$ , the best correlation with respect to test results was achieved with NBR 6118 design criteria (model II with  $\theta$  equals to  $30^\circ$ ).

**Keywords:** reinforced concrete beams, shear design criteria, codes.

### Resumo

A NBR 6118:2003 apresenta modificações significativas nos critérios de dimensionamento ao esforço cortante para vigas de concreto armado. Dois modelos de cálculo são propostos sendo que um deles permite a variação do ângulo  $\theta$  de inclinação das bielas de concreto entre  $30^\circ$  e  $45^\circ$ . Dentro deste cenário, o objetivo deste trabalho é analisar a segurança, precisão e economia destes critérios por meio da comparação com resultados experimentais de vigas, com e sem armadura transversal, e executadas com concreto convencional ( $f_c \leq 50\text{MPa}$ ). Os procedimentos de cálculo preconizados pelo EUROCODE 2:2004 e pelo ACI 318:2005 são também analisados. O estudo inclui ainda vigas fabricadas com concreto de alto desempenho ( $f_c > 50\text{MPa}$ ). Os resultados da análise indicam que a equação da NBR 6118 correspondente à parcela resistente atribuída aos mecanismos complementares ao de treliça necessita de ajuste. Nas vigas com armadura transversal e concreto com  $f_c \leq 50\text{MPa}$ , a melhor correlação obtida foi com o modelo II da NBR 6118 e ângulo  $\theta$  igual a  $30^\circ$ .

**Palavras-chave:** vigas de concreto armado, dimensionamento ao cisalhamento, normas.

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## 1 Introduction

With respect to its previous edition, NBR 6118:2003 [1] presents significant changes in the shear design criteria for reinforced concrete beams. Based on the generalized truss model developed by Morsch, two design procedures are prescribed which differ in terms of the concrete strength contribution  $\tau_c$  and on the value of the angle  $\theta$  corresponded to the concrete struts. In the model I, these values are fixed, while in the model II the portion  $\tau_c$  is a function of the predicted strength and the angle  $\theta$  can vary between  $30^\circ$  and  $45^\circ$ . In this scenario, the objective of this paper is to analyze with respect to safety, precision and economy these criteria through the comparison with experimental results of beams, with and without traverse reinforcement and built with conventional concrete ( $f_c \leq 50$  MPa). The design procedures of EUROCODE 2:2004 [2] and ACI 318:2005 [3] are also analyzed. The investigation also includes beams fabricated with concrete having compressive strength above 50 MPa since the Brazilian construction industry is already producing it and this range of strength is allowed by both EUROCODE 2 and ACI 318. For the comparative study a database was created [4] containing the results of beams tested in laboratories worldwide. Partial analyses, considering the beams, effective depth, the concrete compressive strength, and the amount of flexural and shear reinforcement are also included.

## 2 Methodology

### 2.1 Shear design code criteria for reinforced concrete beams

The shear design criteria for beams ( $b_w \leq 5d$ ) of the three codes under appraisal include two verifications: concrete web crushing and diagonal tension failure. The verification of the web crushing mode is made by comparing the conventional shear stress ( $V_u/b_w \cdot d$ ) to a prescribed value. The strength to diagonal tension failure is equal to the sum of the concrete contribution stress  $\tau_c$  plus  $\tau_{sw}$  corresponded to the shear reinforcement part. The concrete contribution includes the effects of aggregate interlocking, dowel action and the concrete strength above the diagonal cracks [5, 6].

NBR 6118:2003 [1] permits the angle  $\theta$  of the concrete struts to vary between  $30^\circ$  and  $45^\circ$ , while EUROCODE 2:2004 [2] allows the variation from  $21.8^\circ$  to  $45^\circ$ . The designer can choose freely the value of  $\theta$  in the prescribed intervals. In the ACI 318:2005 [3] criteria this angle is constant and equal to  $45^\circ$ . Table [1] presents the design equations corresponded to the two verifications for beams with vertical stirrups.

The analysis of table [1] reveals, for beams with shear reinforcement, the lack of consensus of these code procedures with respect to the diagonal tension failure. While NBR 6118 and ACI 318 consider both the concrete and the transverse reinforcement con-

**TABLE 1 – Shear design equations for reinforced concrete beams ( $b_w \leq 5d$ ) according to the codes under appraisal**

Code	Web Crushing Failure	Diagonal Tension Failure	
	$\tau_{wc}$ (MPa)	$\tau_c$ (MPa)	$\tau_{sw}$ (MPa)
NBR 6118 Model I	$0.27 \left(1 - \frac{f_c}{250}\right) f_c \text{sen}2\theta$	$\tau_c = \tau_{c0} = 0.126 (f_c)^{2/3}$	$0.9 \rho_w f_y \cot\theta$
NBR 6118 Model II		$\tau_c = \tau_{c0} \left( \frac{\tau_{Rd2} - \tau_{Sd}}{\tau_{Rd2} - \tau_{c0}} \right)$	$0.9 \rho_w f_y \cot\theta$
EUROCODE 2	$0.45 v_1 f_c \text{sen}2\theta^*$	$0^{**}$	$0.9 \rho_w f_y \cot\theta$
ACI 318	$0.83 \sqrt{f_c}$	$0.17 \sqrt{f_c}$	$\rho_w f_y$

\* –  $v_1 = 0.6$  for  $f_c \leq 60$  MPa and  $v_1 = \left(0.9 - \frac{f_c}{200}\right) > 0.5$  if  $f_c > 60$  MPa;

\*\* – EUROCODE 2:  $\tau_c = 0$  for beams with stirrups and  $\tau_c = 0.18 k (100 \rho_1 f_c)^{1/3}$  for beams without stirrups when  $k = 1 + \sqrt{\frac{20}{d}}$  and  $\rho_1 = A_{st} / (b_w \cdot d)$

**TABLE 2 – Web Crushing Failure Database – Beams with Stirrups and  $f_c \leq 50\text{MPa}$** 

References	number of beams	$b_w$ (cm)	$d$ (cm)	$a/d$	$f_c$ (MPa)	$\rho_l$ (%)	$\rho_w$ (%)	$\rho_w f_y$ (kN/cm <sup>2</sup> )
Placas and Regan (7)	6	6	25	3.4	13 a 46	1.3 a 4	2 a 3.3	1.3 a 2
Haddadin et al (8)	10	18	38	2 a 4.25	13 a 33	1.7	2 a 4.3	0.3 a 0.6
Leonhardt and Walther apud Ramirez and Breen (9)	5	10 a 152	30 a 38	3.3 a 3.5	17 a 21	---	3.3 a 3.5	0.2 a 0.6
Rangan (10)	4	6 a 7	56.3	2.5	30 a 37	8.4 a 10	2.49	0.7 a 1.6
Lee and Watanabe (11)	18	15 a 20	27 a 37	---	20 a 50	---	0.4 a 1.7	0.3 a 1.5
Total: 43 beams								

**TABLE 3 – Diagonal Tension Failure Database – Beams without Stirrups and  $f_c \leq 50\text{MPa}$** 

References	number of beams	$b_w$ (cm)	$d$ (cm)	$a/d$	$f_c$ (MPa)	$\rho_l$ %
Morrow and Viest (12)	12	31	36 a 38	2.8 a 8	15 a 46	1.2 a 3.8
Haddadin et al (8)	3	18	38	2.5 a 4.25	14 a 30	1.7
Placas and Regan (7)	7	15	25	3.4 a 3.6	24 a 30	1 a 4
Mphonde and Frantz (13)	4	15	30	3.6	21 a 42	2.3 a 3.4
Elzanaty et al (14)	5	18	27	4	21 a 40	1 a 2.5
Bazant and Kazemi (15)	18	8	4 a 17	3	43	1.64 a 1.67
Xie et al (16)	1	13	22	3	38	2.1
Adebar and Collins (17)	2	29	28	2.8	46 and 49	2
Yoon et al (18)	1	38	66	3.3	36	2.8
Kulkarni and Shah (19)	5	10	15	3.5 a 5	40 a 43	1.4
Collins and Kuchma (20)	11	30	11 a 92.5	2.5 a 23.9	36 a 50	0.8 a 1
Angelakos et al (21)	5	30	93	2.9	21 a 38	0.5 a 2
Cladera (22)	1	20	36	3	50	2.24
Garcia (23)	1	15	41	3.1	32	2.6
Kani et al apud Cladera (22)	32	15	14 a 109	2.5 a 7	17 a 35	0.5 a 2.8
Ahmad et al apud Cladera (22)	1	13	22	3	40	1
Islam Pam Kwan apud Cladera (22)	3	15	21	2.9 a 3.9	27 a 34	2 a 3.2
González apud Cladera (22)	4	20	31	3.3	40 a 47	2.9
Tompos and Frosh (24)	1	23	43	3	36	1
Total: 117 beams						

tributions, EUROCODE 2 takes into account only the shear reinforcement part. On the other hand, in the evaluation of the concrete contribution  $\tau_c$ , both Brazilian and American codes rely only on the concrete strength, ignoring the dowel action and size effects

considered by EUROCODE 2. It is worth mentioning that each code has a limited value of the concrete compressive strength: for NBR 6118 the upper value is 50 MPa, while ACI 318 and EUROCODE 2 allow values up to 69 and 90 MPa, respectively.

**TABLE 4 – Diagonal Tension Failure Database – Beams with Stirrups and  $f_c \leq 50\text{MPa}$** 

References	number of beams	$b_w$ (cm)	$d$ (cm)	$a/d$	$f_c$ (MPa)	$\rho_l$ %	$\rho_w$ %	$\rho_w f_y$ (kN/cm <sup>2</sup> )
Placas and Regan (7)	40	15	25	3.3 to 7	12 to 34	1 to 4.2	0.1 to 0.4	0.04 to 0.2
Haddadin et al (8)	12	18	38	2.5 to 6	14 to 45	0.8 to 2	0.2 to 0.4	0.07 to 0.2
Elzanaty et al (14)	2	18	25	4	21 to 40	2.5	0.2	0.07
Jonhson and Ramirez (25)	2	31	54	3	36	2.4	0.1	0.03 to 0.07
Berlabi and Hsu (26)	2	15	56	3.7	32 to 36	1.7	0.4	0.14 to 0.15
Sarsam and Al-Musawi (27)	3	18	24	2.5 to 4	39 to 40	2.2	0.1	0.08 to 0.11
Xie et al (16)	1	13	20	3	41	3.2	0.5	0.16
Furlan Jr (28)	1	4	27	4.2	49	0.6	0.2	0.18
Adebar and Collins (17)	6	29	28	2.9	49	2	0.1 to 0.4	0.06 to 0.2
Yoon et al (18)	3	38	66	3	36	2.8	0.1	0.03 to 0.05
Mphonde and Frantz apud Castro (29)	2	15	30	3.6	22 to 40	3.4	0.1	0.03
Collins and Kuchma (20)	1	30	93	3	47	0.8	0.1	0.04
Vidal Filho (30)	1	8	33	3.8	45	3.1	0.1	0.11
Lee and Watanabe (11)	20	18 to 20	36	---	28 to 37	---	0.2 to 1.1	0.06 to 0.4
Angelakos (21)	3	30	93	2.8	21 to 38	0.5 to 1	0.1	0.04
Cladera (22)	3	20	35	3	50	2.3	0.1 to 0.2	0.06 to 0.13
Garcia (23)	5	15	41 to 42	3.1	32 to 43	1 to 2	0.1	0.04 to 0.07
González apud Cladera (22)	12	20	30 to 31	3.25	37 to 45	2.9	---	0.06 to 0.11
Tompos and Frosch (24)	6	23 s 46	43 to 85	3	36 to 43	1.0	0.2 to 0.3	0.08 to 0.15
Etxeberria (31)	3	20	30	3.3	42	3	0.1 to 0.2	0.06 to 0.12

Total: 128 beams

## 2.2 Reinforced concrete beams database

The database for this analysis was originally assembled by RIBEIRO [4] as part of her master's dissertation. It consists of 526 beams tested in laboratories worldwide (references [7] to [41]) with reported shear failures. All beams have longitudinal reinforcement,  $b_w/d$  ratio smaller or equal to 5 and no web openings. There are 308 beams with web reinforcement consisting of vertical stirrups. Beams fabricated with concrete containing light aggregates or fibers are not included. The beams were tested under single or two point transverse loads as well as uniformly distributed loading. Axial loads were not present. The shear span-to-depth ratio ( $a/d$ ) was always greater than 2.

The most important characteristics of the beams in the databank are shown in tables [2] to [6]. Each table corresponds to a shear failure mode, the presence or not of web reinforcement and a range of concrete compressive strength. Each table also presents the number of beams tested by each research team,

their respective geometrical characteristics, the longitudinal and web reinforcement ratios as well as the material mechanical properties. The indicated concrete compressive strength corresponds to 15 x 30 cm cylinder specimens.

## 2.3 Comparative study methodology

The comparative study between the actual and predicted shear capacities can be quantified based on the ratio of the test shear strength  $\tau_{test}$  to the predicted one. The test shear failure stress  $\tau_{test}$  provides close estimate of the true capacity; it is the ratio between the shear failure force divided by the effective web cross-sectional area ( $b_w$  times  $d$ ). For the evaluation of the predicted strength  $\tau_{pred}$ , all material resistance factors were set equal to one. Further, the measured concrete compressive strength  $f_c$  and steel yield stress  $f_y$  of each test specimen were used in determining  $\tau_{pred}$ . For each beam the ratio  $\tau_{test}/\tau_{pred}$  was calculated. Statistical analyses of this ratio include its average  $M$ , the

median  $M_d$ , the standard deviation  $SD$ , the coefficient of variation  $CV$  as well as the maximum and minimum values. The average of  $\tau_{test}/\tau_{pred}$  is used as a measure of the conservative bias of the procedure while the coefficient of variation is taken as an indication of accuracy.

With the objective of evaluating the reliability and of comparing the performance of shear design code equations for reinforced concrete beams, COLLINS [42] developed a demerit point scale methodology. Considering safety, precision and economy, a score is attributed for each range of  $\tau_{test}/\tau_{pred}$  ratio: these scores are shown in table [7]. In this methodology a score value smaller than 0,5 is worse in terms of safety than one greater than 2. On the other hand, a score value equal 2, corresponded to the low safety zone, is attributed to an extremely conservative ratio for being not economical. The total demerit point score of each design code equation is calculated by summing the products of the percentage of  $\tau_{test}/\tau_{pred}$  obtained in each range times the demerit points attributed to that range. The largest the total demerit point score is the worst is the performance of the design equation.

In order to verify its future use in NBR 6118 design procedures, a angle  $\theta$  of the concrete struts equal to  $21,8^\circ$  was included in the comparative study.

### 3 Test Results and Analysis

#### 3.1 Beams built with concrete of $f_c \leq 50$ MPa

##### 3.1.1 Beams with web crushing failure

It is important to point out that, in the literature review, web crushing failures were only found in beams with high ratios of web reinforcement as shown in table [2].

The results of comparative study for this failure mode are presented in table [8]. The NBR 6118:2003 criterion becomes more conservative as the angle  $\theta$  decreases. In terms of accuracy, the equation is equally precise independently of the value of  $\theta$ , since the coefficients of variation are practically the same. With respect to safety, the results (part B of table [8]) are of concern for  $\theta$  equals to  $45^\circ$ : 26% of the  $\tau_{test}/\tau_{pred}$  values are in the low safety range (between 0.65 and 0.85) and 2% in the dangerous zone (between 0.5 and 0.65). Overall, the analysis shows that NBR 6118 model II procedure with  $\theta = 30^\circ$  provides better predicting results in terms of safety, precision and economy for web crushing failures. The possible reduction of  $\theta$  to  $21,8^\circ$  needs more investigation: no  $\tau_{test}/\tau_{pred}$

**TABLE 5 – Diagonal Tension Failure Database - Beams without Stirrups and  $f_c > 50$ MPa**

References	number of beams	$b_w$ (cm)	$d$ (cm)	$a/d$	$f_c$ (MPa)	$\rho_l$ %
Mphonde and Frantz (13)	6	15	30	2.5 to 3.6	75 to 94	3.36
Ahmad et al (32)	18	13	18 to 21	2.7 to 4	61 to 67	1.8 to 6.6
Elzanaty et al (14)	6	18	27	4 to 6	63 to 79	1.2 to 3.3
Jonhson and Ramirez (25)	1	30,48	53,87	3,1	55,85	2,41
Salandra and Ahmad (33)	4	10	17	2,6 to 3,6	54 to 69	1,5
Xie et al (16)	1	13	22	3	99	2,1
Ahmad et al (34)	2	10	18	3,7	73 to 75	1,4
Adebar and Collins (17)	5	29 to 36	18 to 38	2,1 to 4,5	52 to 59	1 to 3
Yoon et al (18)	2	38	66	3,28	67 to 87	2,8
Collins and Kuchma (20)	10	17 to 30	23 to 93	2,5 to 11,3	53 to 99	0,5 to 1
Simplício apud Garcia (23)	2	15	27	3,8	70 to 73	2,33
Cladera (22)	3	20	36	2,2 to 3	61 to 87	2,24
Ahmad et al apud Cladera (22)	1	13	22	3	89	2,1
Islam et al. apud Cladera (22)	7	15	20 to 21	3 to 3,9	50 to 83	2 to 3,2
Kim Park apud Cladera (22)	16	17 to 30	14 to 92	3 to 4,5	54	1 to 4,7
Thorenfeldt and Drangsholt (35)	16	15 to 30	21 to 44	3 to 4	54 to 98	1,8 to 3,23
Garcia (23)	1	15	40	3	70	2,6

Total: 101 beams

**TABLE 6 – Diagonal Tension Failure Database – Beams with Stirrups and  $f_c > 50\text{MPa}$** 

References	number of beams	$b_w$ (cm)	$d$ (cm)	$a/d$	$f_c$ (MPa)	$\rho_l$ %	$\rho_w$ %	$\rho_w f_y$ ( $\text{kN/cm}^2$ )
Placas and Regan (7)	2	15	25	3.4 to 3.6	54 to 57	1.5 to 4.1	0.14 to 0.28	0.06 to 0.11
Jonhson and Ramirez (25)	5	31	54	3.1	51 to 72	2.4	0.07 to 0.14	0.03 to 0.07
Roller and Russell (36)	6	36 to 46	56 to 76	2.5 to 3	72 to 125	1.6 to 2.75	0.08 to 0.23	0.03 to 0.1
Sarsam and Al-Musawi (27)	11	18	23 to 24	2.5 to 4	70 to 80	2.2 to 3.5	0.1 to 0.2	0.08 to 0.15
Fernandes (37)	4	5	28	3.6 to 5.4	58 to 68	4.4 to 6.6	0.25 to 0.38	0.21 to 0.32
Xie et al (16)	4	13	20	3 to 4	90 to 103	4.53	0.49 to 0.76	0.16 to 0.25
Gomiero (38)	2	5	28	3.6	76 to 77	6.7	0.2 to 0.4	0.23 to 0.3
Yoon et al (18)	6	38	66	3.3	67 to 87	2.8	0.08 to 0.24	0.04 to 0.10
Mphonde and Frantz apud Castro (29)	2	15	30	3.6	60 to 83	3.4	0.11	0.03
Kong and Rangan (39)	36	25	20 to 30	2.5 to 3.3	60 to 85	1.7 to 4.5	0.10 to 0.26	0.06 to 0.13
Angelakos (21)	2	30	93	2.9	65 to 80	1	0.08	0.040
Collins and Kuchuma (20)	2	17 to 30	46 to 92	2.5 to 2.7	71 to 74	1 to 3.6	0.1 to 0.15	0.07 to 0.08
Ozcebe et al (40)	13	15	31 to 33	3 to 5	58 to 82	1.9 to 4.4	0.14 to 0.28	0.04 to 0.07
Vidal Filho (30)	2	8	33	3.7	51	3.1	0.22	0.15
Lee and Watanabe (11)	15	15 to 20	27 to 42	---	56 to 112	---	0.14 to 0.89	0.08 to 0.78
Cladera (22)	9	20	35	3.1	61 to 87	2.3 to 3	0.14 to 0.24	0.08 to 0.13
Ahmad et al apud Cladera (22)	3	13	20	3	83 to 88	4.5	--	0.21 to 0.33
Garcia (23)	5	15	40 to 41	3	67 to 71	2.6	0.06 to 0.22	0.05 to 0.12
Simplício apud Garcia (23)	5	15	27 to 35	3.3 to 3.8	66 to 70	2.3 to 3	0.11 to 0.21	0.08 to 0.15
Teoh et al (41)	3	15	66	2.7	89 to 100	3	0.12 to 0.18	0.04 to 0.06

Total: 137 beams

values were found below the appropriate safety range; on the other hand, the procedure becomes less economical with 12% of  $\tau_{test}/\tau_{pred}$  values above 2.

EUROCODE 2 criterion is also more conservative and safer as the angle  $\theta$  decreases, but equally precise (very similar values for the coefficient of variation). ACI 318 procedure is even more conservative and safer: it presents the largest average among all codes under appraisal with all  $\tau_{test}/\tau_{pred}$  values above the appropriate safety range. In terms of economy, ACI has the worst performance with 74% of the values in the conservative and extremely conservative regions.

For web crushing failure, EUROCODE 2 (with  $\theta = 21.8^\circ$ ) and NBR 6118 (with  $\theta = 30^\circ$ ) criteria provide the best predicting results in terms of safety, precision and economy.

The total demerit point score also reveals this fact: 48 points for EUROCODE 2 and 53 points for NBR 6118. On the other hand, ACI 318 has a total score of 90 points. The large percentage (74%) of  $\tau_{test}/\tau_{pred}$  values above the appropriate safety region is the reason for this result.

### 3.1.2 Beams with diagonal tension failure

The strength corresponded to diagonal tension failure is equal to the sum of the concrete contribution plus the web reinforcement part. In spite of the fact that the presence of web reinforcement boosts the concrete contribution [5, 6] in resisting shear, an accurate quantification of this increase in the concrete contribution is difficult to obtain.

**TABLE 7 – Demerit Point Classification of Collins (42)**

Classification	$\tau_{test} / \tau_{pred}$	Score
Extremely dangerous	< 0.50	10
Dangerous	0.50  ---- 0.65	5
Low Safety	0.65  ---- 0.85	2
<b>Appropriate Safety</b>	<b>0.85  ---- 1.30</b>	<b>0</b>
Conservative	1.30  ---- 2.00	1
Extremely Conservative	$\geq 2.00$	2

For this reason, ACI-ASCE 426 committee [5] and ACI 318 [3] assume explicitly that the value of the concrete contribution to the shear strength is the same for beams with or without web reinforcement. Based on this premise, the comparative study with respect to diagonal tension failure is divided into beams without web reinforcement and beams with vertical stirrups. This way an evaluation of the code design equations for the concrete contribution ( $\tau_c$ ) can be accomplished.

### 3.1.2.1 Beams without web reinforcement

In beams without web reinforcement, the shear strength is reduced to the concrete contribution part  $\tau_c$ . Consequently, a single value for the shear strength is predicted according to NBR 6118 procedure. This value is independent of the concrete strut angle  $\theta$ . EUROCODE 2 has a distinct formulation for beams without web reinforcement (table [1]) which is also independent of the angle  $\theta$ . Therefore a single value for the shear strength was also obtained according to EUROCODE 2.

The results of the  $\tau_{test} / \tau_{pred}$  ratio are shown in table [9]. They were obtained from the comparison with test results of 117 beams without stirrups. The analysis of these results reveals that ACI 318 design procedure is the most conservative one, with the largest average value for  $\tau_{test} / \tau_{pred}$  ratio. NBR 6118 and EUROCODE 2 criteria have similar average values; but EUROCODE 2 is more precise since it presents a significantly smaller coefficient of variation (14,1% versus 29,7%). The demerit scale (part B of table [9]) confirms these results: the percentage values, obtained with the Brazilian code procedure, below the appropriate safety range are much larger in relation to the others codes. This fact is also reflected in the total demerit point score of NBR 6118 equation, which is much larger. This finding reveals that the equation prescribed by NBR 6118 for the concrete contribution to shear strength is not satisfactory in terms of safety. Partial analyses, shown in table [10] and in figure [1], indicate a poorer performance in terms of safety of NBR 6118 equation for beams with effective depth larger

than 60 cm, with longitudinal reinforcement ratio smaller than 2% and with concrete compressive strength above 30 MPa. These results show the influences of the size effect and longitudinal reinforcement ratio which are not considered in NBR 6118 design criteria. The inclusion of these effects in the Brazilian code equation is recommended.

A careful analysis is necessary when comparing EUROCODE 2 and ACI 318 results. While ACI has 14% of  $\tau_{test} / \tau_{pred}$  values in the dangerous and low safety ranges, EUROCODE 2, in spite of not having any result in the dangerous zone, has a large percentile (23%) of results in low safety range. The sum of the dangerous and low safety region demerit scale score of EUROCODE 2 sums up to 46 against 37 of ACI 318. On the other hand, 48% of the ACI results are above the appropriate safety zone against only 1% for EUROCODE 2, which leads to a much smaller partial demerit scale score for EUROCODE 2: 1 versus 51 for ACI 318.

The overall analysis of the results indicates EUROCODE 2 criteria as the best ranked; it is also the only one that explicitly considers the effects of size and longitudinal reinforcement ratio in the design equation. NBR 6118 formulation, on the other hand, is not suitable; it has the worst performance among the appraised codes.

### 3.1.2.2 Beams with vertical stirrups

The comparative results are presented in table [11]. More conservative values of  $\tau_{test} / \tau_{pred}$  ratio are obtained with NBR 6118 design criteria as the angle  $\theta$  increases. In terms of precision, the Brazilian code equations are equally accurate (very similar values for the coefficient of variation) independent of the model or  $\theta$  value used. With respect to safety, 88% of the  $\tau_{test} / \tau_{pred}$  results are larger than 0.85 when model I or II with  $\theta = 45^\circ$  are employed. A similar percentile (86%) is also obtained with model II and  $\theta = 30^\circ$ . But with  $\theta = 30^\circ$ , a larger percentage of  $\tau_{test} / \tau_{pred}$  values was found in dangerous (6%) and extremely dangerous (2%) range against 7% in the dangerous zone for  $\theta$  equal to  $45^\circ$ . The use of model II with  $\theta = 45^\circ$  has proven to be not economical: 66% of the values of  $\tau_{test} / \tau_{pred}$  ratio are above 1.3. The overall analysis of these results indicates NBR 6118 model II procedure with  $\theta = 30^\circ$  as the most appropriate. This fact is also shown in the total demerit point score, proposed by COLLINS [42]. Based on the obtained results, a possible reduction of the angle  $\theta$  to  $21.8^\circ$  into current NBR 6118 design criteria is not justified.

EUROCODE 2 and ACI 318 results are also shown in table [11]. The analysis indicates a similar behavior between ACI 318 and NBR 6118 criteria with  $\theta = 45^\circ$ ; which take into account the concrete contribution part  $\tau_c$ . EUROCODE 2, on the other hand, provides extremely conservative results, since it does not consider the concrete contribution part in the design equation. The EUROCODE 2 conservativeness becomes even more evident when comparison is done with respect to NBR 6118 results for  $\theta$  equals to  $30^\circ$  or  $45^\circ$ .

Partial analysis results (table [12]) reveal significant increases in the percentage values of  $\tau_{test} / \tau_{pred}$  below the appropriate safety range when NBR 6118 criteria are used in beams with effective depth  $d > 60$  cm or longitudinal

reinforcement ratio  $\leq 2\%$ . These results show the value of the concrete contribution part  $\tau_c$  also affects the performance of NBR 6118 criteria in beams with web reinforcement. Similar response, but in a smaller scale, was also observed when the EUROCODE 2 and ACI 318 criteria are analyzed.

The relationship between web reinforcement ratio  $\rho_w$  and concrete compressive strength is also observed in partial analysis results (parts C and D of the table [12]), when NBR 6118 design criteria are used. For beams fabricated with concrete having compressive strength above 30 MPa, larger percentages of  $\tau_{test} / \tau_{pred}$  results were found in the ranges below the appropriate safety zone. Therefore more

web reinforcement is needed in this case. This fact suggests a possible modification in the minimum web reinforcement ratio prescribed by NBR 6118 for beams built with concrete having compressive strength above 30 MPa.

In spite of being less conservative, NBR 6118 model II procedure with  $\theta = 30^\circ$  is the most appropriate criterion, followed by ACI 318 and EUROCODE 2 with  $\theta = 21.8^\circ$ . This finding is also illustrated in figure [2], where the NBR criterion has the largest percentage of results in appropriate safety range (between 0.85 and 1.3). The demerit point scale evaluation (table [11]) also confirms this result since it attributes the smallest value of total demerit point score to this criterion.

**TABLE 8 – Results of Web Crushing Failure – Beams with Stirrups and  $f_c \leq 50\text{MPa}$**

Part A – Statistical Analysis				
Statistical Parameters		Average	Median	Coefficient of Variation
NBR 6118	Model I	1.02	0.93	24.36 %
	$\theta=45^\circ$	1.02	0.93	24.36 %
	$\theta=30^\circ$	1.17	1.07	24.62 %
	$\theta=21.8^\circ$	1.47	1.34	24.62 %
EUROCODE 2	$\theta=45^\circ$	0.90	0.83	25.62 %
	$\theta=30^\circ$	1.04	0.96	25.35 %
	$\theta=21.8^\circ$	1.30	1.20	25.62 %
ACI 318		1.52	1.41	25.64 %

Part B – Demerit Point Classification								
$\tau_{test} / \tau_{pred}$	NBR 6118				EUROCODE 2			ACI 318
	Model I	$\theta=45^\circ$	$\theta=30^\circ$	$\theta=21.8^\circ$	$\theta=45^\circ$	$\theta=30^\circ$	$\theta=21.8^\circ$	
< 0,50	0	0	0	0	0	0	0	0
0,50   -- 0,65	2*	2	0	0	9	0	0	0
0,65   -- 0,85	26	26	12	0	42	28	2	0
0,85   -- 1,30	58	58	59	39	47	56	56	26
1,30   -- 2,00	14	14	29	49	2	16	40	58
$\geq 2,00$	0	0	0	12	0	0	2	16
<b>Total Demerit Point Score</b>	<b>76**</b>	<b>76</b>	<b>53</b>	<b>73</b>	<b>131</b>	<b>72</b>	<b>48</b>	<b>90</b>

\* – Percentage values of  $\tau_{test} / \tau_{pred}$  results  
 \*\* –  $76 = (0 \times 10) + (2 \times 5) + (26 \times 2) + (58 \times 0) + (14 \times 1) + (0 \times 2)$



TABLE 9 – Results of Diagonal Tension Failure – Beams without Stirrups and  $f_c \leq 50\text{MPa}$ 

Part A – Statistical Analysis			
Statistical Parameters	NBR 6118	EUROCODE 2	ACI 318
Average	0.95	0.94	1.28
Median	0.95	0.92	1.28
Coefficient of Variation	29.71 %	14.10 %	28.79 %
Part B – Demerit Point Classification			
$\tau_{\text{test}} / \tau_{\text{pred}}$	NBR 6118 %	EUROCODE 2 %	ACI 318 %
< 0.50	4	0	0
0.50   –0.65	10	0	3
0.65   –0.85	21	23	11
0.85   –1.30	55	76	38
1.30   –2	7	1	45
$\geq 2.00$	3	0	3
<b>Total Demerit Point Score</b>	<b>145*</b>	<b>47</b>	<b>88</b>
* $145 = (4 \times 10) + (10 \times 5) + (21 \times 2) + (55 \times 0) + (7 \times 1) + (3 \times 2)$			

### 3.2 Beams built with concrete of $f_c > 50\text{MPa}$

Since EUROCODE 2 and ACI 318 procedures allow the use of concrete with compressive strength above 50 MPa and the Brazilian construction industry is already capable of producing it, a comparative analysis of shear design criteria for beams built with concrete compressive strength above 50 MPa is presented. For this study, the NBR 6118 design criteria were extrapolated. The beam test results found in the literature review showed a single failure mode in shear: diagonal tension. Consequently only this failure mode is analyzed next.

#### 3.2.1 Beams with diagonal tension failure

##### 3.2.1.1 Beams without web reinforcement

Table [13] presents the results of the comparative study corresponded to 101 beams with no web reinforcement tested in laboratories worldwide. The analysis of the  $\tau_{\text{test}} / \tau_{\text{pred}}$  ratios reveals that ACI criteria provides more

conservative results with the largest average and the smallest percentage of values (10%) below the appropriate safety zone. EUROCODE 2 procedure, on the other hand, has the smallest dispersion of the results around the average and therefore is the most precise. The results obtained with the extrapolation of NBR 6118 design criteria are of concern: it produces the smallest average value (0.86) and the largest percentage of results (> 50%) below the appropriate safety range. This is also reflected in the total demerit point score of NBR 6118 equation: 229 versus 84 and 121 for EUROCODE 2 and ACI 318, respectively. These findings reaffirm that the equation prescribed by NBR 6118 for the concrete contribution to shear strength is not satisfactory in terms of safety also for beams built with concrete having compressive strength  $f_c > 50\text{MPa}$ .

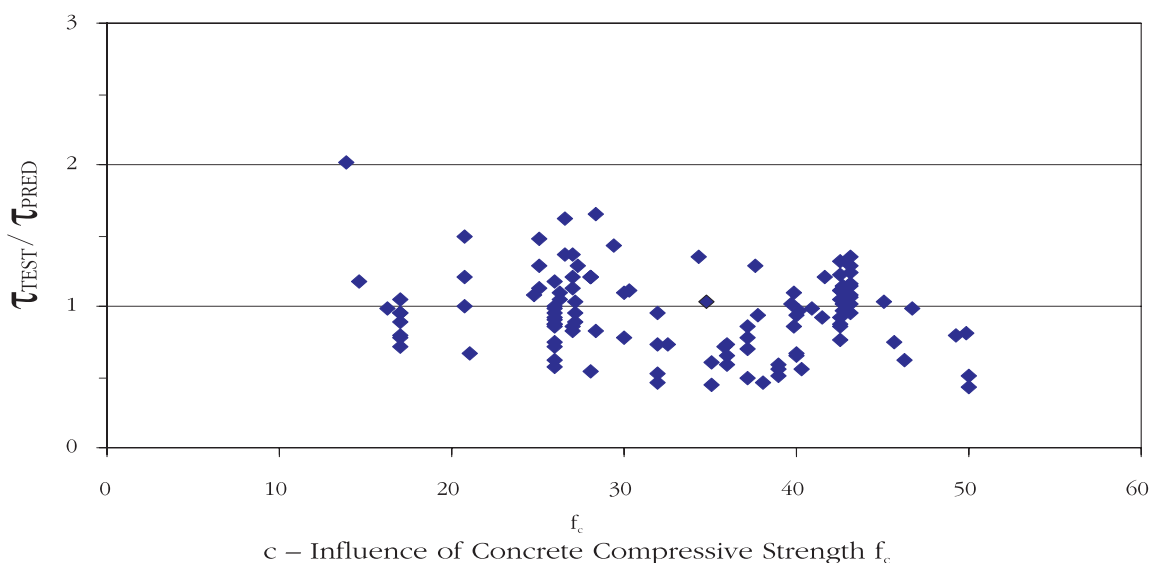
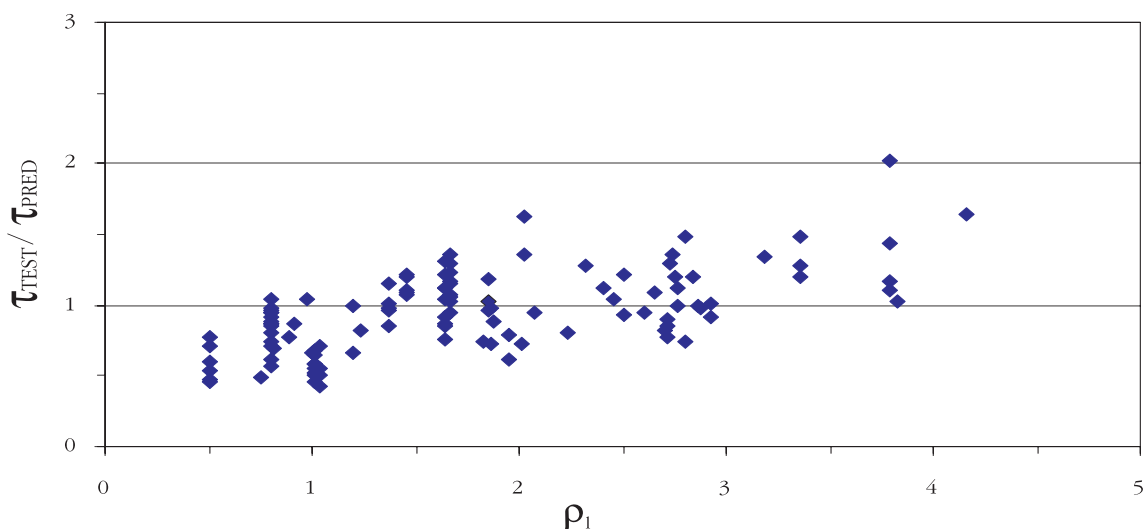
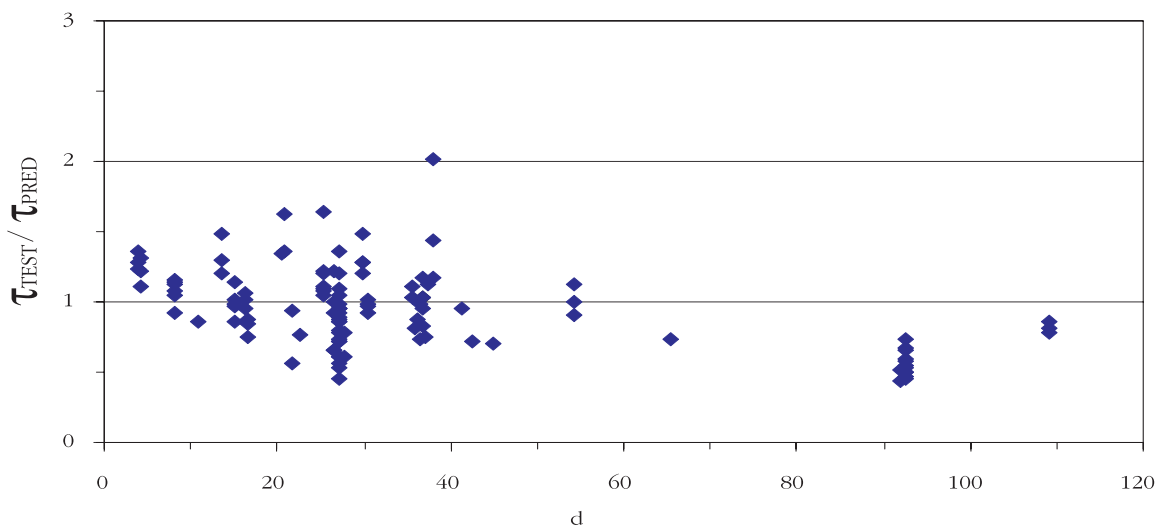
##### 3.2.1.2 Beams with vertical stirrups

The results for diagonal tension failures in beams with vertical stirrups are shown in table [14]. With respect to the results obtained for beams with  $f_c \leq 50\text{MPa}$  (table [11]), the extrapolation of the NBR 6118 criteria provides less

TABLE 10 – Partial Analysis Results of Diagonal Tension Failure Beams without Stirrups and  $f_c \leq 50\text{MPa}$ 

Part A – Effective Depth Influence–Demerit Point Classification								
Code	d (cm)	< 0.50	0.50 –0.65	0.65 –0.85	0.85 –1.30	1.30 –2.00	≥ 2.00	Total Demerit Point Score
NBR 6118	≤ 60	1*	6	17	65	10	1	86**
	> 60	24	35	35	6	0	0	485
EUROCODE 2	≤ 60	0	0	21	78	1	0	43
	> 60	0	0	35	65	0	0	70
ACI 318	≤ 60	0	1	5	39	52	3	73
	> 60	0	18	47	35	0	0	184
* – Percentage values of $\tau_{\text{test}} / \tau_{\text{pred}}$ results								
** 86 = (1 x 10) + (6 x 5) + (17 x 2) + (65 x 0) + (10 x 1) + (1 x 2)								
Part B – Longitudinal Reinforcement Ratio Influence – Demerit Point Classification								
Code	$\rho_l$ (%)	< 0.50	0.50 –0.65	0.65 –0.85	0.85 –1.30	1.30 –2.00	≥ 2.00	Total Demerit Point Score
NBR 6118	≤ 1	11	15	30	44	0	0	245
	$1 < \rho_l \leq 2$	4	15	19	58	4	0	157
	> 2	0	0	13	63	21	3	53
EUROCODE 2	≤ 1	0	0	22	78	0	0	44
	$1 < \rho_l \leq 2$	0	0	33	67	0	0	66
	> 2	0	0	11	87	2	0	24
ACI 318	≤ 1	0	7	19	70	4	0	77
	$1 < \rho_l \leq 2$	0	4	15	31	50	0	100
	> 2	0	0	0	26	66	8	82
Part C – Concrete Compressive Strength Influence – Demerit Point Classification								
Code	$f_c$ (MPa)	< 0.50	0.50 –0.65	0.65 –0.85	0.85 –1.30	1.30 –2.00	≥ 2.00	Total Demerit Point Score
NBR 6118	≤ 30	0	6	17	62	13	2	81
	> 30	8	14	22	51	5	0	199
EUROCODE 2	≤ 30	0	0	15	83	2	0	32
	> 30	0	0	30	70	0	0	60
ACI 318	≤ 30	0	0	8	45	41	6	69
	> 30	0	6	14	33	47	0	105

**FIGURE 1 – Diagonal Tension Failure Beams without Stirrups ( $f_c \leq 50$  MPa)  
NBR 6118:2003 Partial Analysis**



conservative and accurate values. Independent of the employed Brazilian design criterion, a larger percentage of results exists in the zones below the appropriate safety range. A reduction in the angle  $\theta$  to  $21.8^\circ$  produces even larger percentage of results in the low safety, dangerous and extremely dangerous ranges. This strongly suggests that this reduction is not appropriate. Overall, NBR 6118 model II procedure with  $\theta = 45^\circ$  is more suitable in terms of safety and economics; it also has the smallest value in the total demerit point score.

EUROCODE 2 provides results which are the most conservative, imprecise and anti-economical in relation to the

other codes under appraisal. This finding is due to the fact that EUROCODE 2 does not take into account the concrete contribution part in the shear strength of beams with web reinforcement. Similar performance was found between ACI 318 and NBR 6118 with  $\theta = 45^\circ$  with a slight conservativeness in the American code procedure.

The results of partial analyses (table [15]) reveal, also in this case, significant increases in the percentage values of  $\tau_{test}/\tau_{pred}$  below the appropriate safety range when NBR 6118 criteria are used in beams with effective depth  $d > 60$  cm or longitudinal reinforcement ratio  $\leq 2\%$ . These effects influence also ACI 318 performance.

**TABLE 11 – Results of Diagonal Tension Failure - Beams with Stirrups and  $f_c \leq 50\text{MPa}$**

Part A – Statistical Analysis								
Statistical Parameters		Average	Median	Coefficient of Variation				
NBR 6118	Mod I	1.33	1.38	27.37%				
	$\theta=45^\circ$	1.43	1.47	28.46%				
	$\theta=30^\circ$	1.15	1.18	26.50%				
	$\theta=21.8^\circ$	0.99	1.00	26.09%				
EUROCODE 2	$\theta=45^\circ$	3.69	3.50	33.80%				
	$\theta=30^\circ$	2.13	2.02	33.80%				
	$\theta=21.8^\circ$	1.48	1.40	33.80%				
ACI 318		1.50	1.52	25.28%				
Part B – Demerit Point Classification								
$\tau_{test}/\tau_{pred}$	NBR 6118				EUROCODE 2			ACI 318
	Mod. I	$\theta=45^\circ$	$\theta=30^\circ$	$\theta=21.8^\circ$	$\theta=45^\circ$	$\theta=30^\circ$	$\theta=21.8^\circ$	
< 0.50	1*	0	2	7	0	0	1	0
0.50   -0.65	6	7	6	4	0	0	2	2
0.65   -0.85	5	5	6	14	0	2	4	6
0.85   -1.30	31	22	49	65	1	6	31	15
1.30   -2.00	54	60	37	10	6	39	46	73
$\geq 2.00$	3	6	0	0	93	53	16	4
<b>Total Demerit Point Score</b>	<b>110**</b>	<b>117</b>	<b>99</b>	<b>128</b>	<b>192</b>	<b>149</b>	<b>106</b>	<b>103</b>
* – Percentage values of $\tau_{test}/\tau_{pred}$ results								
** 110 = (1 x 10) + (6 x 5) + (5 x 2) + (31 x 0) + (54 x 1) + (3 x 2)								

The relationship between web reinforcement ratio  $\rho_w$  and concrete compressive strength is also observed in partial analysis results when NBR 6118 design criteria are used. The analysis reveals a significant improvement of NBR

**TABLE 12 – Partial Analysis Results of Diagonal Tension Failure Beams with Stirrups and  $f_c \leq 50\text{MPa}$**

**Part A – Effective Depth Influence – Demerit Point Classification**

Code	d (cm)	< 0.50	0.50 –0.65	0.65 –0.85	0.85 –1.30	1.30 –2.00	≥ 2.00	Total Demerit Point Score
NBR 6118 Mod I	≤ 60	0*	2	4	32	59	3	83**
	> 60	9	55	9	27	0	0	383
NBR 6118 $\theta = 45^\circ$	≤ 60	0	2	4	21	66	7	98
	> 60	0	64	9	27	0	0	338
NBR 6118 $\theta = 30^\circ$	≤ 60	1	2	5	52	40	0	70
	> 60	18	46	18	18	0	0	446
EUROCODE 2 $\theta = 30^\circ$	≤ 60	0	0	2	4	38	56	154
	> 60	0	0	9	27	37	27	109
EUROCODE 2 $\theta = 21.8^\circ$	≤ 60	1	2	2	32	47	16	103
	> 60	0	9	27	28	27	9	144
ACI 318	≤ 60	0	1	2	14	78	5	97
	> 60	0	9	55	18	18	0	173

\* – Percentage values of  $\tau_{test} / \tau_{pred}$  results

\*\* 83 = (0 x 10) + (2 x 5) + (4 x 2) + (32 x 0) + (59 x 1) + (3 x 2)

**Part B – Longitudinal Reinforcement Ratio Influence – Demerit Point Classification**

Code	$\rho_l$ (cm)	< 0.50	0.50 –0.65	0.65 –0.85	0.85 –1.30	1.30 –2.00	≥ 2.00	Total Demerit Point Score
NBR 6118 Mod I	≤ 2	3	20	13	36	28	0	184
	> 2	0	0	2	29	63	6	79
NBR 6118 $\theta = 45^\circ$	≤ 2	0	23	13	28	33	3	180
	> 2	0	0	2	22	66	10	90
NBR 6118 $\theta = 30^\circ$	≤ 2	8	20	13	36	23	0	229
	> 2	0	0	3	50	47	0	53
EUROCODE 2 $\theta = 30^\circ$	≤ 2	0	0	8	10	38	44	142
	> 2	0	0	0	2	32	66	164
EUROCODE 2 $\theta = 21.8^\circ$	≤ 2	3	5	8	33	38	13	135
	> 2	0	0	0	23	56	21	98
ACI 318	≤ 2	0	5	21	28	46	0	113
	> 2	0	0	0	9	82	9	100

**TABLE 12 – Partial Analysis Results of Diagonal Tension Failure Beams with Stirrups and  $f_c \leq 50\text{MPa}$   
(continuation)**
**Part C – Shear Reinforcement Ratio Influence with Concrete Strength  $f_c \leq 30\text{ MPa}$   
Demerit Point Classification**

Code	$\rho_w$ (%)	< 0.50	0.50 –0.65	0.65 –0.85	0.85 –1.30	1.30 –2.00	$\geq 2.00$	Total Demerit Point Score
NBR 6118 Mod I	$\leq 0.1$	0	0	8	23	69	0	85
	$0.1 < \rho_w \leq 0.2$	0	0	0	22	57	21	99
	$> 0.2$	0	0	0	8	92	0	92
NBR 6118 $\theta = 45^\circ$	$\leq 0.1$	0	0	8	15	69	8	101
	$0.1 < \rho_w \leq 0.2$	0	0	0	7	72	21	114
	$> 0.2$	0	0	0	0	92	8	108
NBR 6118 $\theta = 30^\circ$	$\leq 0.1$	0	0	8	31	61	0	77
	$0.1 < \rho_w \leq 0.2$	0	0	0	43	57	0	57
	$> 0.2$	0	0	0	31	69	0	69
EUROCODE 2 $\theta = 30^\circ$	$\leq 0.1$	0	0	0	0	8	92	192
	$0.1 < \rho_w \leq 0.2$	0	0	0	0	36	64	164
	$> 0.2$	0	0	0	15	62	23	108
EUROCODE 2 $\theta = 21.8^\circ$	$\leq 0.1$	0	0	0	8	46	46	138
	$0.1 < \rho_w \leq 0.2$	0	0	0	29	64	7	78
	$> 0.2$	0	0	8	61	31	0	47
ACI 318	$\leq 0.1$	0	0	0	8	92	0	92
	$0.1 < \rho_w \leq 0.2$	0	0	0	7	79	14	107
	$> 0.2$	0	0	0	8	85	7	99

**Part D – Shear Reinforcement Ratio Influence with Concrete Strength  $30 < f_c \leq 50$   
Demerit Point Classification**

Code	$\rho_w$ (%)	< 0.50	0.50 –0.65	0.65 –0.85	0.85 –1.30	1.30 –2.00	$\geq 2.00$	Total Demerit Point Score
NBR 6118 Mod I	$\leq 0.1$	0	17	18	17	44	4	173
	$0.1 < \rho_w \leq 0.2$	4	12	0	47	37	0	137
	$> 0.2$	0	4	4	44	48	0	76
NBR 6118 $\theta = 45^\circ$	$\leq 0.1$	0	17	17	13	49	4	176
	$0.1 < \rho_w \leq 0.2$	0	15	0	42	39	4	122
	$> 0.2$	0	4	3	26	63	4	97
NBR 6118 $\theta = 30^\circ$	$\leq 0.1$	0	22	17	22	39	0	183
	$0.1 < \rho_w \leq 0.2$	7	9	7	54	23	0	152
	$> 0.2$	4	3	0	70	23	0	78
EUROCODE 2 $\theta = 30^\circ$	$\leq 0.1$	0	0	0	0	26	74	174
	$0.1 < \rho_w \leq 0.2$	0	0	4	11	31	54	147
	$> 0.2$	0	0	7	12	70	11	106
EUROCODE 2 $\theta = 21.8^\circ$	$\leq 0.1$	0	0	0	3	43	44	131
	$0.1 < \rho_w \leq 0.2$	0	4	12	23	54	7	112
	$> 0.2$	4	7	4	67	15	3	104
ACI 318	$\leq 0.1$	0	0	17	22	52	9	104
	$0.1 < \rho_w \leq 0.2$	0	4	12	22	58	4	110
	$> 0.2$	0	4	4	18	74	9	102

6118 criteria for larger web reinforcement ratios.

The overall analysis of the results (table [14]) indicates that ACI 318 and NBR 6118 model II criterion with  $\theta = 45^\circ$  are the most suitable procedures when safety, precision and economy are lumped together. EUROCODE 2 criterion is extremely safe and consequently not economical since it has the largest percentage of  $\tau_{test}/\tau_{pred}$  results in the conservative and extremely conservative zones. These findings are also illustrated in figure [3] which shows the largest percentage of  $\tau_{test}/\tau_{pred}$  values in appropriate safety range (between 0.85 and 1.3) when the NBR 6118 model II criterion with  $\theta = 45^\circ$  is used as well as the large dispersion of the results obtained with EUROCODE 2 procedure.

## 4 Conclusions

The objective of this paper was to analyze with respect to safety, precision and economy NBR 6118:2003 shear design criteria for reinforced concrete beams. The study was done through the comparison with experimental results of beams, with and without traverse reinforcement and built with conventional concrete ( $f_c \leq 50$  MPa). EUROCODE 2:2004 and ACI 318:2005 shear design criteria are also analyzed. Beams fabricated with high-performance concrete ( $f_c > 50$  MPa) are also investigated. Partial analyses, considering the beams, effective depth, the concrete compressive strength, and the amount of flexural and shear reinforcement are also included. The main conclusions of the comparative study are presented next.

### 4.1 Beams built with concrete of $f_c \leq 50$ MPa

#### 4.1.1 Web crushing failure

For the web crushing failure, EUROCODE 2:2004 with  $\theta = 21,8^\circ$  and NBR 6118:2003 with  $\theta = 30^\circ$  procedures are the most suitable ones in terms of safety, precision and economy. ACI 318:2005 criterion is very conservative, since it produces 74% of  $\tau_{test}/\tau_{pred}$  results above the appropriate safety zone.

#### 4.1.2 Beams with diagonal tension failure

For the beams without web reinforcement, EUROCODE 2:2004 criterion is the best ranked. The comparative analysis also shows that the NBR 6118:2003 formulation for the concrete contribution stress  $\tau_c$  is not suitable. The equation needs adjustments which should include explicitly the influences of the effective depth of the beams and the flexural reinforcement ratio.

For beams with vertical stirrups, NBR 6118:2003 model II criterion with  $\theta = 30^\circ$  is the best ranked, in spite of being less conservative. Similar performance was achieved when ACI 318:2005 and EUROCODE 2:2004 with  $\theta = 21,8^\circ$  criteria were used. This Brazilian code criterion provides even better results for beams fabricated with concrete having compressive strength up to 30 MPa. The analysis revealed, on the other hand, a poorer performance for beams with

**TABLE 13 – Results of Diagonal Tension Failure – Beams without Stirrups and  $f_c > 50$  MPa**

Part A – Statistical Analysis			
Statistical Parameters	NBR 6118	EUROCODE 2	ACI 318
Average	0.86	0.96	1.31
Median	0.84	0.93	1.26
Coefficient of Variation	36.49 %	21.86 %	36.15 %
Part B – Demerit Point Classification			
$\tau_{test}/\tau_{pred}$	NBR 6118 (%)	EUROCODE 2 (%)	ACI 318 (%)
< 0.50	8	0	5
0.50   --0.65	16	5	2
0.65   --0.85	29	26	3
0.85   --1.30	36	62	46
1.30   --2	11	7	33
$\geq 2.00$	0	0	11
Total Demerit Point Score	229*	84	121
* 229 = (8 x 10) + (16 x 5) + (29 x 2) + (36 x 0) + (11 x 1) + (0 x 2)			

effective depth larger than 60 cm or with longitudinal reinforcement ratio smaller than 2%.

### 4.2 Beams built with concrete of $f_c > 50$ MPa

#### 4.2.1 Beams with Diagonal Tension Failure

In the case of beams without web reinforcement, EUROCODE 2:2004 has the best ranked criterion. The extrapolation of NBR 6118 procedure is not suitable when compared to the other design code procedures under appraisal. For beams with web reinforcement, the overall analysis of the results indicates ACI 318:2005 and NBR 6118:2003 model II criterion with  $\theta = 45^\circ$  as the most appropriate. It is important to point out that the extrapolation of NBR

6118:2003 procedures produced less conservative and precise results in relation to beams built with concrete of  $f_c \leq 50$  MPa.

### 5 Notation

- a = shear span;
- $b_w$  = web width;
- d = effective depth;
- $f_c$  = concrete compressive strength;
- $f_y$  = shear reinforcement yield strength;
- s = stirrups spacing;
- $V_u$  = test failure shearing force;
- $\theta$  = angle of concrete struts;
- $\rho_w$  = shear reinforcement ratio =  $A_{sw} / (b_w \cdot s)$ ;
- $\rho_l$  = longitudinal reinforcement ratio =  $A_{sl} / (b_w \cdot d)$ ;

**TABLE 14 – Results of Diagonal Tension Failure Beams with Stirrups and  $f_c > 50$  MPa**

**Part A – Statistical Analysis**

Statistical Parameters		Average	Median	Coefficient of Variation
NBR 6118 (2003)	Mod I	1.18	1.11	41.64 %
	$\theta=45^\circ$	1.22	1.17	36.97 %
	$\theta=30^\circ$	1.04	1.00	36.56 %
	$\theta=21.8^\circ$	0.92	0.88	37.13 %
EUROCODE 2 (2003)	$\theta=45^\circ$	4.64	4.24	55.21 %
	$\theta=30^\circ$	2.68	2.45	55.21 %
	$\theta=21.8^\circ$	1.86	1.70	55.21 %
ACI 318 (2005)		1.49	1.41	37.49 %

**Part B – Demerit Point Classification**

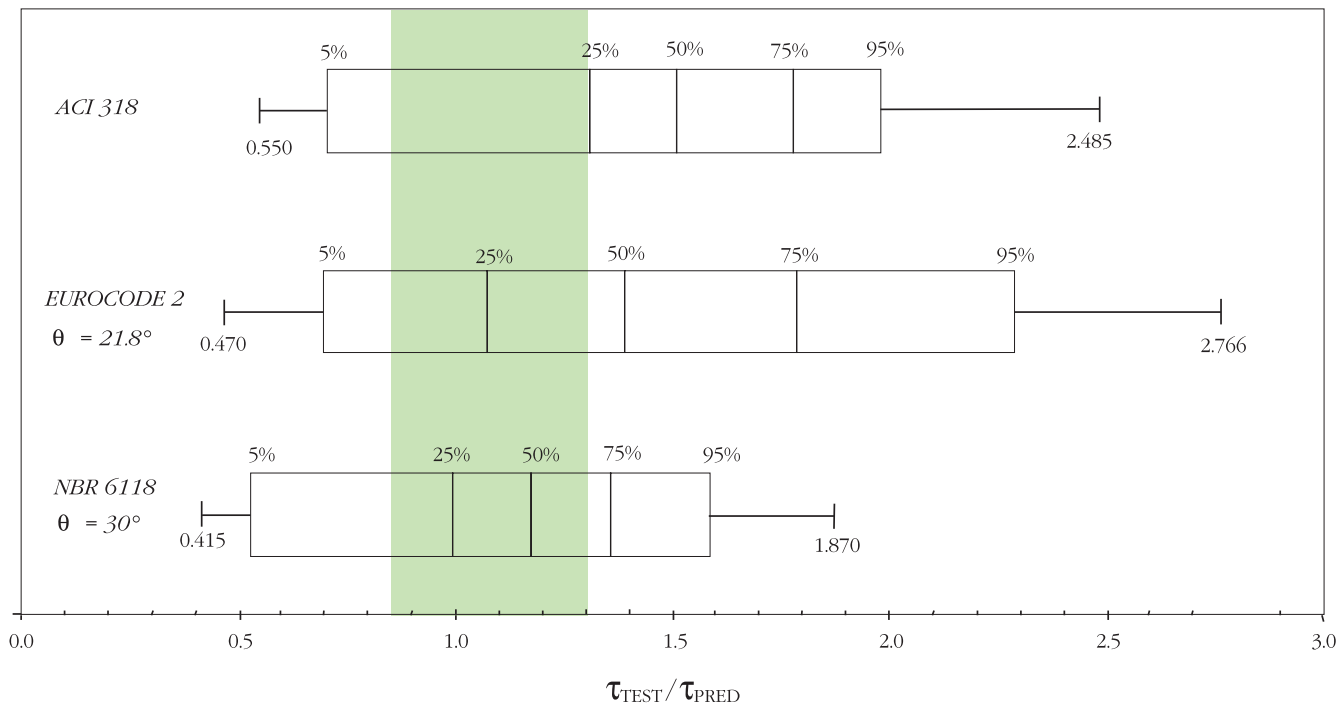
$\tau_{test}/\tau_{pred}$	Mod. I	NBR 6118			EUROCODE 2			ACI 318
		$\theta=45^\circ$	$\theta=30^\circ$	$\theta=21.8^\circ$	$\theta=45^\circ$	$\theta=30^\circ$	$\theta=21.8^\circ$	
< 0,50	2*	1	2	3	0	0	0	0
0.50   -0.65	6	2	4	8	0	0	2	1
0.65   -0.85	10	10	20	37	0	1	4	2
0.85   -1.30	56	55	60	45	0	7	17	34
1.30   -2.00	22	28	12	5	5	19	45	55
$\geq 2.00$	4	4	2	2	95	73	32	8
Total Demerit Point Score	100**	76	96	153	195	167	127	80

\* – Percentage values of  $\tau_{test}/\tau_{pred}$  results

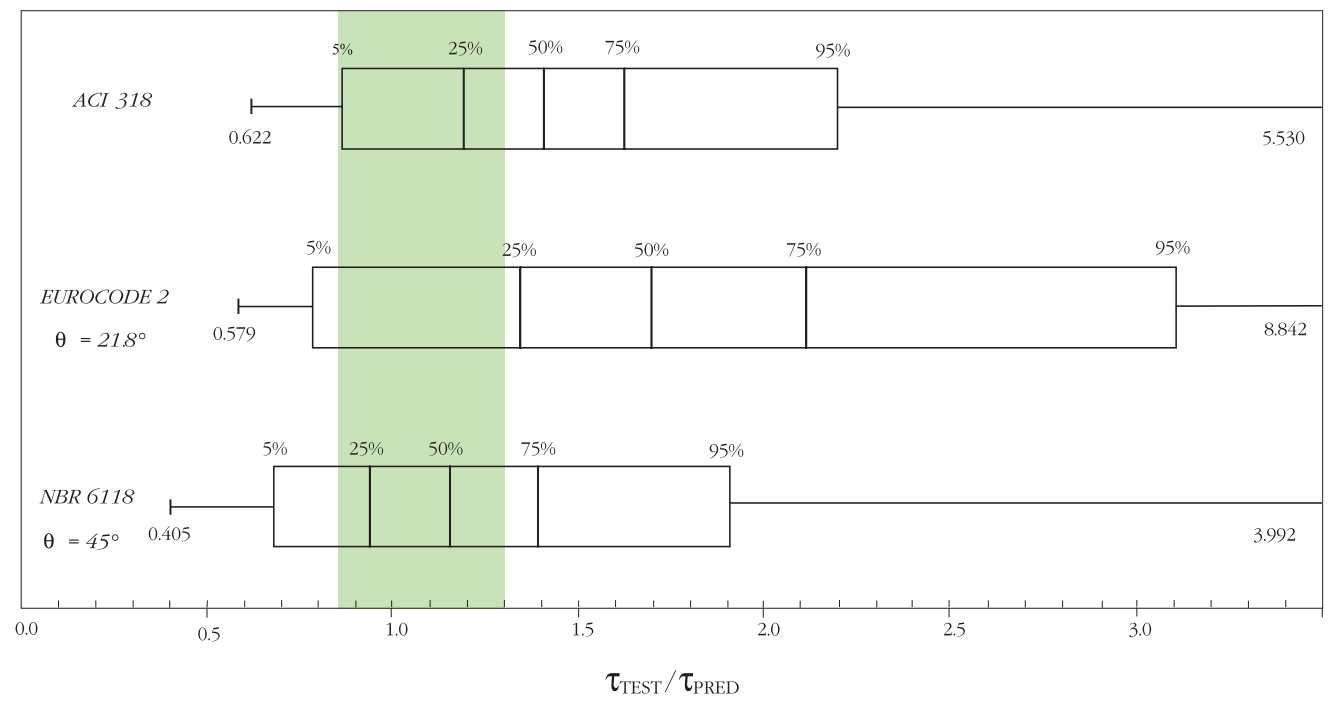
\*\* 100 = (2 x 10) + (6 x 5) + (10 x 2) + (56 x 0) + (22 x 1) + (4 x 2)



**FIGURE 2 – Percentage of  $\tau_{test}/\tau_{pred}$  ratio – Diagonal Tension Failure – Beams with Stirrups and  $f_c \leq 50$  MPa**



**FIGURE 3 – Percentage of  $\tau_{test}/\tau_{pred}$  ratio – Diagonal Tension Failure – Beams with Stirrups and  $f_c > 50$  MPa**



$\tau_c$  = concrete contribution stress to the shearing strength;  
 $\tau_{SW}$  = web reinforcement contribution stress to the shearing strength;

$\tau_{test}$  = test failure shear stress =  $V_u / (b_w \cdot d)$ ;  
 $\tau_{pred}$  = predict shear stress

**TABLE 15 – Partial Analysis Results of Diagonal Tension Failure Beams with Stirrups and  $f_c > 50$  MPa**

**Part A – Effective Depth Influence – Demerit Point Classification**

Code	d (cm)	< 0.50	0.50 –0.65	0.65 –0.85	0.85 –1.30	1.30 –2.00	≥ 2.00	Total Demerit Point Score
NBR 6118	≤ 60	1*	2	6	59	25	5	71**
Mod I	> 60	6	29	30	35	0	0	265
NBR 6118	≤ 60	1	0	6	57	31	5	63
θ = 45°	> 60	6	12	41	41	0	0	202
NBR 6118	≤ 60	1	2	16	66	13	2	69
θ = 30°	> 60	12	17	53	18	0	0	311
EUROCODE 2	≤ 60	0	0	1	8	19	72	165
θ = 30°	> 60	0	0	0	0	18	82	182
EUROCODE 2	≤ 60	0	3	4	18	43	32	130
θ = 21.8°	> 60	0	0	0	12	59	29	117
ACI 318	≤ 60	0	0	1	30	60	9	80
	> 60	0	6	12	65	17	0	71

\* – Percentage values of  $\tau_{test}/\tau_{pred}$  results

\*\* 71 = (1 x 10) + (2 x 5) + (6 x 2) + (59 x 0) + (25 x 1) + (5 x 2)

**Part B – Longitudinal Reinforcement Ratio Influence – Demerit Point Classification**

Code	$\rho_l$ (%)	< 0.50	0.50 –0.65	0.65 –0.85	0.85 –1.30	1.30 –2.00	≥ 2.00	Total Demerit Point Score
NBR 6118	≤ 2	18	28	18	27	0	9	374
Mod I	> 2	0	4	11	62	21	2	67
NBR 6118	≤ 2	18	9	37	18	9	9	326
θ = 45°	> 2	0	1	9	65	22	3	51
NBR 6118	≤ 2	27	19	27	18	0	9	437
θ = 30°	> 2	0	4	22	63	11	0	75
EUROCODE 2	≤ 2	0	0	0	0	9	91	191
θ = 30°	> 2	0	0	0	5	20	75	170
EUROCODE 2	≤ 2	0	0	0	9	64	27	118
θ = 21.8°	> 2	0	0	3	18	44	35	120
ACI 318	≤ 2	0	9	18	46	18	9	117
	> 2	0	0	1	34	59	6	73

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**TABLE 15 – Partial Analysis Results of Diagonal Tension  
Failure Beams with Stirrups and  $f_c > 50$  MPa (continuation)**

Part C – Shear Reinforcement Ratio Influence with Concrete Strength $50 < f_c \leq 70$ MPa Demerit Point Classification								
Code	$\rho_w$ (%)	< 0.50	0.50 –0.65	0.65 –0.85	0.85 –1.30	1.30 –2.00	$\geq 2.00$	Total Demerit Point Score
NBR 6118 Mod I	$\leq 0.1$	0	43	28	29	0	0	271
	$0.1 < \rho_w \leq 0.2$	0	3	16	62	16	3	69
	$> 0.2$	0	0	0	57	38	5	48
NBR 6118 $\theta = 45^\circ$	$\leq 0.1$	0	0	57	29	14	0	128
	$0.1 < \rho_w \leq 0.2$	0	0	3	75	19	3	31
	$> 0.2$	0	0	0	43	47	10	67
NBR 6118 $\theta = 30^\circ$	$\leq 0.1$	0	14	57	29	0	0	184
	$0.1 < \rho_w \leq 0.2$	0	3	19	66	9	3	68
	$> 0.2$	0	0	5	76	19	0	29
EUROCODE 2 $\theta = 30^\circ$	$\leq 0.1$	0	0	0	0	0	100	200
	$0.1 < \rho_w \leq 0.2$	0	0	0	0	9	91	191
	$> 0.2$	0	0	5	19	38	38	124
EUROCODE 2 $\theta = 21.8^\circ$	$\leq 0.1$	0	0	0	0	29	71	171
	$0.1 < \rho_w \leq 0.2$	0	0	0	9	63	28	119
	$> 0.2$	0	14	5	38	38	5	128
ACI 318	$\leq 0.1$	0	0	0	71	29	0	29
	$0.1 < \rho_w \leq 0.2$	0	0	0	41	56	3	62
	$> 0.2$	0	0	0	24	71	5	81
Part D – Shear Reinforcement Ratio Influence with Concrete Strength $f_c > 70$ MPa Demerit Point Classification								
Code	$\rho_w$ (%)	< 0.50	0.50 –0.65	0.65 –0.85	0.85 –1.30	1.30 –2.00	$\geq 2.00$	Total Demerit Point Score
NBR 6118 Mod I	$\leq 0.1$	14	15	21	29	14	7	285
	$0.1 < \rho_w \leq 0.2$	0	6	8	64	22	0	68
	$> 0.2$	0	0	4	54	29	13	63
NBR 6118 $\theta = 45^\circ$	$\leq 0.1$	14	7	29	29	14	7	261
	$0.1 < \rho_w \leq 0.2$	0	3	14	58	25	0	68
	$> 0.2$	0	0	0	50	42	8	58
NBR 6118 $\theta = 30^\circ$	$\leq 0.1$	21	8	21	29	21	0	313
	$0.1 < \rho_w \leq 0.2$	0	8	14	72	6	0	74
	$> 0.2$	0	0	25	54	17	4	75
EUROCODE 2 $\theta = 30^\circ$	$\leq 0.1$	0	0	0	0	0	100	200
	$0.1 < \rho_w \leq 0.2$	0	0	0	0	8	92	192
	$> 0.2$	0	0	0	13	49	38	125
EUROCODE 2 $\theta = 21.8^\circ$	$\leq 0.1$	0	0	0	0	43	57	157
	$0.1 < \rho_w \leq 0.2$	0	0	0	6	50	44	138
	$> 0.2$	0	0	4	46	29	21	79

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