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Shear and concrete tensile strength in the design concept of strut-and-tie models

Cisalhamento e resistência à tração do concreto no conceito de projeto de modelos de escoras e tirantes

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Abstract

The shear capacity of structural concrete members has puzzled researchers and designers since the beginning of reinforced concrete, despite the fact that the truss model had already early been proposed as a design model. In the last 20 years many efforts were made to re-introduce models for designing structural concrete in terms of strut-and-tie models. This was enhanced by damages and failures of structures. A brief review is given on the design concepts in present codes and a modern design concept based on strut-and-tie models is presented. This concept addresses discontinuity regions (D-regions) with the same emphasis as B-regions with the design for shear and flexure. The present state of the shear design is outlined and a design method presented for members with stirrups, where the strut angle depends on the magnitude of the shear force. The shear capacity of members without shear reinforcement is treated with special emphasis on the size effect.

Keywords: codes; concrete tensile strength; shear design; size effect; structural concrete; strut-and-tie models

Resumo

A capacidade ao cisalhamento de elementos estruturais de concreto tem confundido pesquisadores e projetistas desde o início do concreto armado, apesar do fato de que o modelo de treliça tenha já desde o início sido proposto como um modelo de cálculo. Nos últimos 20 anos muitos esforços foram feitos para reintroduzir modelos para o cálculo de concreto estrutural em termos de modelos de escora e tirante. Isto foi acentuado por danos e falhas em estruturas. Uma breve revisão sobre os conceitos de projeto em códigos atuais é feita e um conceito moderno de projeto com base em modelos de escora e tirante é apresentado. Esse conceito aborda regiões de descontinuidade (Regiões D) com a mesma ênfase que as regiões B, tendo em vista o projeto para cisalhamento e flexão. O estado atual do projeto ao cisalhamento é descrito e um método para elementos reforçados com estribos é apresentado, onde o ângulo da escora depende da magnitude da força cortante. A capacidade ao cisalhamento de elementos sem armadura ao cisalhamento é tratada, com ênfase especial no efeito escala.

Palavras-chave: códigos; resistência à tração do concreto; cálculo ao cisalhamento; concreto estrutural; modelos de escoras e tirantes

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1 Introduction

1.1 Brief historical review

In the very early years of reinforced concrete ("Eisenbeton") already remarkable structures were built, although not much was known about this new material and the guidelines or codes only consisted of few pages. Well known examples are:

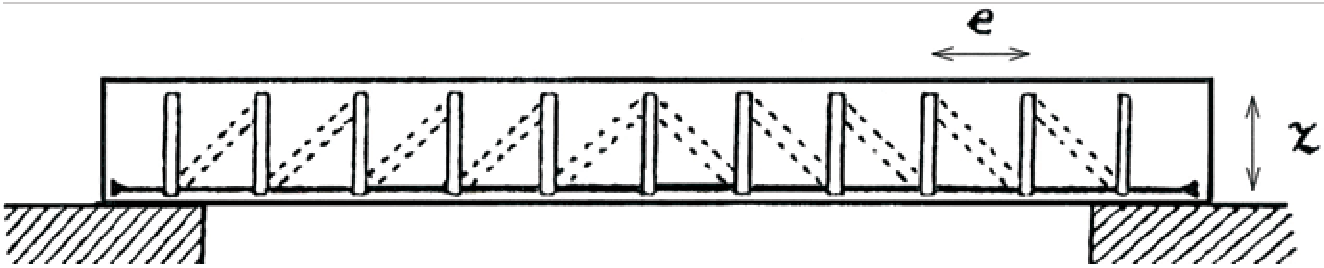
- the bridges over the in Zuoz (1901) and the Rhine in Tavanasa (1905) built by Maillart;
- the Risorgimentobridge in Rome (1911) built by Hennebique;
- the bridge over the Isar near Grünwald (1904), the bridge over the Gmündertobel (1908) near Teufen (Kanton Appenzell) and the railway bridge Rosenstein over the river Neckar in Stuttgart (1911), all built by Mörsch.

These pioneers of structural concrete only knew very simple models for the analysis, which however provided a clear understanding of the flow of forces in the structure: these were the arch and the truss. Thus the designers extended these models also to cover the new composite material, like demonstrated in Fig. 1 by the truss models for members with shear reinforcement proposed by Ritter (1899) and Mörsch (1912). Even for members without shear reinforcement models were proposed, like the inclined strut in Fig. 2a (fully complying with theory of plasticity) proposed by Faber (1916) or something like the "tooth model" in Fig. 2b proposed by von Thullie (1905).

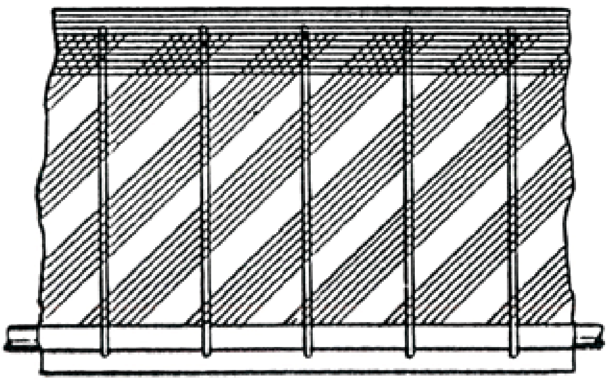
1.2 Damages and failures

Now about 100 years later the designers can utilize refined methods for the analysis and can use extremely powerful computers in order to calculate all action effects for multiple load cases. Yet despite of this, many severe damages of structures demonstrated that this does not necessarily coincides with a clearer understanding of the flow of forces. All this was discussed at the IABSE Colloqium „Structural Concrete", April 1991 in Stuttgart [IABSE (1991a, b)]. This is demonstrated by some examples, like those presented by Breen (1991), Podolny (1985), Leonhardt (1970, 1979) as well as Schlaich and Reineck (1993). Also the need for using clear terminology and models in design of structural concrete was pointed out Breen (1991) and Schlaich (1991). Some of the main points of the Summarizing statement of this IABSE Colloquium [IABSE 1991 b] were:

Figure 1 – Historical models for members with shear reinforcement

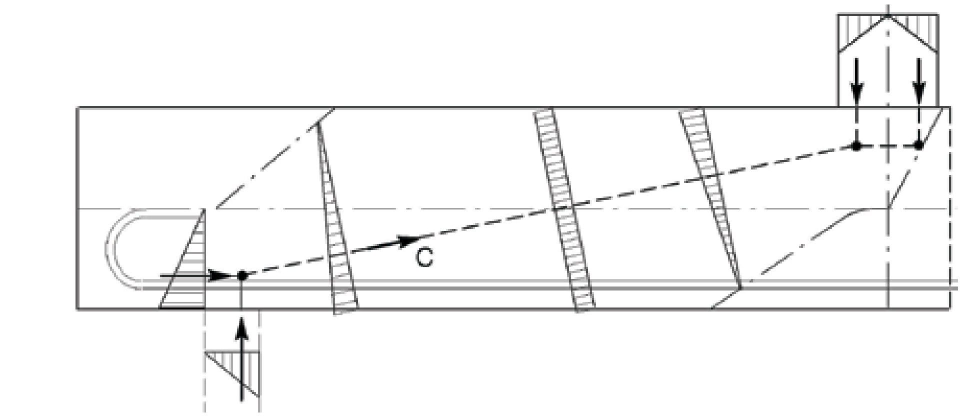


a truss model proposed by W. Ritter (1899)

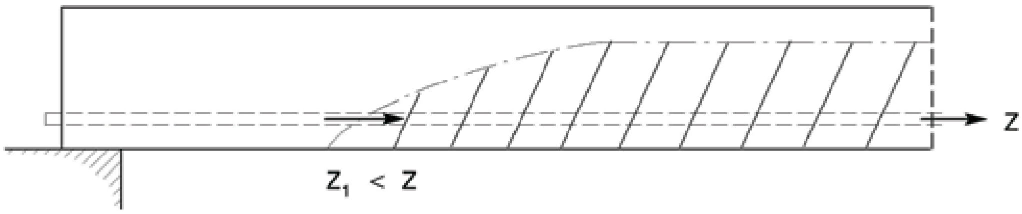


b truss model according to Emil Mörsch (1912)

Figure 2 – Historical models for members without shear reinforcement



a model proposed by O. Faber (1916)



b model proposed by Max von Thullie (1905)

9 Failures of actual structures vividly show that overall structural integrity is heavily dependent on proper dimensioning and detailing especially at geometrical or load discontinuity regions (D), and that at the nodes.

11 The analysis techniques utilized should be commensurate with the assumptions and the required information.

12 In the dimensioning process highly transparent models should be used to emphasize the flow of forces. In Regions with linear strain distributions (B- regions) the internal state of stress can be determined from sectional forces (M, N, V) or from truss models including stress fields. In regions with nonlinear strain distributions (D-regions) the internal state of stress may be determined from strut-and-tie models.

These statements are certainly still valid and should especially be considered for writing codes.

2 Codes and modern design concepts

2.1 Concepts of present codes

The unsatisfactory state of practice is obviously somehow connected to the codes, and this is summarized here briefly with reference to Reineck (1999).

Undoubtedly the Eurocodes have to be mentioned first when discussing the development towards a modern code, since it and brought a major step forward towards a consistent code concept. This especially refers to the fact that the set of codes cover different materials including the soil with same principles and with the same basis of the safety concept. It is really also noteworthy that these codes were developed by and for different countries with quite different codes and engineering traditions.

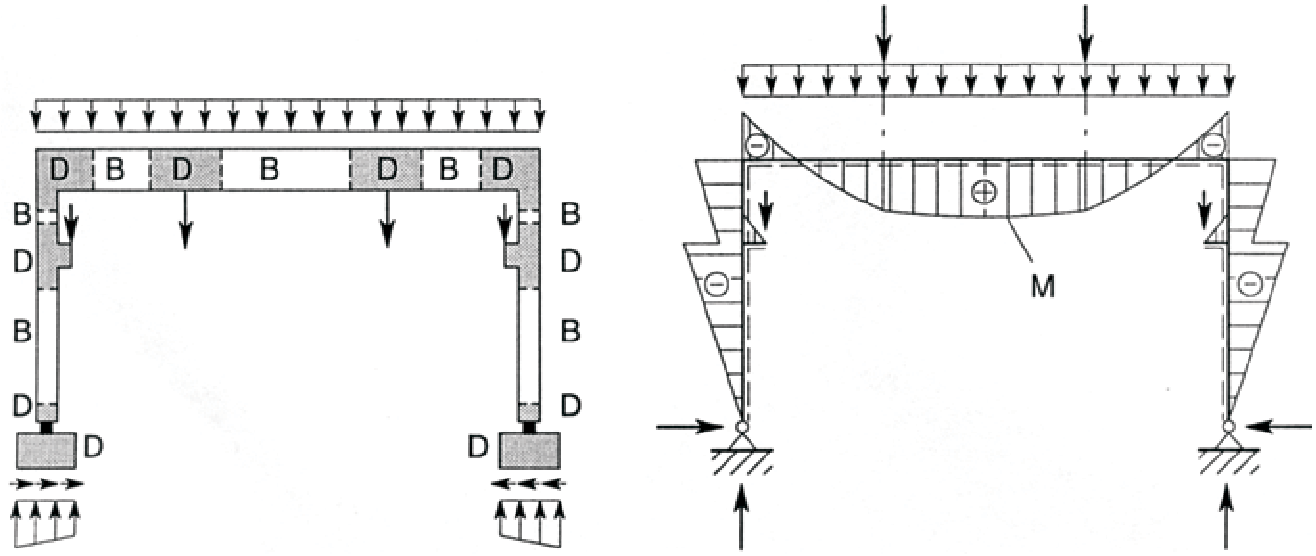
The principles for the design are very clearly defined in the EC2, part 1, and they are undisputed, like:

- It shall be verified that no relevant Limit State is exceeded.
- All relevant design situations and loading cases shall be considered.
- Calculations shall be performed using appropriate design models involving all relevant variables.

In this context it is especially noteworthy that the whole structure is addressed and not the sections.

However, the procedures for checking the Ultimate Limit State in section 4 of EC 2 are based on separate checks of sections for the action effects, like for bending moment and axial force, shear force and torsion moment. Contrary to the above principles, the checking procedures focus on

Figure 3 – Examples for B- and D- regions in a structure according to Schlaich et al.(1987) and Schlaich (1991)



sections. The subsequently following detailing rules are meant to secure this procedure and the overall safety of the structure.
The danger of such design procedure is now obvious from the above mentioned damages of structures. The overall flow of forces may be overlooked and critical regions are not covered by checking sections for the action effects, which are normally gained from an ordinary beam analysis. Especially the regions with discontinuities due to the loading or/and the geometry, the D-regions, are not dimensioned but left to be covered by detailing rules. The latter, however, only deal with some special cases, like e.g. frame corners or corbels, so that most present codes do not give any general guidance on other problems. Only recently the design with strut-and-tie models was included in some codes, like e.g. in App. A of ACI 318 and in EC2 and the German DIN 1045-1.

2.2 The FIP Recommendations "Practical design of structural concrete"

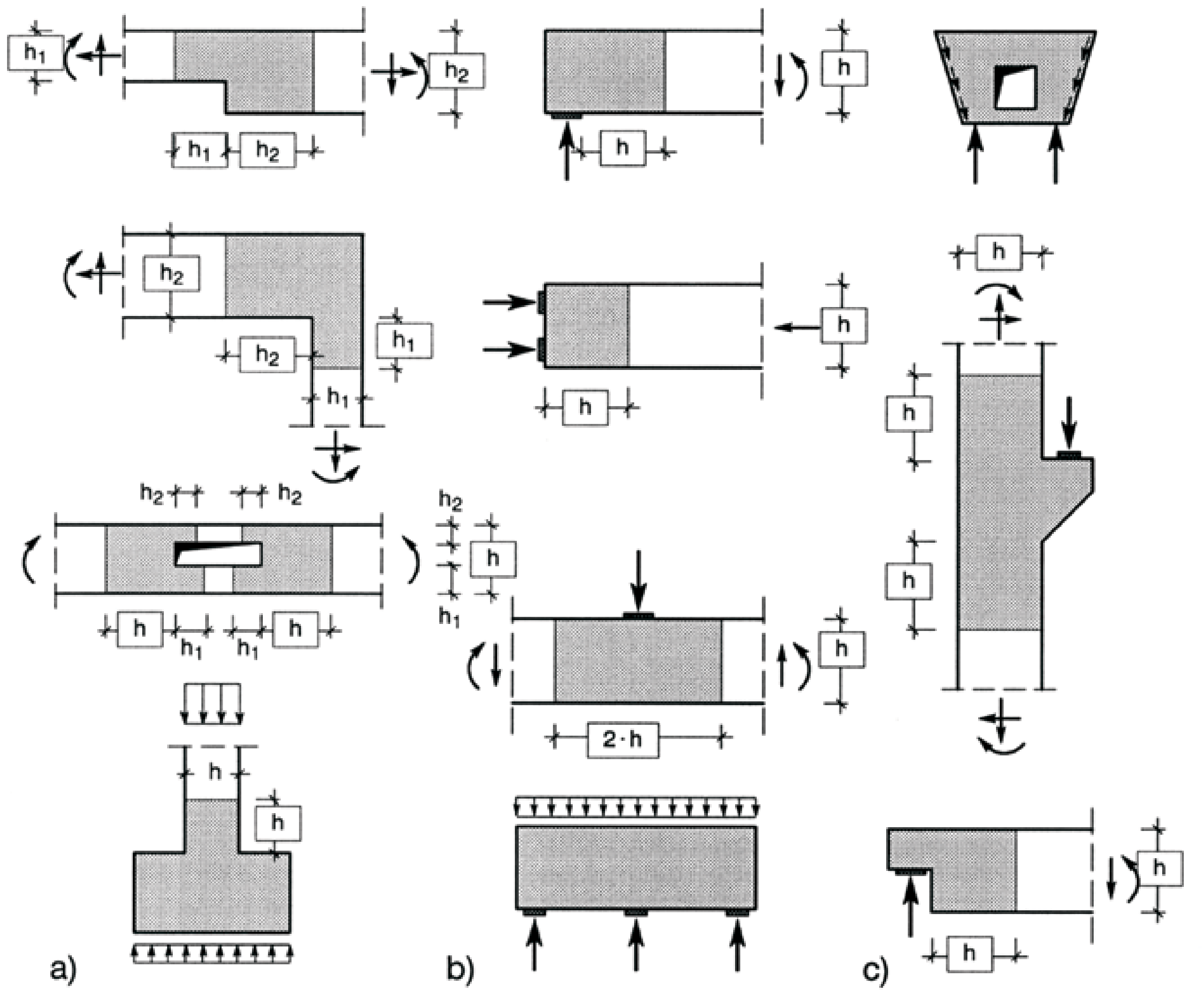
The above shortages of present codes were overcome by the design concept of the FIP Recommendations "Practical design of structural concrete" (1999), which were written by a small group with strong representation of designers from practice. These recommendations can be regarded as a major step forward to a modern design concept.
These FIP Recommendations 1999 are a revision of the edition from 1984, and they are based on the CEB-FIP Model Code 1990. However, some further developments were made, and this especially refers to the full implementation of the design concept of strut-and-tie models.
The first step is to clearly discern between the B- and the D-regions, as defined in Fig. 3 and 4.
The next step is obvious from the list of contents shown in Fig. 5. After stating the principles and defining the material characteristics as well as the technological and durability requirements in the chapters 1 to 4, the elements of strut-and-tie models are defined in chapter 5. In this chapter also the basic requirements for bond, anchorages and splices are given (see Fig. 6 a), because these are fundamental design requirements and cannot be regarded as "detailing rules". Therefore, a clear and consistent basis is given with all requirements and principles for the dimensioning and detailing of sections and members in B- and D-regions, before any application rules are given.
The list of contents of chapter 6 on the design at ULS is given in Fig. 6b. After stating the general requirements and definitions in section 6.1, the sections 6.2 and 6.3 briefly describe the actions and action effects as well as the requirements for the structural analysis. This is a further special feature of the FIP Recommendations 1999 that the analysis does not form a separate chapter but it is directly related to the chapters for ULS and SLS. This is directly followed by the sections with the dimensioning requirements, i.e. the section 6.4 "Design of B-regions" and section 6.5 "Design of D-regions". This emphasizes that both sections are closely related and are equally important.
The survey of the contents of section 6.5 is given in Fig. 7 and demonstrates already that the treatment of D-regions plays an important role in the chapter 6 on dimensioning of members. This is in contrast to present codes where these problems are scarcely addressed. A few items are presently dealt with in codes as shear problems, like e.g. the case of a point load near a support, but they can prop-

erly only dealt with by looking at the whole D-region. This section 6.5 covers the most common problems occurring in practice and should be of great help for designers.
All these D-regions listed in Fig. 7 are dealt with on basis of the elements of strut-and-tie models defined in chapter 5. Especially the nodes have to be considered when modeling, so that the designer is faced with the important problems of e.g. especially the anchorages. In present codes all these items are regarded as detailing matters which have to be solved by the draftsmen. The use of strut-and-tie models leads to a systematic dimensioning of many of these so-called detailing rules and thus to a better understanding. Misunderstandings between designer and draftsmen are avoided and the quality of the structure and its details greatly improved, so that the use of strut-and-tie models in the framework of this new code concept can be regarded as an important improvement in the quality control of the design. In the following some examples are given which should demonstrate these points.

3 Some examples for the design of D-regions with strut-and-tie models

Finding a strut-and-tie model (STM) for a given geometry and loading of a member or a D-region is the first and major task

Figure 4 – Different types of discontinuity regions (D-regions) according to Schlaich et al.(1987) and Schlaich (1991)



a geometrical
b statical
c geometrical and statical

for the design engineer. The subsequent analysis of the forces and the check of the stresses then is relatively straightforward. The different modelling methods are [Schlaich et al. (1987)]:

- using standard examples and adapting them to the given geometry and forces, like the well known corbels or deep beams;
- using linear elastic stress distributions in decisive sections to determine the location of major struts or ties;
- applying the load path method.

The load path method is explained briefly with the following two examples in Fig. 8 and 9.

For the example in Fig. 9 several more strut-and-tie models can be shown, see Reineck (2002 a, b), which demonstrates that there is not a single solution but several engineers may come to different solutions. These differences are small if the geometry of the model is orientated by the linear elastic stress distributions. However, if the model is freely selected, the differences may be significant and this may lead to different forces of ties and thus to different amounts of required reinforcement at possibly different locations.

All this poses the question regarding the uniqueness of strut-and-tie models for given loads and geometry of a D-region. The reason for this problem is, that for a strut-and-tie model only 2 conditions must be fulfilled: equilibrium and strength limits for the elements of strut-and-tie models. These two

Figure 5 – Contents of the FIP Recommendations 1999 "Practical Design of Structural Concrete"

1 – Principles
2 – Material characteristics
3 – Prestressing
4 – Technological details and durability requirements
5 – Strength of ties, struts and nodes of strut-and-tie models
6 – Ultimate Limit State Design
7 – Serviceability Limit State
8 – Structural members and structures

Figure 6 – Contents of chapters 5 and 6 of the FIP Recommendations 1999 "Practical Design of Strutural Concrete"

5 – Strength of ties, struts and nodes of strut-and-tie models
5.1 Strength of steel ties
5.2 Strength of struts
5.3 Strength of concrete ties
5.4 Transfer of forces by friction across interfaces
5.5 Strength of nodes and anchorages
5.6 Splices of reinforcements
5.7 Special rules for bundled bars

a contents of chapter 5

6 – Ultimate Limit State Design
6.1 General
6.2 Actions and action effects
6.3 Structural analysis
6.4 Design of B-regions
6.5 Design of D-regions
6.6 Design of slender compressed members
6.7 Design of slabs
6.8 Plate elements
6.9 Fatigue

b contents of chapter 6

Figure 7 – Survey on contents of section 6.5 on D-regions in the FIP Recommendations (1999)

6.5 – Design of discontinuity regions (D-regions)

6.5.1 Requirements and general criteria for modelling
6.5.2 Statical discontinuities: beam supports and corbels
6.5.2.1 Direct supports of beams
6.5.2.2 Indirect supports
6.5.2.3 Point load near a support and corbels
6.5.3 Deep beams
6.5.4 Deviation of forces
6.5.5 Frame corners and beam-column connections
6.5.5.1 Frame corners with negative (closing) moment
6.5.5.2 Frame corners with positive (opening) moment
6.5.5.3 Beam-column connection for an external column
6.5.6 Half joints and steps in members
6.5.7 Point loads in direction of member axis and anchorage zones of prestressing reinforcements
6.5.7.1 D-regions at end-support of a rectangular members
6.5.7.2 End-support of a beam with flanges
6.5.7.3 Interior anchor zones and construction joints with prestressing anchors

conditions comply with the static solution of the theory of plasticity, where generally the compatibility is not fulfilled, i.e. a mechanism is not found. A unique solution can only be expected if compatibility is fulfilled. However, any consideration of compatibility requires the calculation of strains and deformations. In order to avoid this complication, Schlaich et al. (1987) recommended to orientate the model by the stress fields of a linear elastic analysis. This also has the advantage, that the model can also be used for checking the serviceability limit state, i.e. crack widths and deformations. The fact that in design different strut-and-tie models can be found for a given problem has puzzled many engineers and especially code makers when STM was presented as a design tool. Perhaps the reason is that structural engineers are predominantly trained analytically and thus believe in only one correct solution. However, this is only true under a given clear conditions, like for example in case of the analysis of a structure for given geometry and loading according to linear elastic theory.
Contrary to this attitude in analysis however, in design an engineer is accustomed to select from a variety of solutions, and for the same task she/he has many options to satisfy the given conditions and the requirements for safety and economy and of quality. This is demonstrated by the many different types of bridges which a designer may consider in the conceptual design or in the first design phases of a bridge design.

4 D-regions of beams

Beams are not only very common members in buildings and bridges, but they also serve as basis for defining the fundamental rules in design, as explained above. Therefore, the common D-regions of beams are briefly addressed in the following, and the truss model forms the basis for the design. The basic elements of the truss model are shown in Fig. 10:

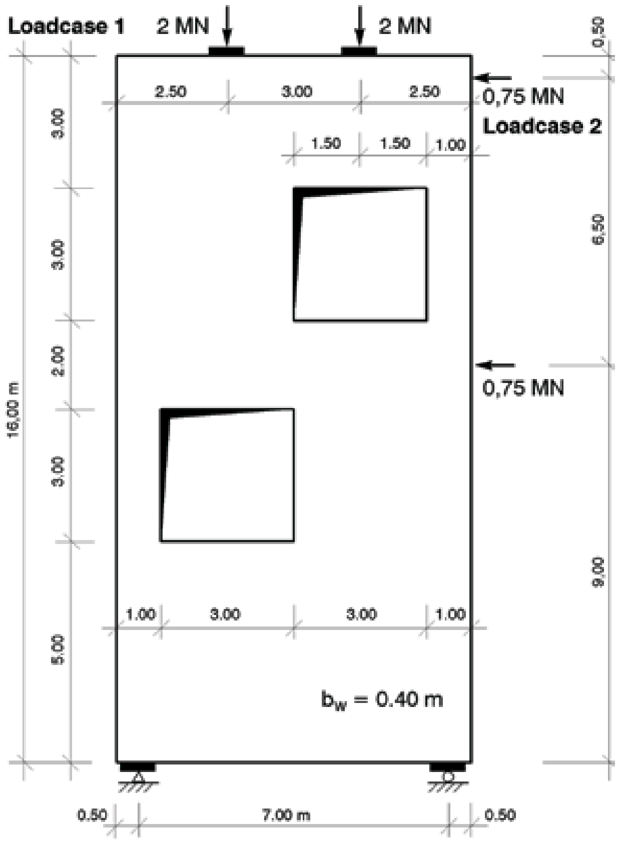
- the ties, either representing the longitudinal reinforcement concentrated in the tension chord or the distributed closely spaced stirrups in the web;
- the struts in the B-regions representing uniaxial compression fields, either as a prismatic stress field in the compression chord or as an inclined compression field in the web;
- the struts in the D-regions representing fan-shaped compression fields;
- the nodes, either as CCC-nodes (like at the loading point) or as TCC-nodes (like at the end support) or as TTC-nodes (like at the connections of the stirrups and the tension chord).

The forces in the model can easily be calculated if the inner lever arm z and the angle θ of the inclined struts are known. The transition from the B- region to the D-region, like that at an end support shown in Fig. 11, is then also clearly defined and needs no further assumptions. For example, the angle θ_A of the inclined strut representing the fan-shaped compression field at the end support follows from the geometry assuming that the stirrups are constantly distributed. With this angle the force in the tension chord is known, which has to be anchored at the end support; for common geometries and for an angle of $\theta = 30°$ this force is $F_{sA} = 1{,}2 \cdot A$.
The strut-and-tie model in the beam does not change if the end support is monolithically connected with a column, as shown in Fig. 12. The support is provided by the compression zone at the column edge. For the design of the top reinforcement the section 1-1 is decisive, but not the edge of the support.

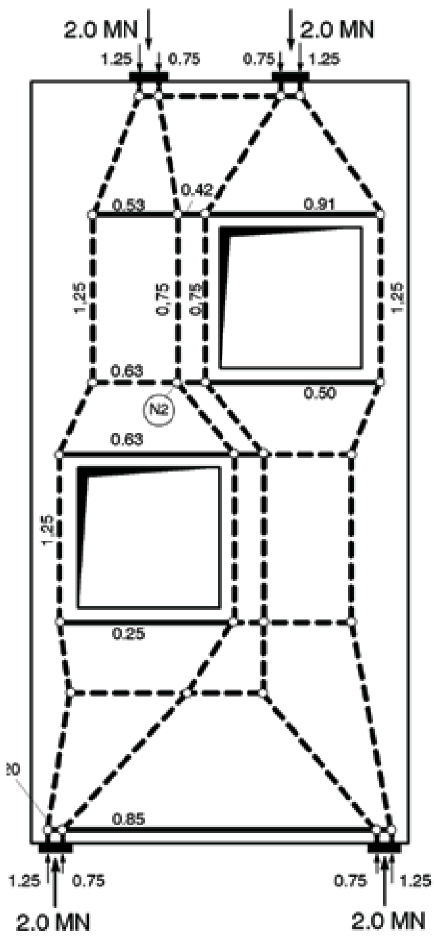
The same model shown in Fig. 11 also applies to the so-called indirect support (Fig. 12), and consequently the shear design does not change, contrary to rules in many codes. This D-region is a very critical and its improper treatment in the design and also in codes in the past even led to damages and almost failures of structures. The strut-and-tie models clearly shows that hanging-up reinforcement is required, and this has to placed in the within the connection region of both webs, contrary to the rules in many codes. The strut-and-tie model also shows that the node at the support of beam (I) is a T-T-C-node, which is very unfavourable for the anchorage of the bottom reinforcement, because transverse tension reduces the bond strength. Considering this problem of anchorage, it is also evident that the stirrups for the hanging-up reinforcement must be placed as shown in the Fig.12 a, which is in continuation of the stirrups in beam (I).

If a point load is near an end support the design problem is statically indeterminate, because there are two load paths, as shown in Fig. 14. The nearer the load is located to the support the higher the load component which is directly transferred to the support. The magnitude of the load to be transferred by stirrups is determined empirically and different formulae are given in different codes. The FIP Recommendations (1999) present the formula given in Fig. 14, where no stirrups are required from a distance nearer than a = z/2 from the support axis. For loads very near or over the support horizontal stirrups are required. This model also provides a consistent transition to the design of corbels, as shown in Fig. 15. The model in Fig. 15a is used to determine the top reinforcement, whereas the refined model in Fig. 15b shows that horizontal reinforcement should be provided. If the distance a or a_c increases the model of Fig. 14 applies and stirrups are required.

Figure 8 – Shear wall with 2 openings

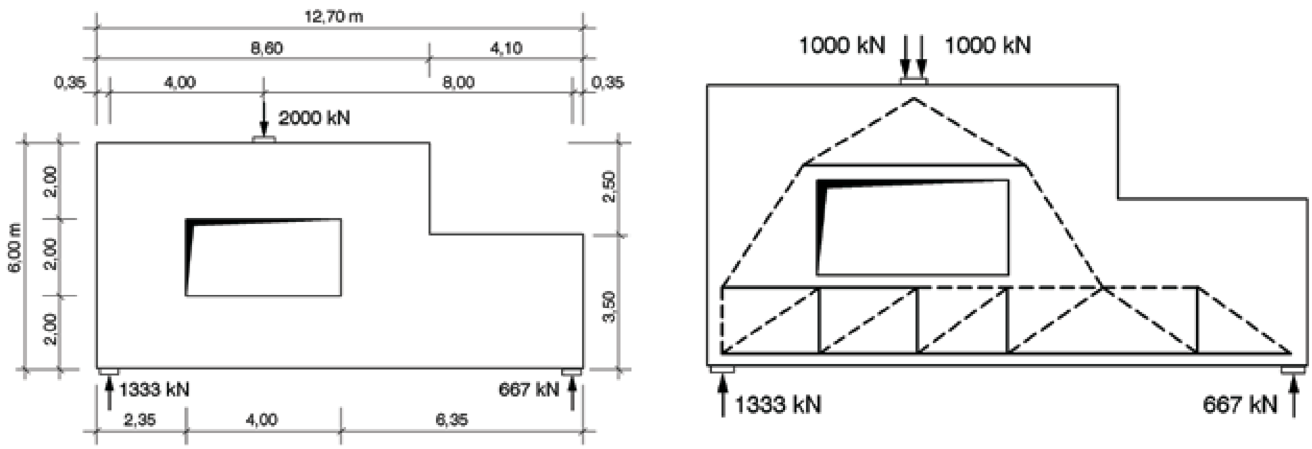


a design task



b strut-and-tie model for load case 1

Figure 9 – Deep beam with opening



a design task

b a possible strut-and-tie model

Figure 10 – The strut-and-tie model or truss model for beams and its elements

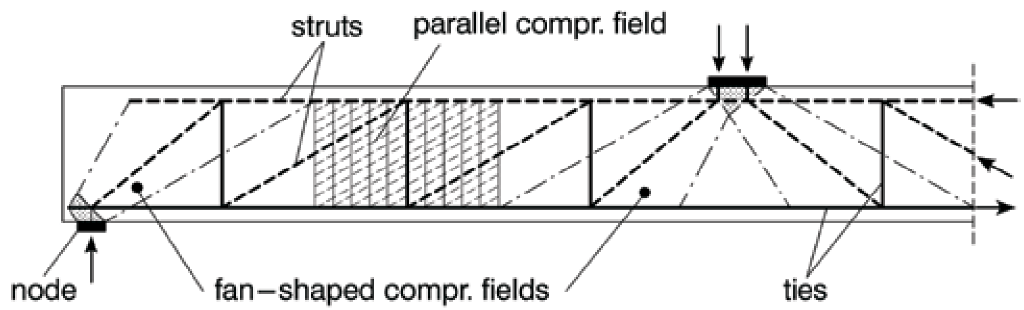


Figure 11 – Strut-and-tie model for an end support and forces

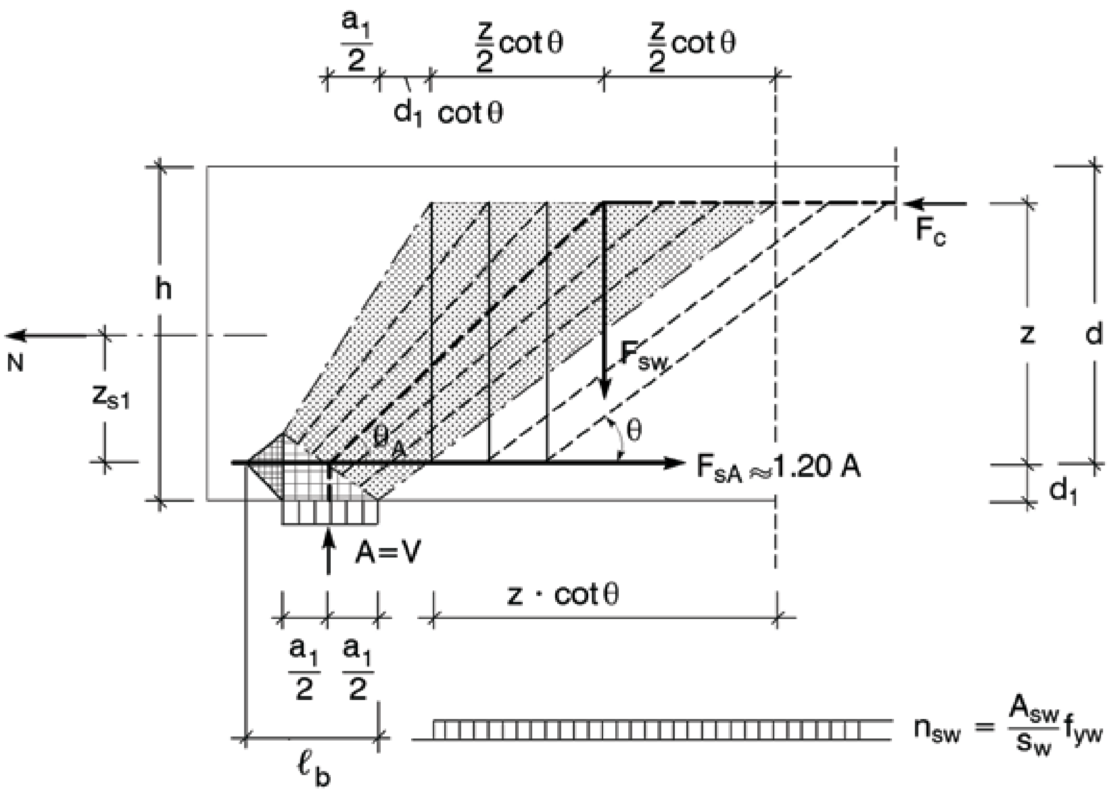


Figure 12 – Frame corner at an end support

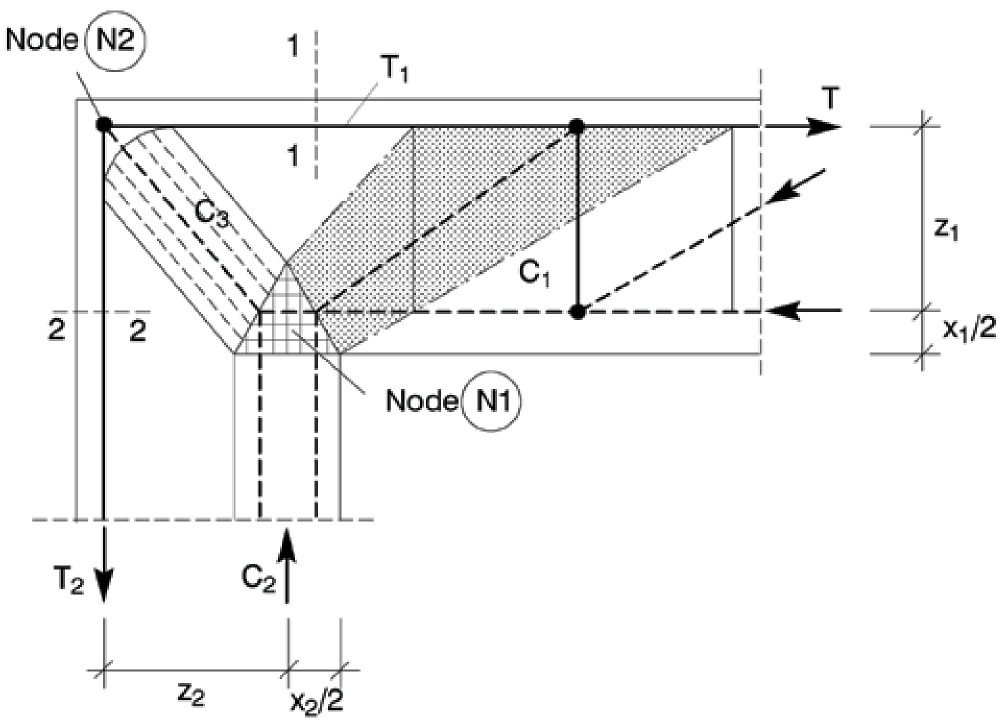
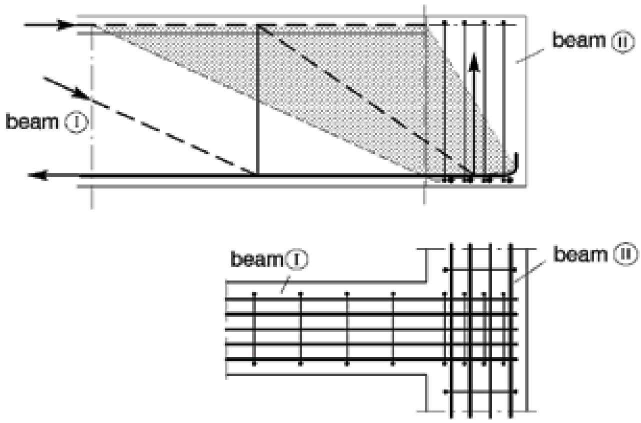
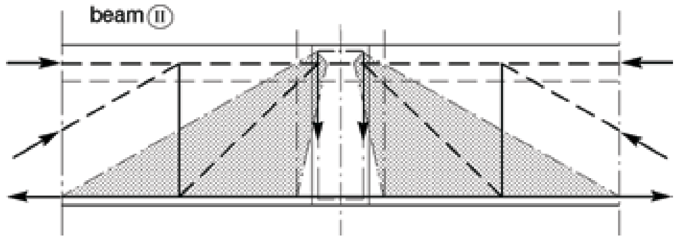


Figure 13 – Strut-and-tie model for an indirectly supported beam



a indirect end – support and arrangement of stirrups



b STM in supporting beam

Figure 14 – Strut-and-tie model for a point load near an end support

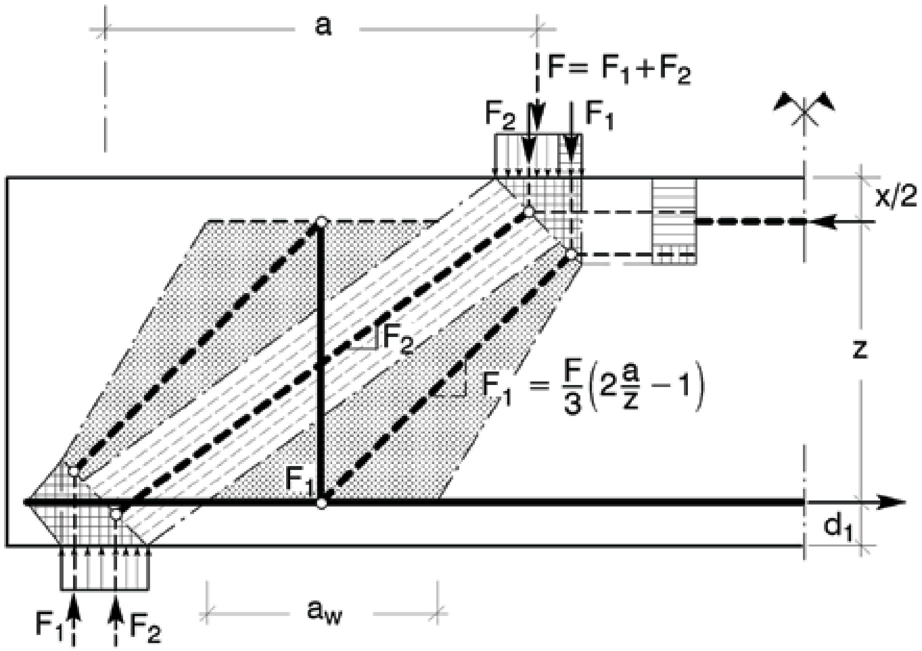
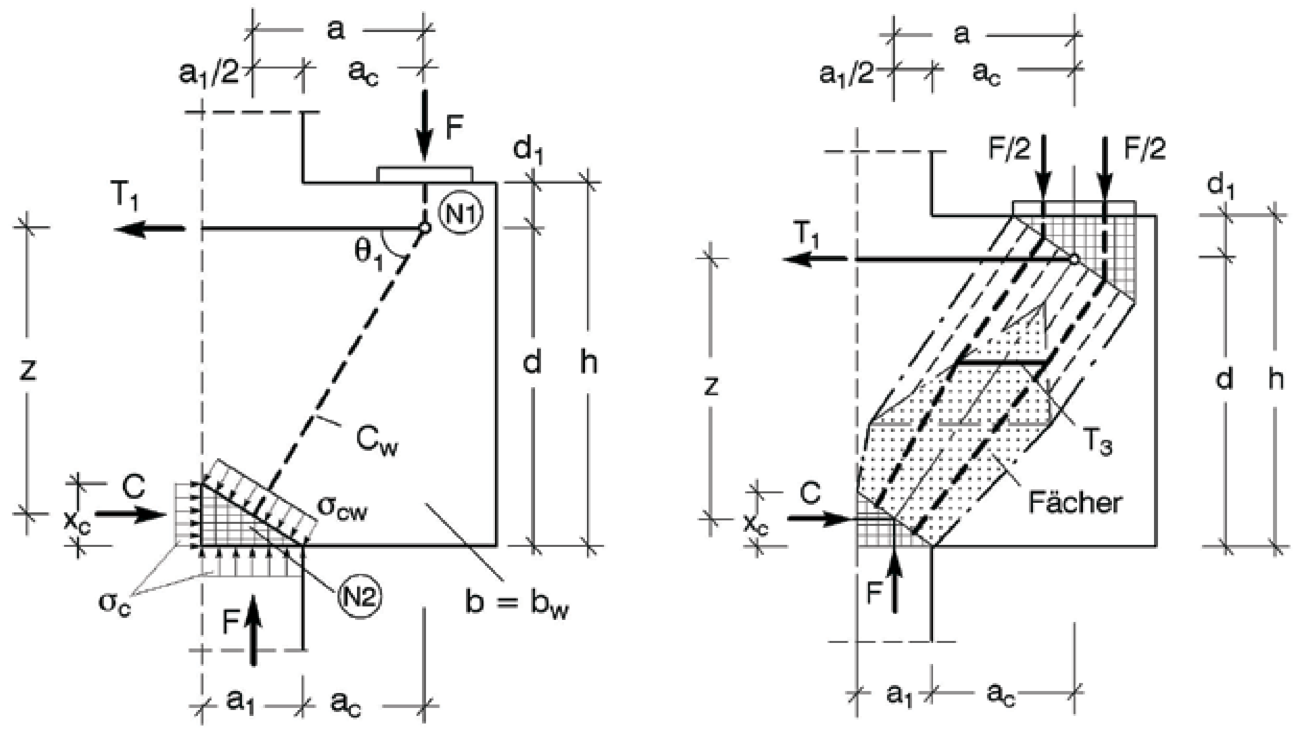


Figure 15 – Design of corbels



A strut-and-tie model

B refined model with stress fields

5 Shear of members with shear reinforcement

As stated before, the forces in the truss model of Fig. 10 can easily be calculated if the inner lever arm z and the angle θ of the inclined struts are known. The inner lever arm follows from the flexural design, and for simplicity it is assumed to be constant in the region with shear force. The angle θ of the inclined struts is determined by the shear design.
The required amount of stirrups follows form the vertical equilibrium of the section parallel to the inclined struts, and the shear force carried by the stirrups is given by equation (1).

$$V_{u,sy} = \frac{A_{sw}}{s_w} f_{yw} \cdot z \cdot \cot\theta = n_{sw} \cdot z \cdot \cot\theta \qquad (1)$$

where: n_{sw} = stirrup forces kN/m
It is convenient for the presentation in dimensioning diagrams to write this Eq. in a dimension free format (equation (2)):

$$\upsilon_u = \frac{V_u}{b_w\, z\, f_{cwu}} = \rho_w \frac{f_{yw}}{f_{cwu}} \cot\theta = \omega_w \cdot \cot\theta \qquad (2)$$

$$\text{where: } \omega_w = \rho \cdot f_{yw} / f_{cwu} \quad \text{and} \quad \rho_w = A_{sw} / (b_w \cdot s_w) \tag{3}$$

The upper limit for the shear force is determined by the strength f_{cwu} of the concrete in the inclined struts:

$$\upsilon_{u,max} = 1 /(\cot\theta + \tan\theta) \tag{4}$$

This relationship is a circle in a ω-υ-diagram like that shown in Fig. 16 presented in a design format. The different lines in the diagram give the relationships for r.c.- and p.c.-members according to the FIP Recommendations (1999).
The Fig. 17 gives the comparison with tests on reinforced concrete beams collected in an extended database by Reineck, Kuchma et al. (2005). The design proposal represents well the general trend of the data and it is safe.

6 Shear capacity of members without shear reinforcement

Since long it has been known that members without shear reinforcement can fail in shear well below the flexural capacity, as demonstrated e.g. by Leonhardt and Walther in their well known test series from 1962 (Fig. 18). The failure is brittle and without almost any warning. In design this is considered by a separate check of the capacity of members without shear reinforcement by defining a limiting design value for the shear force of such members. The different design equations in codes were derived purely empirically and consider the main parameters differently; some consider only the concrete strength (like e.g. ACI 318), whereas most codes consider additionally the depth d (size effect) and the reinforcement ratio (like MC 90, EC2 and DIN 1045-1). Some codes also consider additionally the moment-shear force-ratio or slenderness a/d (like JSCE).
It must be pointed out that members without shear reinforcement occur in practice as slabs. Beams should always be

Figure 16 – Dimensioning diagram for the FIP Recommendations !999 for reinforced and prestressed concrete members or members with axial compression for members with vertical stirrups

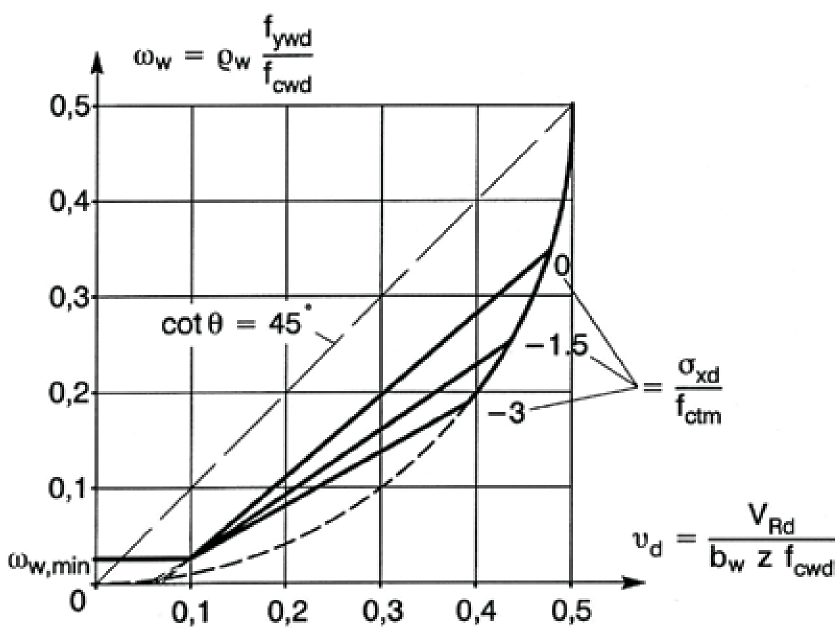


Figure 17 – Comparison of shear tests on reinforced concrete beams with the design proposal of the FIP Recommendations (1999) or DIN 1045-1 (stirrups with plain bars (o), ribbed stirrups (x))

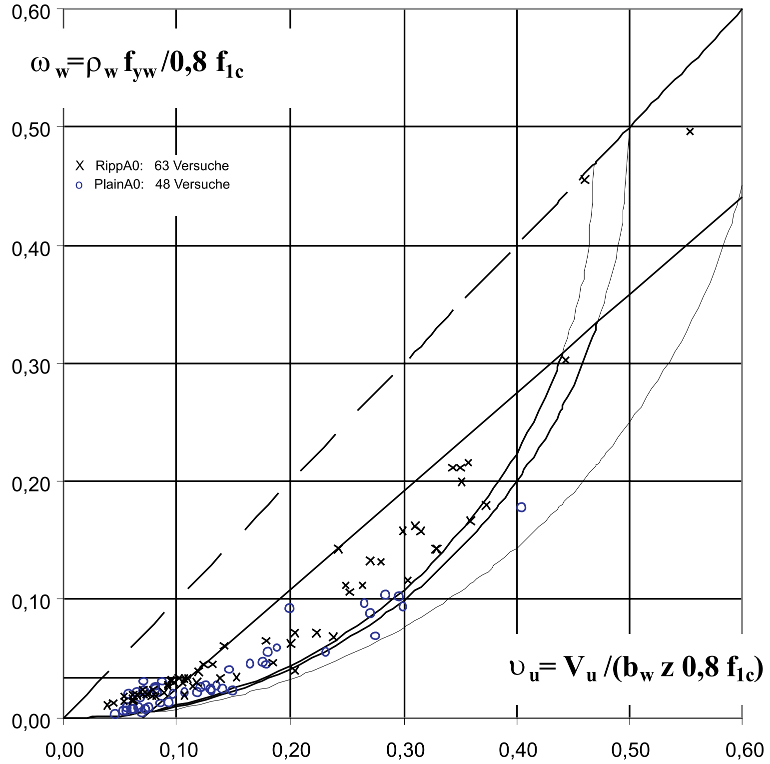
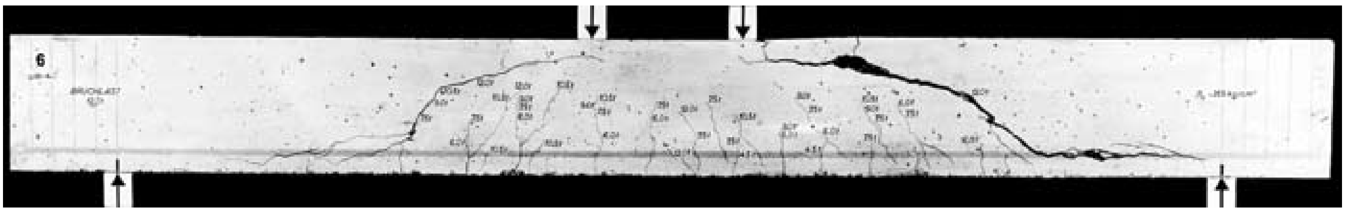


Figure 18 – Foto of beam 6 without stirrups after failure (Leonhardt and Walther (1962))



provided with a minimum amount of stirrups, and most codes contain relevant provisions.

This type of failure cannot be predicted by the theory of plasticity, and this is demonstrated in Fig. 19: the only model for this load case is the direct load transfer to the supports by two inclined struts tied together by the longitudinal reinforcement at the bottom. However, in the load stages preceding failure inclined cracks have developed extending to the compression zone and thus these inclined struts cannot form and develop their capacity. This failure type of members without stirrups does not comply with the basic assumption of the theory of plasticity that the materials are ductile.

The role of all the above mentioned parameters is also known since the 1960´s, but the size effect was ignored for many years in codes, and still is until now in some codes, like ACI 318. The experimental evidence is demonstrated in Fig. 20, where the model safety factor $\gamma_{mod} = V_{u,test}/V_{u,cal}$ is plotted versus the depth d [mm] for Eq.(11-3) of ACI 318. The average value for γ_{mod} as well as the 5%-fractile value decrease considerably with increasing depth. For simplicity the 5%-fractile values listed there were calculated assuming a normal distribution; this is only a rough but safe measure. Therefore, the number of unsafe tests are listed in the following for the first three ranges:

- range A with $d < 200$ mm (8 in): 3<7,8 (5% of 156);
- range B with $200 < d < 300$ mm (12 in): 23>19,15 (5% of 383); lowest $\gamma_{mod} = 0,69$;
- range C with $300 < d < 600$ mm (24 in):13>6,15 (5% of 123); lowest $\gamma_{mod} = 0,60$.

Figure 19 – Crack pattern at failure of test beam OA-2 by Bresler and Scordelis to demonstrate the unsafe application of the theory of plasticity

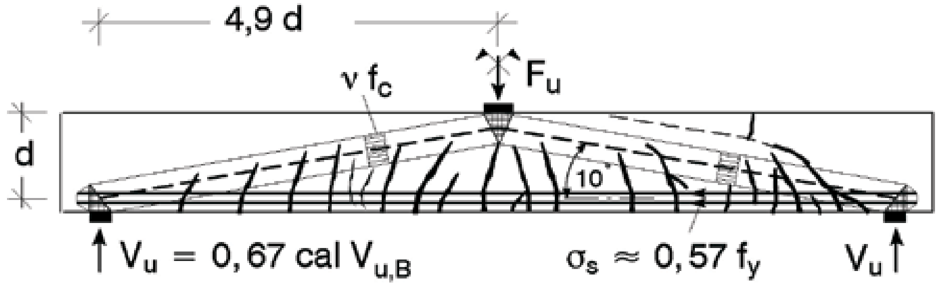
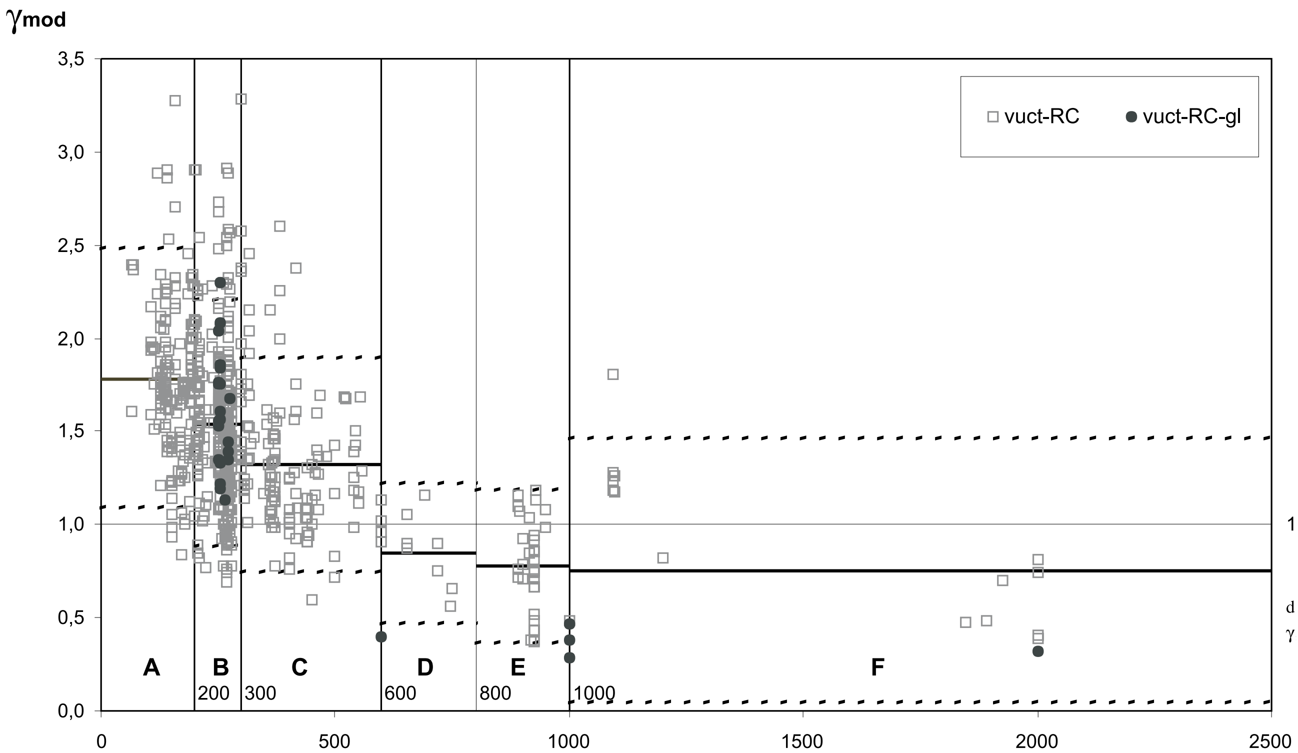


Figure 20 – Model safety factor $\gamma mod = V_{u,test} / V_{u,cal}$ for Eq. (11-3) of ACI 318 plotted versus the effective depth and statistical results for different groups for the new database with 728 tests (□ beams with point loads; • beams with distributed load)



Statistical Value	All Tests	Range A	B	C	D	E	F
n	728	156	383	123	14	32	20
m	1,4864	1,7812	1,5413	1,3189	0,8450	0,7753	0,7513
s	0,4713	0,4225	0,4023	0,3497	0,2296	0,2507	0,4324
v	0,3171	0,2372	0,2610	0,2652	0,2718	0,3233	0,5756
5%	0,7110	1,0862	0,8796	0,7436	0,4672	0,3630	0,0400
95%	2,2617	2,4763	2,2031	1,8941	1,2228	1,1877	1,4627

Figure 21 – Tooth model with the shear force components explaining the shear failure of members without stirrups (Reineck (1991 a,b))

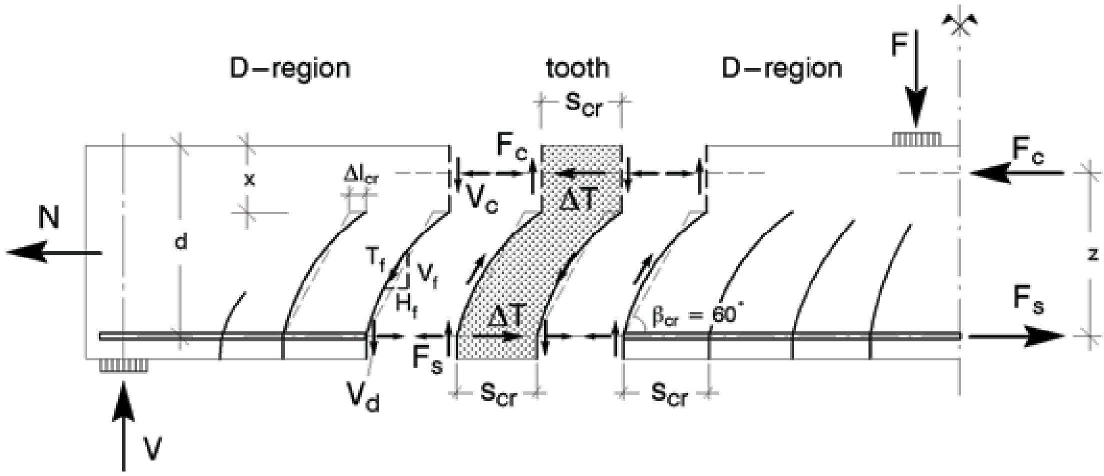
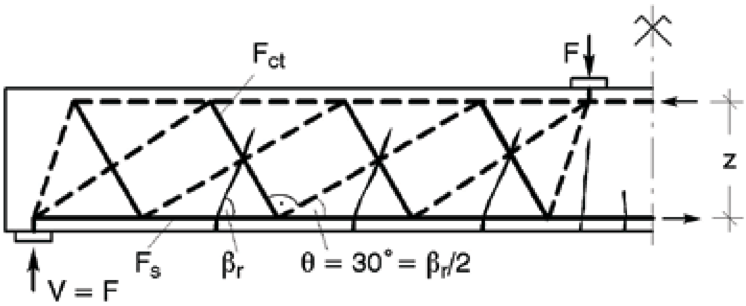


Figure 22 – Truss model with inclined concrete ties for members without stirrups demonstrating the role of the concrete tensile strength (Reineck (1991 b))



It can be concluded that only slabs with depths of roughly $d < 200$ mm (8 in) are safe according to Eq.(11-3) of ACI 318. The ranges B and C and all further increasingly unsafe.
The size effect can be explained by the influence of the crack width in the tooth model (Fig. 21), as further explained by Reineck (1991 a,b). The larger the crack width the less is the capacity of the friction forces along the crack, which are the main component for the transfer of the shear force. Thereby, the role of the reinforcement ratio cannot be ignored, because the crack width increase with decreasing reinforcement ratio. This means that the shear capacity can only be assessed if the strains and the crack spacing are known, so that the crack width of the major cracks can be calculated in the middle of the web.
This tooth model requires refined analyses of the kinematics and assumptions for the constitutive laws, so that as such it is beyond the concept of strut-and-tie models. However, the main load transfer can be described by the truss model shown in Fig. 22, where a biaxial tension-compression field in the web transfer the shear force. The model not only visualizes the flow of forces but it can be used in design and is helpful in practice; e.g. for staggering the longitudinal reinforcement the force in the tension chord can be calculated following truss analogy, and it is given by equation (5).

$$F_s = M/z + 0{,}58 \cdot V \quad (5)$$

The truss model for members without transverse reinforcement is the basic model for investigating the flow of the forces and the shear transfer in slabs. For these members the modeling with concrete tension fields is indispensable. This is especially important for the transition of the B-regions to the D-regions of these members, like e.g. for a slab with dapped beam ends.

7 References

ACI = American Concrete Institute
BuStb = Beton- und Stahlbetonbau
CEB = Comité Eurointernational du Beton (until 1998)
DAfStb = Deutscher Ausschuß für Stahlbeton
fib = Fédération Internationale du Béton (since 1998; merged from CEB and FIP)
FIP = Fédération Internationale de la Précontrainte (until 1998)
IABSE; IVBH = International Association for Bridge and Strutural Engineering
JSCE = Japan Society of Civil Engineering
PCI = Prestressed Concrete Institute

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