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Nonlinear Probabilistic Analysis of Precast Prestressed Girders Made Continuous

Análise Probabilística Não-Linear de Vigas Contínuas Pré-Fabricadas e Pré-Esforçadas


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Abstract

This article analyzes the behavior in service of bridge decks which are built with precast beams, connected at supports through ordinary reinforcement bars, without continuity prestress cables. In this type of structure, there is significant change in stress over time. This is mainly due to delayed concrete deformation, resulting from creep and shrinkage. As creep and shrinkage are two of the properties of concrete that show the greatest variation, the structural response is likely to show significant variability. The main objective of the study is to assess the range of variation in structural response of this type of structure. In particular, it seeks to assess the importance of this variability in terms of structural performance during service. In order to achieve this, a probabilistic analysis is carried out, based on a Monte Carlo simulation.

Keywords: prestressed concrete bridges; non-linear analyses; Monte Carlo simulation.

Resumo

Neste artigo é tratada a análise do comportamento em serviço de tabuleiros construídos com vigas pré-fabricadas, ligadas na zona dos apoios com armaduras ordinárias, isto é, sem pré-esforço de continuidade. Nestas obras há uma evolução temporal significativa dos esforços e tensões instalados, evolução essa que é motivada essencialmente pelas deformações diferidas do betão, por fluência e retracção. Como a fluência e a retracção são das propriedades do betão com maior variabilidade, a resposta estrutural terá também uma variabilidade significativa. O objectivo principal deste trabalho consiste na avaliação da variabilidade da resposta estrutural deste tipo de obras, e da importância dessa variabilidade sobre o desempenho da estrutura durante a fase de serviço. Para atingir tal objectivo, é realizada uma análise probabilística, através de uma simulação de Monte Carlo.

Palavras-chave: pontes em betão pré-esforçado; análise não-linear; simulação de Monte Carlo

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1 Introduction

This analysis uses numerical methods to evaluate the behavior in service of continuous bridge decks constructed with precast beams, and to assess the variability in structural response. The beams referred to are approximately equal in length to each bridge span, and connected at the supports without continuity prestress cables.

For the numerical analysis of the behavior of this type of structure, the correct modeling of concrete behavior is important in order to be able to realistically translate the behavior of tensioned concrete and delayed deformations from creep and shrinkage. Another important aspect is the variability of material properties and environmental factors, such as average temperature and relative humidity. This is important because stress and strain vary over time, and are highly dependent on delayed deformations (creep and shrinkage), because of the sequence in which the structure is constructed. Because quantifying creep and shrinkage is more approximate than quantifying other properties of concrete, the structural response is also likely to show significant variability.

Monitoring the behavior of reinforced or prestressed concrete structures, under serviceability limit state, usually involves deterministic calculations. These are based on the average values of several parameters, relating to load as well as strength. This procedure is adequate for structures that are not sensitive to the variation of these parameters. However, for structures that are sensitive to variability in creep and shrinkage, a deterministic analysis may be inadequate, once the structural response moves away from the average values. Various authors [1-4] have highlighted the advantages of using a probabilistic approach to describe the properties of concrete, in studies of the delayed behavior of concrete bridges. Some authors [5-6] have studied the influence of variation in deformation from creep and shrinkage, in the type of structure dealt with here. They used parametric analyses, where different relevant parameters gave a range of different values. This type of analysis, however, does not quantify the probability associated with the occurrence of each of the results obtained.

This study uses the Monte Carlo method to calculate structural response in service, to describe not only average values, but also variability in structural response. This results in a characteristic value (for example, corresponding to the 5% fractile) for variables such as:

- the bending moment at the joining section, at different points in time;
- stress on the extreme fibers of the most critical sections;
- crack opening in the most critical sections.

This type of analysis illustrates whether variability in structural response is important in this type of structure, and consequently, whether it should be taken into consideration. It also evaluates whether performance in service is satisfactory, in view of the structural response. Structural performance can be considered satisfactory, if it can be guaranteed that tension stresses around the prestressing tendons (that is, decompression) will not occur, and if crack width remains within limit values. This evaluation would need to consider not only average values but also characteristic values of the structural response.

This study does not analyze behavior in ultimate limit state. This is because variability in creep and shrinkage has less importance in ultimate limit state, as stresses within the structure can be redistributed.

2 Analytical modeling

2.1 Concrete under Tensile Stress

Previous studies of this type of structure have illustrated the importance of considering the tension stiffening effect, for example, the experimental study carried out by Miller et al [7]. This effect needs to be taken into account in order to correctly predict the stiffness of tensioned reinforced concrete elements after cracking, through numerical modeling. The force (N) -average strain (ε_m) relationship, of a reinforced concrete tie in tension is represented by the graphic in Figure 1a. This graphic shows a reduction of the axial force after cracking, which is controlled by the dissipation of fracture energy in the concrete. It shows the average stress retained by the concrete to be equal to 40% of its tensile strength, f_{ct}, during the stabilized cracking stage (as shown in MC90 [8]). It also shows the plastification of the tie, following reinforcement yielding in the cracked section.

The effect of tension stiffening may be translated in numerical models through a modification of the stress (σ_c) -average strain (ε_m) relationship, applied to the concrete surrounding the tension reinforcement. The value of the stress retained by concrete is given in the ratio between the tensile force retained by the concrete, ΔF_c, and the area of the tie cross-section, A_c, resulting in the diagram in Figure 1b.

In Figure 1, ε_r represents the tie cracking strain and ε_y represents the yielding strain in the reinforcement.

2.2 Delayed Concrete Deformation

Deformations from creep and shrinkage are quantified in this study using FIB model [9], which was adopted by EC2 [10]. This model includes improvements, compared to the model previously proposed in MC90 [8]. The following alterations need to be highlighted: the explicit inclusion of a term relating to autogenous shrinkage; the inclusion in the creep model of parameters that allow a better estimate of high-performance concrete behavior. The change over time in the modulus of elasticity of concrete is also determined following FIB model [9].

2.3 Structural Discretization and Numerical Implementation

Structural analysis is carried out using the DIANA software. Structural discretization is carried out using beam finite elements, in accordance with the Mindlin theory, which are numerically integrated along the beam axis and in its cross-section (Figure 2). In order to model the behavior of concrete, a strain decomposition model is used, which allows for creep, shrinkage and cracking effects all to be handled simultaneously. This model represents the association in series, of concrete and cracks, where concrete has linear visco-elastic behavior, as represented in Figure 2. The following relationship holds in each point of the model:

$$\varepsilon = \varepsilon_e + \varepsilon_{cr}$$

where ε represents the total strain, ε_e represents the elastic (or visco-elastic) strain of concrete between cracks, and ε_{cr} represents the strain relative to the opening of cracks. In this way, the value of crack width, w_k, at each point of the structure, may be computed through the multiplication of ε_{cr} (result of the numerical analysis) by the maximum distance between cracks, $l_{s,max}$ (quantified in accordance with EC2 [10]). A description of this constitutive model may be found in the bibliography [11-12].
The reinforcement, both ordinary and prestressed, was modeled using embedded reinforcement elements. The deformation of reinforcement elements is calculated from the displacement field of the concrete finite element, in which they are embedded. The stress-strain relationship of the reinforcement element is modeled using a bilinear diagram, which represents an elastic-perfectly plastic behavior.
Constructive sequence is studied using a phased analysis, where the finite element model is changed at every new stage, by including new elements, adding new parts to the cross-section and modifying the connection to the supports.

2.4 Verification

In order to validate the procedure used in the analysis, a comparison was made between the results obtained from the numerical model and the experimental results obtained by Mattock [13]. In the laboratories of the Portland Cement Association, Mattock tested two continuous beams, with a scale of 1:2, each composed of two precast girders linked in continuity at the central support. The results are presented in the reference Sousa [14]. This comparison validated the analysis procedure, as there was strong agreement between experimental and numerical results. Where this was not the case, the differences could be explained.

3 Probabilistic analysis

3.1 Probabilistic Analysis Methodology

The Monte Carlo simulation of structural behavior is carried out in 3 stages:
1) simulation of the random basic variables, using their probability distributions to generate sets of random values;
2) structural analysis, used to determine the structural response for each of these sets of values;
3) statistical analysis of the sample, composed of the set of observations for each output variable (force, stress and strain), in order to estimate the probability distribution of each output variable (result).
The first stage was based on the Latin Hypercube method. This stratified sampling method can achieve a reduction in the number of observations necessary to estimate the statistical parameters that characterize structural response. It can at the same time maintain a similar level of precision compared with conventional methods, as shown by Bazant and Liu [15] and Florian [16]. This procedure has been used by several authors [2,15,17].
The second stage, structural analysis, used successive deterministic calculations, which were based on the non-linear behavior models mentioned earlier.
In the third stage, simulation results are assumed to represent a random sample of the complete set of possible responses. Based on that assumption, for each output variable (for example, stress, displacement or crack width at a given point in time) an estimate of its mean and variability can be determined. If an output variable is shown to conform to a Normal distribution (using the Normality test or the Kolmogorov-Smirnov test, see Aivazian et al [18]), and the number of simulations is large enough for the sample to be considered representative, then the characteristic values C_k and C_{1-k}, associated to the k and 1-k fractiles of the variable's statistical distribution, can be calculated as follows:

$$C_k = \mu - f \cdot \sigma$$
$$C_{1-k} = \mu + f \cdot \sigma$$

where μ and σ are the mean and standard deviation of the

sample. The 5% and 95% characteristic values are obtained for f=1.645 (see Montgomery and Runger [19]).
If an output variable is shown not to conform to a Normal distribution, characteristic structural response values are estimated using its sample's cumulative frequency curve.
In this type of analysis it is important to know the number of simulations required to adequately estimate the probability of a limit state being exceeded. Some authors [20] suggest values between 1/P and 10/P, where P is the probability of exceeding the limit state. So, for ultimate limit state analyses with a probability of failure around 10^{-4}, somewhere between 10000 and 100000 runs would be necessary. For serviceability limit state analyses and a probability of around 0.05, the number of simulation runs should be between 20 and 200. Other authors, such as Bazant and Liu [15], suggest that a number of simulation runs equal to twice the number of random basic variables are enough for analyses of time delayed effects under service conditions, in structures where cracking is not likely to occur, or in analyses where cracking is ignored.

3.2 Random Basic Variables

Another important issue in the Monte Carlo simulation is the identification of the critical parameters which should be considered random basic variables, and how to determine their statistical properties.
One specific aim of this work is to assess the importance of the variability of the parameters which affect concrete deformation due to creep and shrinkage. The following parameters are considered random basic variables:
- relative humidity, ;
- average temperature, T;
- concrete compressive strength, f_{cm};
- Ψ^F parameter, which represents the uncertainty in the creep model;
- Ψ^R parameter, which represents the uncertainty in the shrinkage model;
- prestress force.

The first three parameters are linked to the external uncertainty involved in determining delayed concrete deformations. The other two parameters represent internal uncertainty and errors in the theoretical predictive models, due to the variability inherent in predicting concrete deformation due to creep and shrinkage. That variability is taken into account by introducing the Ψ^F and Ψ^R parameters in the theoretical formulae used to determine creep and shrinkage, respectively. The creep coefficient, $\phi(t,t_0)$, thus becomes:

$$\phi(t,t_0)=\Psi^F\cdot\phi_0\cdot\beta_c(t,t_0)$$

where ϕ_0 is the notional creep coefficient and $\beta_c(t,t_0)$ is a coefficient to describe the development of creep with time after loading. Shrinkage strain, $\varepsilon_{cs}(t)$, is then determined as follows:

$$\varepsilon_{cs}(t)=\Psi^R\left[\varepsilon_{cas}(t)+\varepsilon_{cds}(t,t_s)\right]$$

where $\varepsilon_{cas}(t)$ and $\varepsilon_{cds}(t,t_s)$ represent, respectively, autogenous shrinkage and drying shrinkage. Parameters Ψ^F and Ψ^R have a mean value of 1 and are time-independent.
In terms of the spatial variability of parameters, it is assumed that average temperature and relative humidity are the same for every element of concrete. With regard to concrete compressive strength, creep and shrinkage, two different concrete elements are considered: the precast beam and the slab-diaphragm element. The decision to treat these two concrete elements independently of each other is because they are built in two separate casting operations.
All random variables are assumed to follow a Normal distribution, and are characterized by their mean and coefficient of variation. It is also assumed that all random variables are independent. Table 1 summarizes the mean and coefficient of variation adopted for each variable. RH, T, f_{cm} and prestress mean values were taken from one practical design situation. The coefficient of variation of each basic variable was obtained from the literature.
It is assumed that the geometric properties of the structure, the properties of steel and the variable actions (traffic loads and temperature variation) are deterministic and known.

3.3 Dependent Random Variables

A number of parameters are computed at each run, as a function of the random basic variables:
- the creep coefficient is a function of Ψ^F, RH, T and f_{cm};
- shrinkage strain is a function of Ψ^R, RH, T and f_{cm};
- the concrete modulus of elasticity, E_{ci}, for loading at the age of 28 days, is computed as a function of f_{cm}, as follows [8]:

$$E_{ci}=2.15\cdot10^4\left(f_{cm}/10\right)^{1/3}$$ (E_{ci} and f_{cm} in MPa);

- concrete average tensile strength, f_{ct}, is computed as a function of f_{cm}, as follows [8]:

$$f_{ct}=1.40\left[\left(f_{cm}-8\right)/10\right]^{2/3}$$ (f_{ct} and f_{cm} in MPa).

4 Case studies

4.1 Description of the Case Study Structures

The probabilistic analysis methodology described earlier was applied to three bridge decks, each with two continuous spans of 20, 25 and 30 m respectively. Each bridge deck is constituted of five beams, spaced 2.25 m apart. The slab has a thickness of 0.22 m. The beams are prestressed, with strands of 0.6 in (1.52 cm) diameter and a cross section area equal to 1.5 cm^2. Negative moment reinforcement is constituted of straight bars embedded in the slab. Positive moment reinforcement is constituted of reinforcing bars with hook ends, embedded in the ends of precast girders, as represented in Figure 3. The beams are initially mounted on temporary supports, as represented in Figures 3 and 4. These supports are removed after the slab and diaphragm concrete have hardened and the diaphragm then becomes supported by a single bearing pad. The existence of a "tooth" on top of the precast beams improves the shear strength of the join between the concrete of the precast beams and of the diaphragm.
Figures 4, 5 and 6, and Table 2 present the geometry and reinforcement layout of the three bridge decks under analysis. Table 3 presents the mechanical properties of the steel. The preliminary design for the amount of prestressing steel to be used did not take into account the continuity effect; it was determined from a deterministic analysis of a simple span beam behavior. This procedure has been adopted by various authors, as evident from the analysis of the results of a survey, presented by Hastak et al [21].

4.2 Actions and Constructive Sequence

This study shows the results for the beam closest to the edge of the deck, because it is the one subject to the highest loads. The following actions were considered to be acting on the structure:
- individual weight of the structural members;
- other permanent gravity loads such as asphalt coating and side walks (7.87 kN/m applied on the edge beam);
- differential temperature variation (top fiber at a higher temperature than the bottom fiber) quantified in accordance with EC1 [22];
- traffic loads, composed of type I vehicles as defined by Portuguese standards [23] (Figure 7).

Using a 3D finite element model, it was shown that for the case studies, traffic loads could be simulated using three concentrated 100 kN forces, with a 1.5 m spacing between them, applied at mid-span (see Sousa [14]).
Figure 8 shows the constructive sequence and the sequence according to which the differential temperature variation and traffic loads were applied. The prestress is transferred to the precast beams 3.5 days after the casting operation. The slab and diaphragm are cast 30 days later. It is assumed that these elements start performing a structural function one day after the casting operation. At that time, temporary supports are removed from the numerical model and the continuous beam becomes supported on the permanent bearing pads. As shown in Figure 8, the effect of variable actions was considered at two different points in time:
- immediately after the end of the construction stage, because the effect of these actions may cause cracking, which influences the development of stresses over time;
- 20000 days after construction, in order to assess the long term effect of variable actions.

4.3 Results: Bending Moment at the Supports

The Monte Carlo simulation undertaken provides an estimate of the statistical distribution of the resulting forces, at each point in time. Figure 9 shows the change over time of the bending moment at the supports of the 25 m span structure. The average and characteristic values are presented, corresponding to the fractiles of 5% and of 95%. In this Figure the results of a visco-elastic linear analysis (without consideration of cracking effects) are also presented. The results are widely scattered, which is shown by the fact that the 5% and 95% fractiles are far apart. This Figure also shows that the variability of the structural response increases over time and is dampened by the effects of cracking.
Figure 10 shows the statistical distribution of the bending moment at the supports, due to the long-term effects of differential temperature variation and traffic loads, for a 25 m span structure. A deterministic calculation, assuming that concrete has linear elastic behavior, would give rise to the following bending moments:
- +655 kNm, under the effect of differential temperature variation;
- -685 kNm, under the effect of traffic loads.

The histogram in figure 10a which describes temperature effects, shows a significant dispersion of observations. It is shown that the bending moment induced by temperature variation can be very different from that which is obtained from a linear elastic analysis. This is because, since this type of load represents a deformation imposed on the structure, the resulting stresses depend significantly on whether or not cracking takes place.
Figure 10b illustrates the effect of traffic loads, and shows a significant number of occurrences of bending moment val-

ues similar to the values resulting from a linear elastic analysis. The mean value of bending moment due to traffic loads represents a 88% redistribution, with respect to the bending moment that would result from a linear elastic deterministic analysis.

4.4 Results: Verification of Serviceability Limit State.

According to European standards [8,10], checking structural performance at serviceability limit state requires:
- verification of the decompression limit around prestressing tendons;
- verification of the crack width limit of 0.3 mm in sections without prestressing tendons.

In places where corrosion of reinforcement may take place as a result of concrete carbonation, such verifications should be carried out under the following quasi-permanent load combination:

$$G + 0.2 \cdot Q + 0.3 \cdot T$$

where G represents permanent loads, Q represents traffic loads and T represents temperature variation. Q and T, should not be considered when they produce a favorable effect. As may be seen in Figure 8, the loading sequence under analysis includes quasi-permanent load combinations at the end. This analysis has shown that the lower fiber at the midspan section is representative of decompression problems for the whole span, because it is at that point, around the prestress strands, that the lowest tensile stresses occur. Likewise, it is at the lower and upper fibers of the central support section that the greatest crack widths occur. Therefore only the results for these two sections are presented.
Table 4 shows the most significant results of the deterministic calculation of the case studies.
Table 5 summarizes most relevant results of the probabilistic analysis. It presents the probability of decompression at the lower fiber in the midspan section, that is, the occurrence of tensile stresses at that fiber. It also presents the probability of the existence of crack widths, w_k, greater than 0.2 mm and 0.3 mm, at the central support section. Figure 11 shows, for the structure with spans of 25 m, how the estimates evolved as the number of simulation runs increased. Having concluded that 150 runs are enough to obtain good estimates, this number was adopted for other simulations.
Table 5 shows that, for the amount of prestress adopted, the probability of exceeding decompression limit state at the midspan section is 0.15, 0.10 and 0.03 for structures with spans of 20, 25 and 30 m, respectively. It should be noted that in the design process, this limit state is usually checked using a deterministic analysis, resulting in $P(\sigma>0) = 0.50 = 50\%$. The probability of exceeding this limit state was relatively low for all the case studies in this analysis (a probability of 0.05 means that the limit state is satisfied, based on the characteristic values associated to the 5% or 95% fractiles). Table 5 also shows that for quasi-permanent load combinations, the probability of crack widths w_k greater than 0.3 mm occurring at the supports, is 0.06 at most. It can thus be concluded that this limit state is satisfied, even for characteristic values corresponding to the 6% fractile.
This analysis has already concluded that the structures in question perform well in service, even in terms of characteristic values (corresponding to the 5% or 95% fractiles, or similar). It can therefore be concluded with greater confidence than would result from conventional structural design methods, that this type of structure (without continuity prestress between beams) performs well during service. However, this study was limited to structures with two equal spans, beam spacing of 2.25 m, and deck slenderness ratio (ratio of span length to deck height) of approximately 18. This means that the conclusions cannot be directly applied to other structural configurations.
Another conclusion can be drawn in relation to prestress design criteria for serviceability limit state, if the continuity effect is disregarded. Authors such as Clark and Sugie [5] have highlighted that due to structural response variability, rigorous viscoelastic analyses that are deterministic, would not be justifiable on this type of structure. Instead, simplified design methods should be used, which take into account the effects of structural response variability. However, simple robust tools for this type of analysis do not appear to have been developed to date. Prestress was quantified in this study using a deterministic analysis, and disregarding the continuity effect between spans, a criterion that was adopted in various states in the USA, see Hastak et al [21]. Having shown that this type of analysis leads to good structural performance in service for the case studies, it is reasonable to conclude that these simplified design criteria can adequately take account of response variability, in this type of structure. However, further research is necessary in this field. It would be particularly interesting to widen these analyses to other structural configurations.

5 Conclusions

This study has lead to the following conclusions:
1. Structural response variability in continuous bridge decks built with precast beams is significant. It should thus be taken into account.
2. The variability in change over time of the bending mo-

ment (at the support section) is reduced by the effects of cracking.
3. The structures analyzed with spans of 20, 25 and 30 m showed satisfactory performance in service, even when considering the structural response values associated to the 5% and 95% fractiles.
4. The design of prestress, disregarding continuity effects, appears to be a good simplified criterion for analyzing variability in structural response in this type of structure. However, further research is necessary in this area.
5. The method described in this paper could potentially be applied to the analysis of structures that are sensitive to cracking and to variability in creep and shrinkage.
6. Further research is necessary, to be able to conclude unequivocally whether a probabilistic approach is essential for the design of this type of structure in service.

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Table 1 – Mean and coefficient of variation for basic random variables

Basic random variable	Mean	Coefficient of variation (%)
Relative humidity, RH	70%	15
Mean temperature, T	15°C	15
Concrete compressive strength, f_{cm} (beam)	53 MPa	9.18
Concrete compressive strength, f_{cm} (slab and diaphragm)	38 MPa	12.80
Creep model uncertainty, ΨF, (beam)	1	30
Creep model uncertainty, ΨF, (slab and diaphragm)	1	30
Shrinkage model uncertainty, ΨR, (beam)	1	35
Shrinkage model uncertainty, ΨR, (slab and diaphragm)	1	35
Initial prestress	1372 MPa	5

Table 2 – Geometry and reinforcement of precast girders

L (m)	20	25	30
H1 (m)	0.130	0.150	0.170
H2 (m)	0.085	0.085	0.085
H3 (m)	0.415	0.730	0.960
H4 (m)	0.110	0.095	0.095
H5 (m)	0.160	0.140	0.140
HV (m)	0.900	1.200	1.450
BI (m)	0.630	0.630	0.630
BW (m)	0.140	0.140	0.180
BS (m)	0.650	1.000	1.000
As,sup (cm^2/m)	14.0	15.7	18.5
As,inf (cm^2)	17.3	20.4	23.6
n	14	17	22

Table 3 Mechanical properties of steel

Modulus of elasticity of steel, Es (GPa)	200
Yield strength of mild steel reinforcement, fsy (MPa)	500
Yield strength of prestressing strands, fpy (MPa)	1674

Table 4 – Summary of most significant results of the deterministic calculation

	Span length (m)		20	25	30
Midspan section	σc (MPa)		-2.21	-2.41	-3.10
Center support section	Top fiber	w_k(mm)	0.01	0.01	0.12
	Bottom fiber	w_k (mm)	---	---	---

Table 5 – Summary of most significant results of the Monte Carlo simulation

	Span length (m)		20	25	30
Midspan section	P(σc>0)		0.15	0.10	0.03
Center support section	Top fiber	P(w_k) >0.2 mm	0.32	0.27	0.25
		P(w_k) > 0.3 mm	0.05	0.03	0.01
	Bottom fiber	P(w_k) > 0.2 mm	0.16	0.06	0.05
		P(w_k) > 0.3 mm	0.06	0.05	0.03

Figure 1 – Force-average strain relationship and concrete stress-average strain relationship for a reinforced concrete tie.

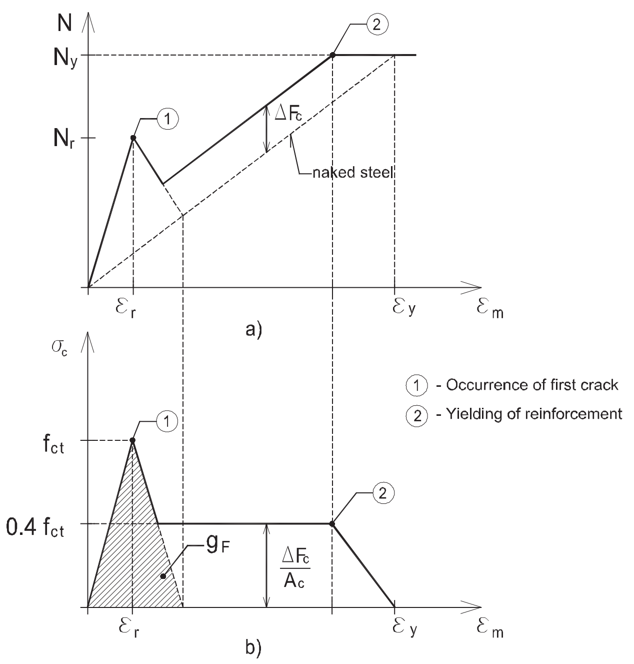


Figure 2 – Structural discretization and a diagram of the constitutive model for concrete

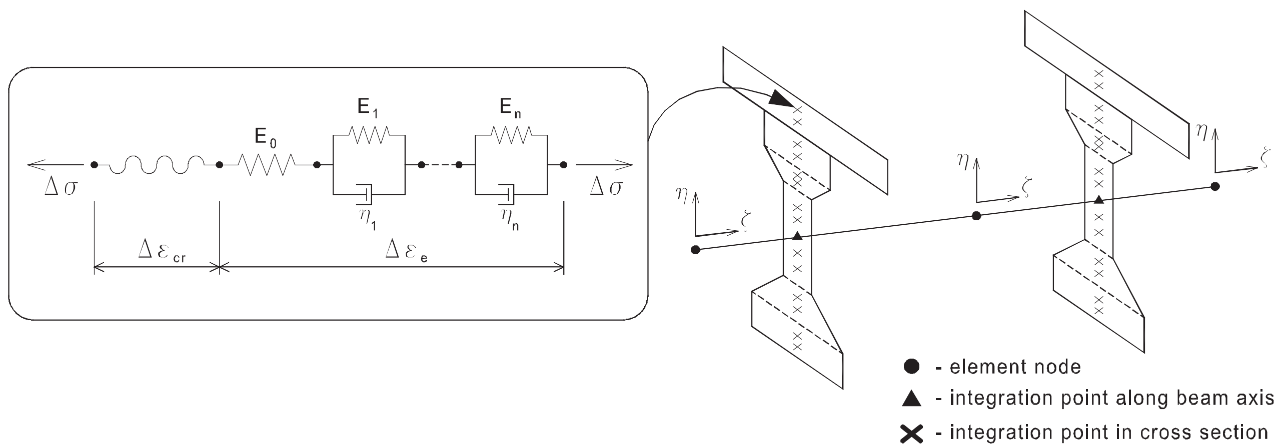


Figure 3 – Continuity connection detail

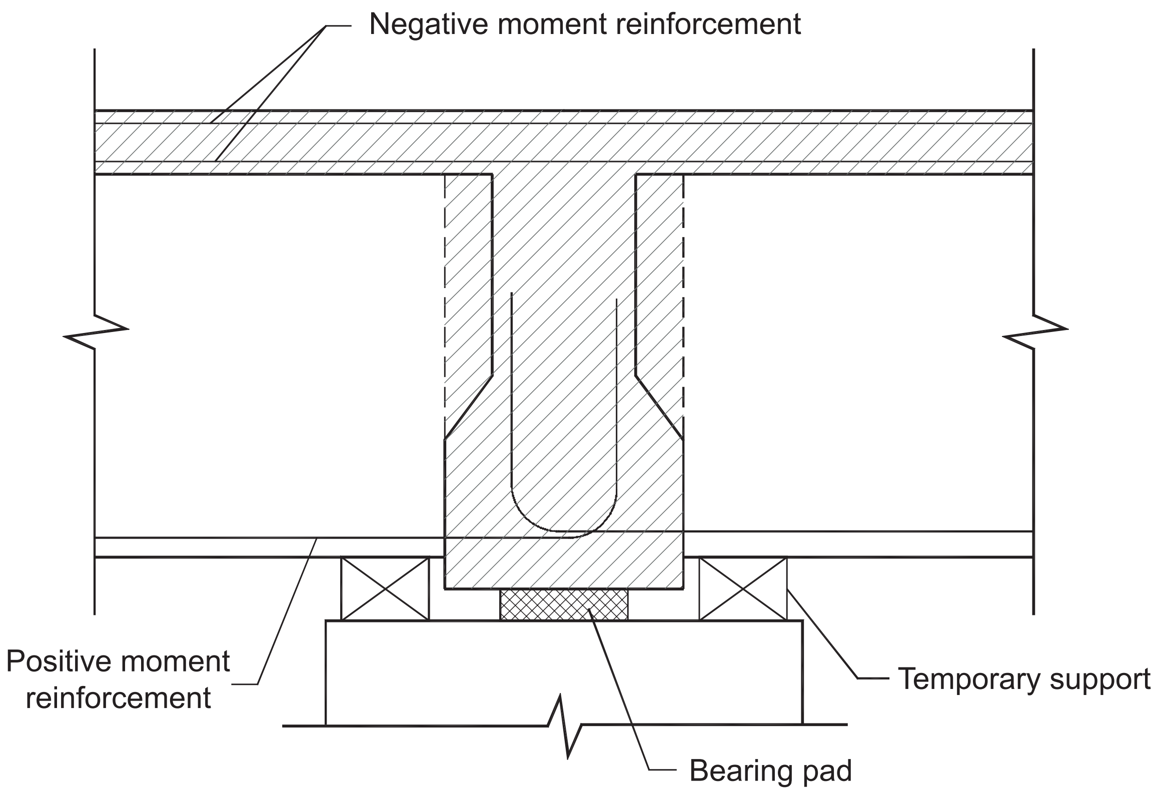
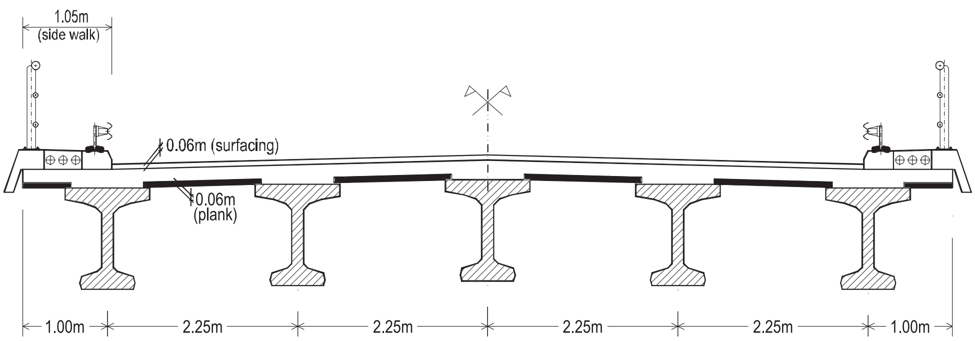


Figure 4 – Bridge decks under analysis: cross and longitudinal sections



L (m)	H (m)
20	0.90
25	1.20
30	1.45

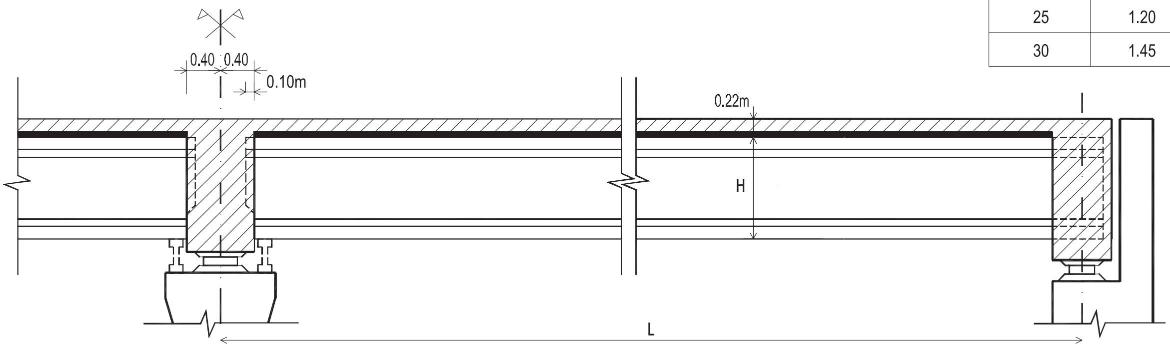


Figure 5 – Cross section of precast girders

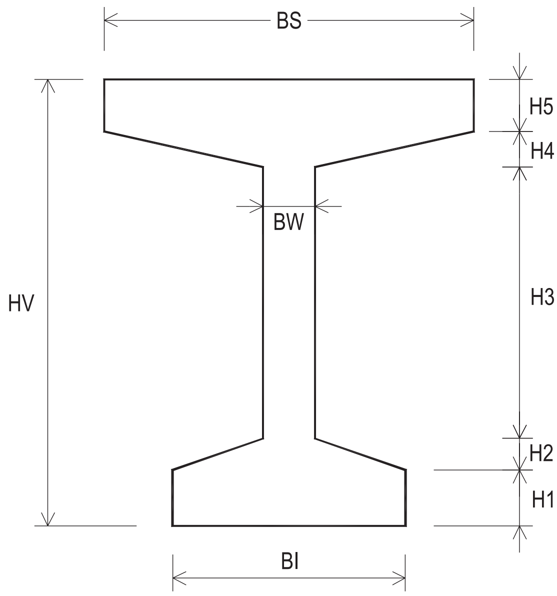


Figure 6 – Longitudinal reinforcement layout: a) midspan section; b) center support section

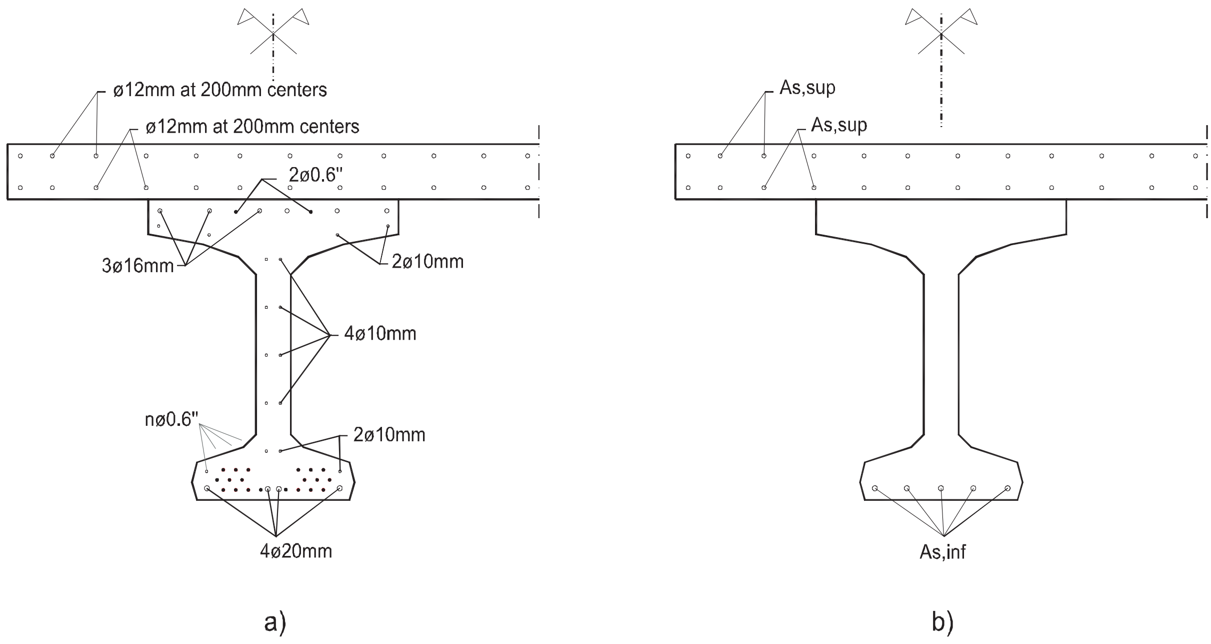


Figure 7 Type I vehicles as defined by Portuguese standards (23)

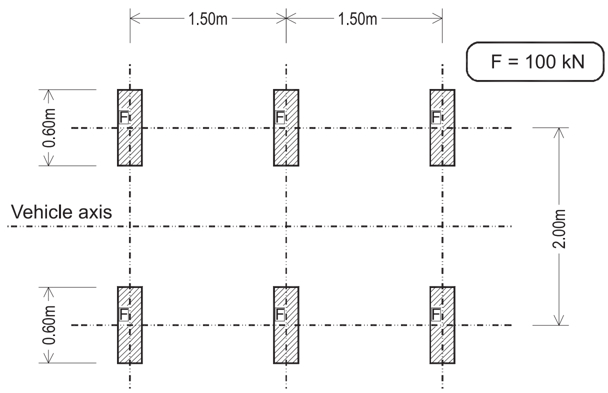


Figure 8 – Constructive sequence and schedule for variable action application

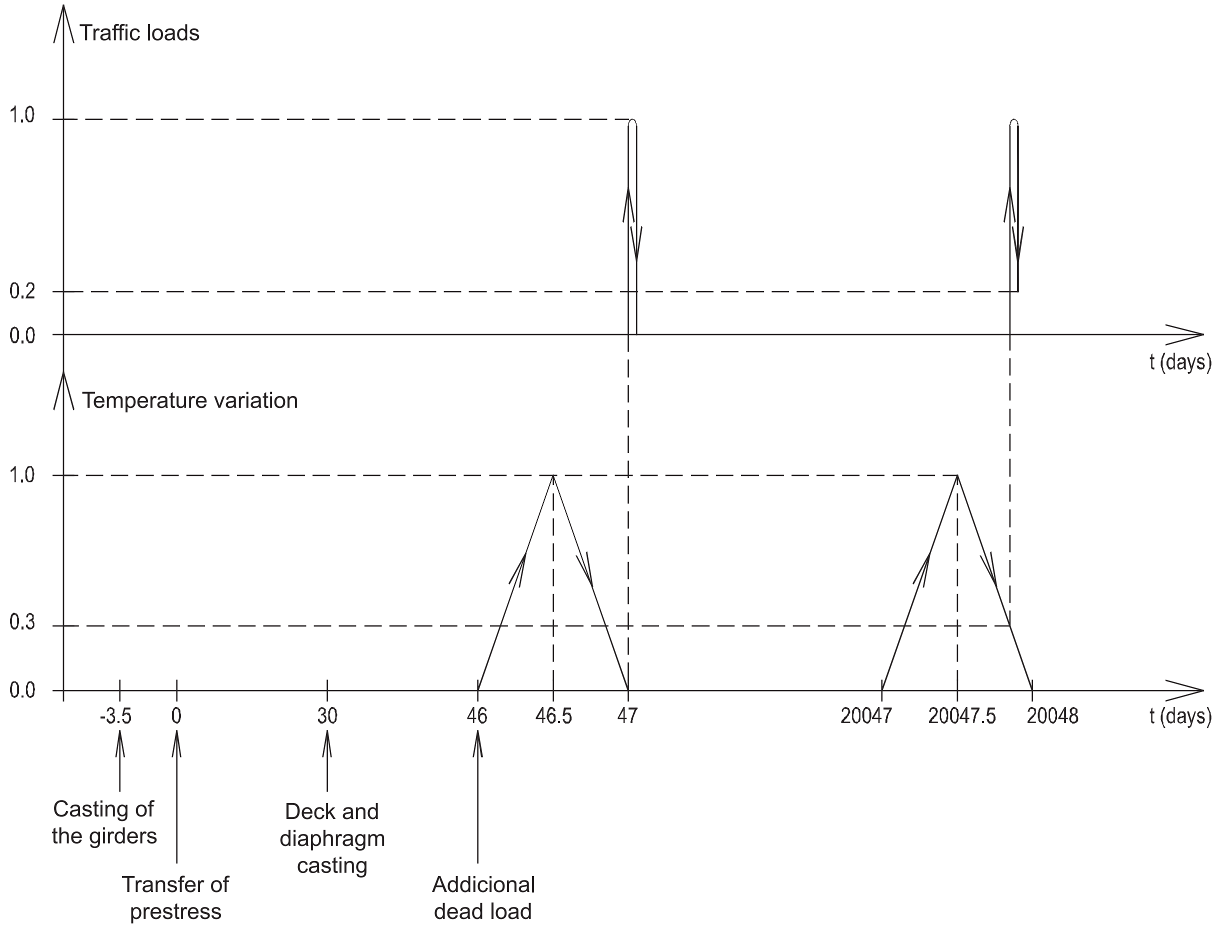


Figure 9 – Time-dependent variation of the center support moment of the 25 m span structure

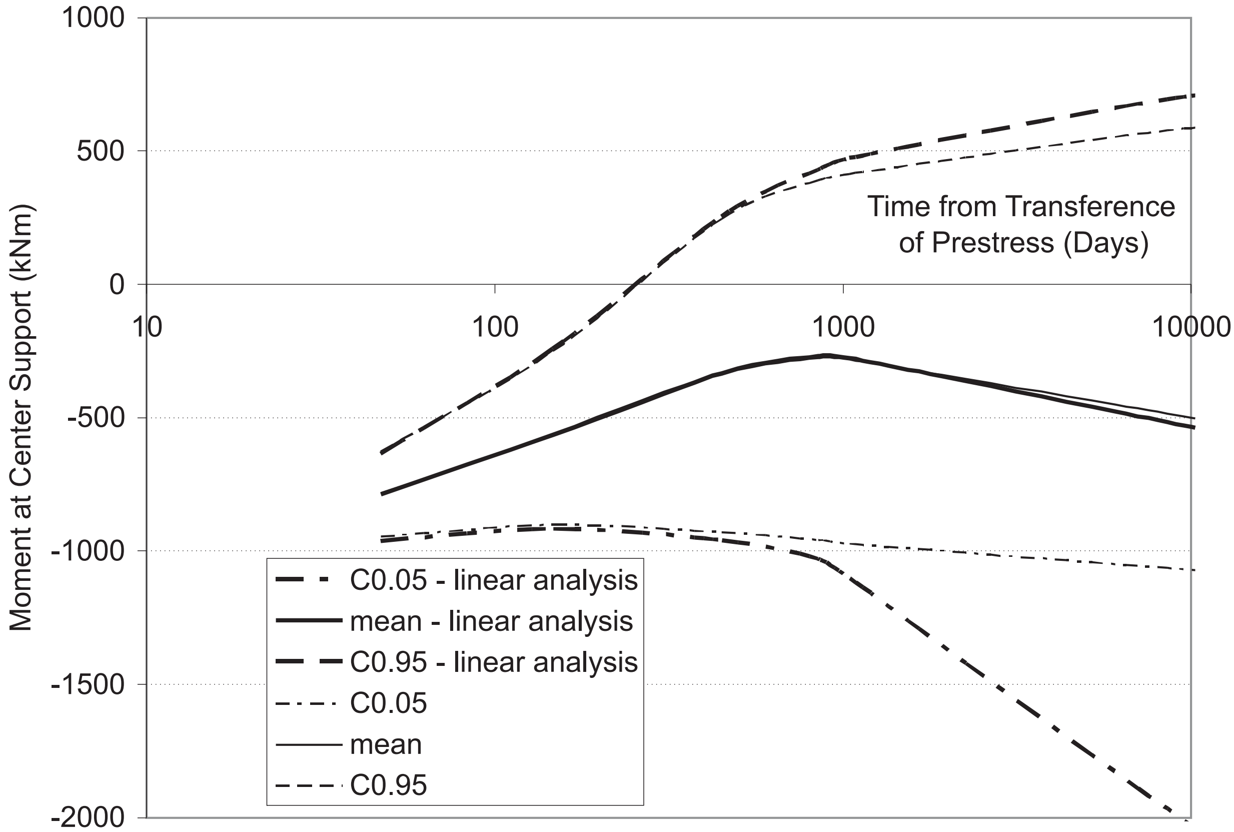
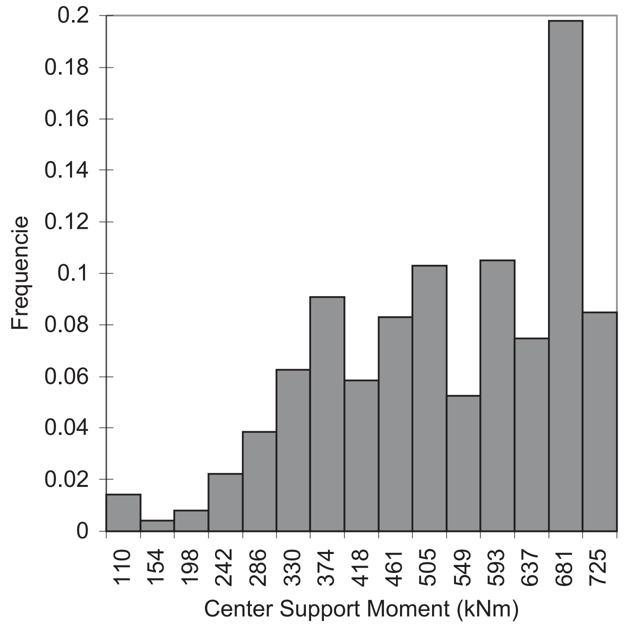
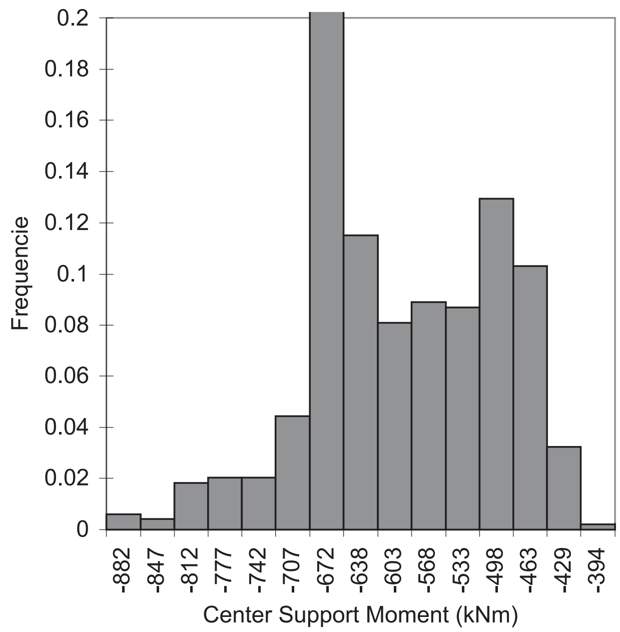


Figure 10 – Statistical distribution of the center support moment (25 m span structure) due to: a) differential temperature variation; b) traffic loads.

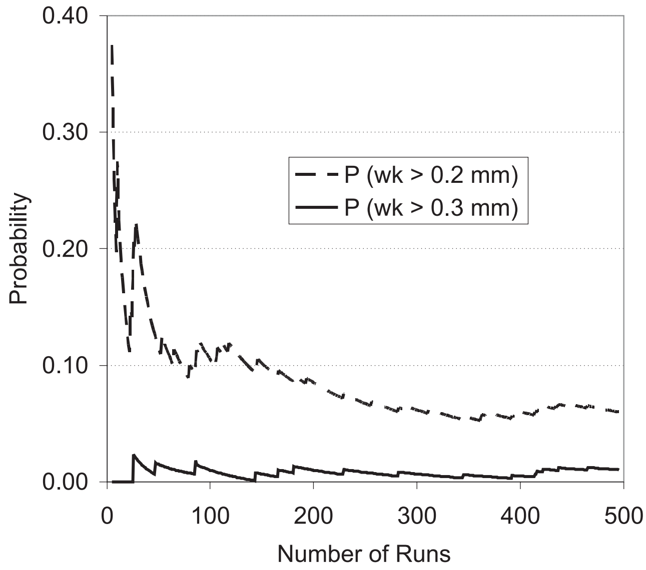


a)

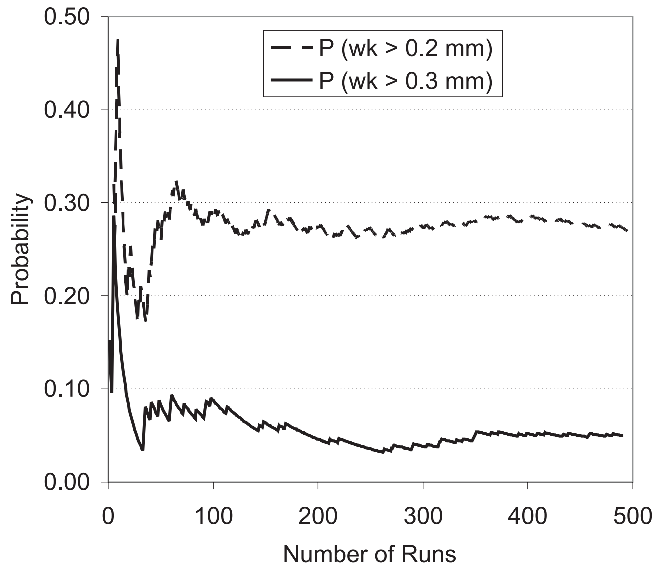


b)

Figure 11 – Probability of occurrence of crack widths greater than 0.20 mm and 0.30 mm at centersupport section (25 m span structure),as a function of the number of simulation runs: a) bottom fiber; b) top fiber.



a)



b)