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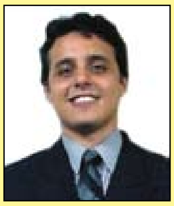
Numerical Analysis of RC Flat Slabs under Flexure and Punching

Análise Numérica de Lajes Lisas de Concreto Armado à Flexão e Punção

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Abstract

The estimates of the current version of the Brazilian code, NBR 6118:2003, and its previous version, NBR 6118:1978, are analysed. The Brazilian code NBR 6118:2003 considers no flexural effects on punching resistance of the slabs, which are higher for elongated columns. For these columns, the punching failure takes place around the column's ends. The intensity and distribution of the shear stresses depends on the configuration of the columns in the slab, and the "λ" flexural factors consider these variables. In addition a numerical analysis (F.E.M.) of a pattern floor of a building built in Belém, modelled with flat slabs and rectangular columns with maximum relation between its sides equal to 4 is presented. The results showed that the flexural factors improved the NBR 6118:2003 estimates and its results' trend. The NBR 6118:1978 code tends to underestimate the results.

Keywords: reinforced concrete, flat slab, punching, rectangular column.

Resumo

São analisadas as estimativas de resistência de acordo com a versão atual da norma brasileira, a NBR 6118:2003, e sua antecessora, a NBR 6118:1978. A NBR 6118:2003 não considera os efeitos da flexão na resistência à punção das lajes, que são mais significativos para pilares alongados. Para esses pilares a ruína por punção tem início em torno de suas extremidades. A intensidade e a distribuição dos esforços solicitantes dependem da configuração dos pilares na laje, e os fatores "l" de flexão consideram tais variáveis. São apresentados os resultados de uma análise numérica (M.E.F.) do pavimento tipo de um edifício construído na cidade de Belém, modelado com lajes lisas e pilares retangulares com relação máxima entre os lados de 4. Os fatores de flexão melhoraram as estimativas da NBR 6118:2003 e a tendência de seus resultados. A NBR 6118:1978 mostrou-se conservadora.

Palavras-chave: concreto armado, laje lisa, punção, pilar retangular.

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1 Introduction

The use of flat slab structural systems, when compared to the conventional structural systems (with beams), presents several advantages that can be immediately identified, as the reduction of the concrete amount, wood formworks consumption and self weight of the construction, besides providing larger agility in the execution of masonries, thick partition walls and installations (electric, hydro-sanitary). Thus, the necessary time for the construction of the enterprise may be reduced. However, flat slab structural systems present as a major disadvantage, punching, which has been quite discussed by the national and international technical communities. Punching is a brittle shear failure that happens in the slab-column connections, which may lead the entire structure to ruin through the progressive collapse.
In spite of the flat slabs design be governed by the punching resistance, experimental researches have been demonstrating that the flexural resistance exercises strong influence over the ultimate punching load of the tested models, fact that is still neglected by some codes. Other codes already consider the contribution of the flexural reinforcement in the slabs' punching resistance, but don't present prescriptions to consider the influence of the boundary conditions in the shear resistance around loads or concentrated areas, mainly when these areas are rectangular and present relationship between the sides greater than 2 (rectangularity index ɪ). This situation becomes really important because, as can be observed in the most of the construction sites of reinforced concrete buildings, the use of cross sections with rectangularity indexes greater than 4 is quite usual.
This paper aims to discuss the recommendations and results from the Brazilian codes NBR 6118:1978 [1] and its new version, NBR 6118:2003 [2], which is use since 2004 for the design of slab-rectangular column connections. To validate the proposals of this paper, the results of a parametric analysis using the Finite Elements Method are presented, besides comparisons with the experimental results of 85 reinforced concrete flat slab tests. The adopted units were N and mm.

1.1 Available information

Researches approaching the influence of the slabs boundary conditions, as the shape of the loaded area and the support conditions are not frequently found in the literature. Aiming to present some important informations related to the study of the symmetrical punching under bending, the results of some researches that consider, simultaneously or not, the rectangularity index of the columns and the boundary conditions of reinforced concrete flat slabs are commented.
One of the first works regarding slab-rectangular column connections found in the literature is the one of FORSSEL and HOLMBERG [3], where square flat slabs supported in the four sides were tested. Two slabs were individually loaded through a pair of columns with circular sections with diameters of 140 mm. In one of those slabs the sections were symmetrically distanced from each other of 400 mm and in other slab of 200 mm. Only one slab presented the load applied through a rectangular section, which was positioned in the center of the slab, with sides dimensions of 25 mm x 300 mm. A significant result of this research is that for one of the analyzed slabs, the punching resistance is well predicted by codes' equations such as the ones presented by NBR 6118:2003, which doesn't consider the rectangularity index of the column, showing that this index is not a so important parameter when the ratio $c_{máx}/d$ is low ($c_{máx} = 2{,}8 \cdot d$).
In 1956, ELSTNER AND HOGNESTAD [4] tested two slabs supported in the two opposed edges and loaded through short square column stretches linked to the slab. A reduction of the resistant capacity of these slabs was observed in relation to the slabs supported in the four edges, and this reduction reached a maximum value of 20%. The authors attributed this reduction to the increasing value of the parameter $\varphi = V_{Exp}/F_{Flex}$, present in the equation 1, proposed for slabs without shear reinforcement.

$$V = \frac{7}{8} \cdot (2{,}3 + \frac{0{,}046 . f_c^{'}}{\varphi}) \cdot b \cdot d \qquad (01)$$

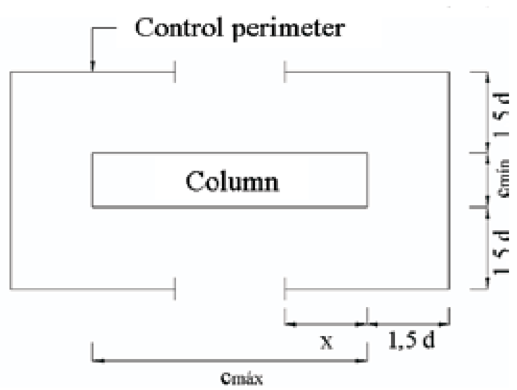
Due to the use of just squared columns, the authors don't make any reference to the influence of the rectangularity index of the columns in the behavior of the slabs. However, these point out the contribution of the bending reinforcement, calculated in agreement with the yield line theory without the consideration of a shear failure, in the punching resistance, that now comes in some codes in function of the reinforcement rate (ρ). $f_c^{'}$ is the compression resistance of the concrete.
Also using square columns, MOWRER and VANDERBILT [5] carried out tests in thin plates ($d = 51$ mm) of reinforced concrete with lightweight gravel. The plates were supported in the four edges and loaded through small column stretches with sides measuring up to $8d$. The results were compared with estimates using several methods, and the best results were observed using a modification of the MOE's equation [6], in which the influence of the dimensions of the loaded area in relation to the effective depth of the slab is considered, being the punching resistance obtained in function of the

relation V_{Exp} / F_{Flex}. The bending resistance is determined by the yield line theory. The modified equation is presented in the equation 2.

$$\frac{V}{u_0 \cdot d \cdot \sqrt{f_c^{'}}} = 0{,}8 \cdot (1 + \frac{d}{c}) - 0{,}44 \cdot \frac{V}{F_{Flex}} \quad (02)$$

The loaded area perimeter length is u_0 and c is the length of a square column side. The major interest in this stage is in the term V / F_{Flex}, which allows the consideration of two effects. One is the influence of the boundary of the slabs. For example, for a square slab simply supported in the edges and equal bending resistance in both directions, it can be deduced that:
a) slabs supported in two opposed edges: $F_{Flex} = 4 \cdot m$;
b) slabs supported in the four edges: $F_{Flex} = 2 \cdot \pi \cdot m$.
Thus, it is noticed that that equation predicts a reduction in the punching resistance if only two edges of the slab are supported. The other effect is the bending resistance inside of any presented structural configuration, predicting an increment in the punching resistance if the yield stress of the flexural reinforcement steel increases, while the rate of this reinforcement is kept constant.
Seeking the best understanding of the influence of columns' rectangularity index in the flat slabs behavior, HAWKINS et al. [7] analyzed square slabs where the columns rectangularity index varied from 1 to 4,33. In most of the tests the load was applied in two edges perpendicular to the largest sides of the columns. The punching resistance reduced considerably when the rectangularity index increased. With the behavior observed of the slabs' tests, HAWKINS et al. agreed with the limit of $0{,}335.\sqrt{f_c^{'}}$, established by the ACI 318 [8] code for the shear resistance.
Another work, which also contributes significantly to the best understanding of the flat slab punching resistance development according to the column section shape, it is of VANDERBILT [9]. Tests were conduced in square slabs supported in the four edges with columns of circular and square sections, monitored with strains gauges in areas immediately below the bottom surface of the slabs, in the vertical position. The author observed that there was a considerable concentration of stresses in areas close to the extremities of the square section columns, while the distribution of stresses around the circular columns was practically uniform. These results indicate the potential of non uniform distribution of the stresses, which can be more intense in function of the column shape and boundary conditions of the slab.
LEONG and TENG [10] investigated the behavior of 20 square flat slabs supported by rectangular columns and loaded in the four edges. The rectangularity index varied from 1 to 5. In general, it was observed that there was a resistance increment when the rectangularity index increased. It is proposed a recommendation or procedure for projects that consider punching around rectangular columns. This procedure is essentially an extension for the code BS 8110 [11] in order to insert the case of the rectangular columns in its recommendations. Then a critical perimeter similar to that one of the code EC2 [12] is adopted, with some considerations. The proposed perimeter is presented in the figure 1.

Figure 1 – Control perimeter proposed to BS8110 by Leong and Teng (9)



Considering the following limits:

$$c_{mín} \leq 600\,m \qquad d \geq 125\,m$$

and

$$x \leq \begin{cases} c_{máx}/2 \\ 2 \cdot c_{mín} \\ 5{,}6 \cdot d - c_{mín}/2 \end{cases}$$

AL-YOUSIF and REGAN [13] tested slabs with rectangular columns presenting rectangularity index of 1 and 5 ($\frac{c_{máx}}{d} = 6{,}25$).

One-way slabs (supported in two edges) and bi-directional slabs (supported in the four edges) were tested. The slabs were supported in two and four edges. The failure loads varied significantly with the boundary conditions, being observed the biggest values for slabs supported in the four edges and the smaller values for slabs supported in two edges. Based on the results from literature, the autors proposed a method to reduce the control perimeter recommended by BS 8110 for an "effective perimeter" (u_{eff}), in agreement with the equation 3.

$$u_{eff} = 2 \cdot \{\lambda_x \cdot (c_x + 3 \cdot d) + \lambda_y \cdot (c_y + 3 \cdot d)\} \quad (03)$$

where

$$\lambda_x = \{\text{one-way or bi-directional slabs}: (1{,}09 - 0{,}03 \cdot c_x / d) \leq 1$$

$$\lambda_y = \begin{cases} \text{one-way slabs}: (1{,}09 - 0{,}09 \cdot c_y / d) \leq 1 \\ \text{bi-directional slabs}: (1{,}09 - 0{,}03 \cdot c_y / d) \leq 1 \end{cases}$$

The variable c_y corresponds to the column dimension parallel the slab's span, if the slab is predominantly designed in one direction. The limitation of this method is that for any value of the column dimension largest than 4,55·d. In this situation, the value of the portion $\lambda_y \cdot (c_y + 3 \cdot d) = 5{,}14 \cdot d$ should be considered for one-way slabs. It is noticed that the smallest c_x value in λ_x and c_y in λ_y, for bi-directional slabs, is 3·d. This observation indicates that the values predicted by BS 8110 for this type of slab and with $c_{x,y}/d < 3$ are considered satisfactory for the authors.

1.2 Codes' Recommendations

NBR 6118:1978, previous version of the current Brazilian code, recommends a control perimeter indifferent for rectangularity indexes smaller or equal to 3. For the other situations, the perimeter becomes independent on the largest side of the column, as showed figure 2.

This perimeter u is used to predict the slabs' resistance, in agreement with the equation 4. It is observed that NBR 6118:1978 doesn't consider the influence of bending reinforcement, but it limits the slabs' resistance, stabilizing the control perimeter for rectangularity indexes above 3.

$$V_{ck} \leq 0{,}32 \cdot \sqrt{f_{ck}} \cdot u \cdot d \quad (04)$$

Considering the influence of the bending reinforcement, independent of its yield stress, the current version of the Brazilian code, NBR 6118:2003, recommends a control perimeter that grows with the dimensions of the columns, in other words, for a slab with the same physical and mechanical characteristics, the shear stress will be constant and the ultimate resistance of the slab will be proportional to the dimensions of the columns, according to the equation 5. Figure 3 shows the perimeter recommended by NBR 6118:2003.

$$V_{ck} = \gamma_c \cdot 0{,}13 \cdot \left(1 + \sqrt{200/d}\right) \cdot (100 \cdot \rho \cdot f_{ck})^{1/3} \cdot u \cdot d \quad (05)$$

where $\gamma_c = 1{,}4$

For the best understanding of the control perimeter and of the relation $c_{máx}/d$ influence in the resistance of the slabs, figure 4 shows the tendencies of the two versions of the Brazilian code when the perimeters are compared. To eliminate the influence of the maximum shear stress allowed, the punching resistance was divided for a reference resistance, establishing a square column with $c = 2 \cdot d$. In the situations where the slab is supported by square columns, with sides dimensions above $2 \cdot d$, the estimates of NBR 6118:1978 grow significantly when the column side increases. In this

Figure 2 – Control perimeter recommended by NBR 6118:1978 code (1)

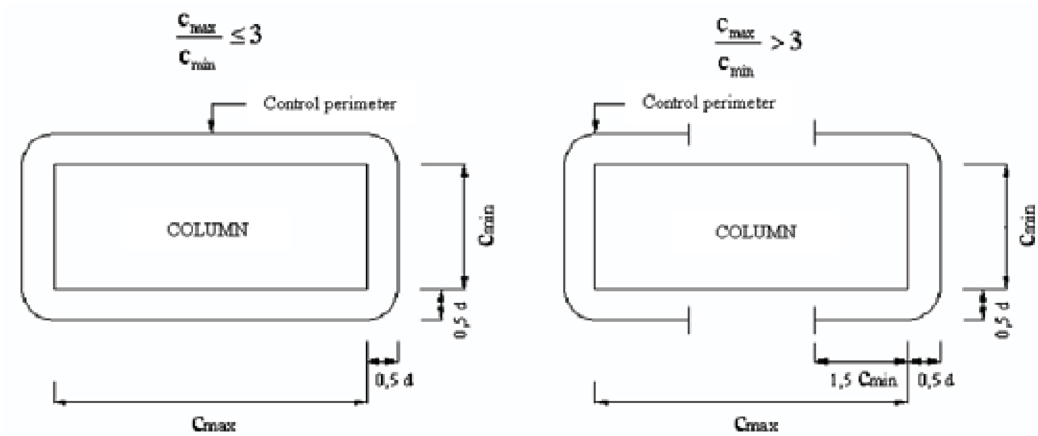
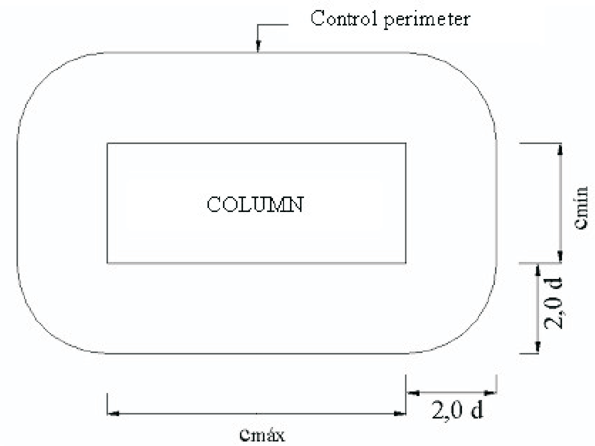


Figure 3 – Control perimeter according to NBR 6118:2003 code (2)

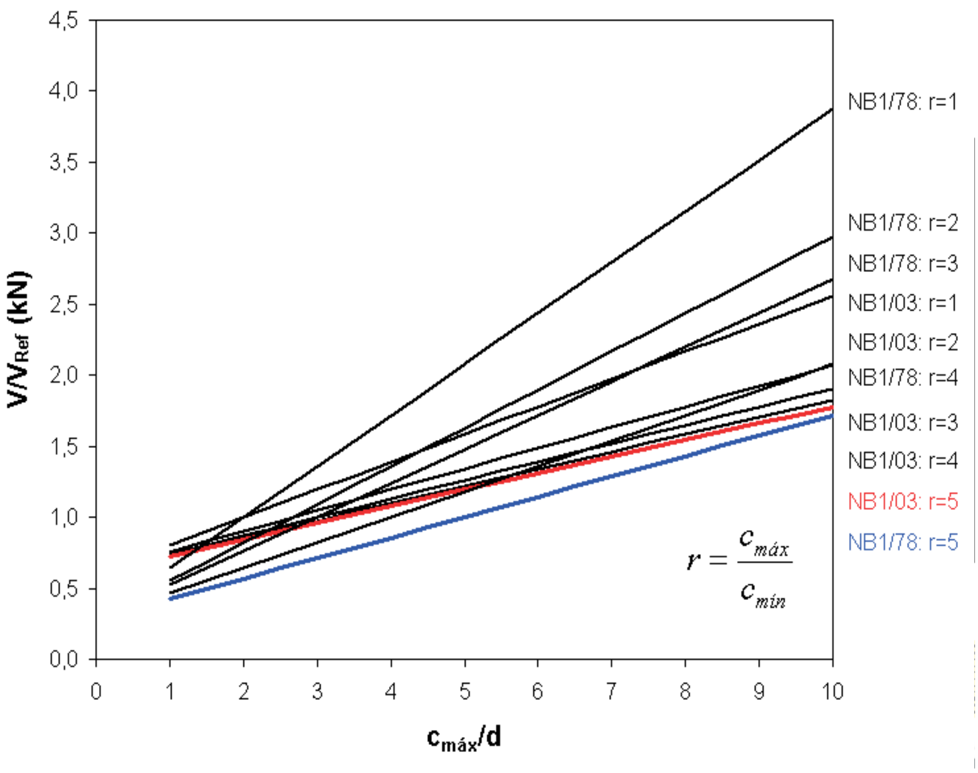


case, the punching failure can occur exactly at the ends of the columns. This also happens with NBR 6118:2003, however more tests are necessary.
Still considering NBR 6118:1978, it is verified again that the influence of the control perimeter is more intense for rectangularity indexes smaller than 3 and relations $c_{máx}/d$ above 4,5, approximately. For rectangularity indexes above 3, the influence of the control perimeter is less important. It is pointed out that the codes' recommendations don't consider the behavior of the slabs to bending. In this case, for example, slabs loaded perpendicularly to the largest sides of the column should consider a control perimeter smaller, what is clearly demonstrated in the work of HAWKINS et al.. The additional curve for this situation would be much less tilted in relation to the others in figure 4, indicating the imprecise consideration of a continuous perimeter around a column with high rectangularity index, because failure occurs around the ends of the column, drawing a perimeter significantly smaller.

2 Bending effects in punching

To illustrate the mark of this work, it's presented the analysis of a construction in execution in the city of Belém. The building is of a commercial type, with 4 floors constituted by post-tensioned concrete ribbed slabs supported by columns disposed as can be seen in figure 5. The average use of this type of structural system has been verified (post-tensioned concrete ribbed slabs with lattice reinforcement supported in rectangular columns) in the country as an alternative used by designers to avoid the risk of punching. It is noticed in the figure the absence of solid slab strips in the traverse direction. For this slab type, ribbed with trusses reinforcement, the tensile stresses due to shear force are satisfactorily absorbed by the bars positioned in the trusses' diagonals, inexistent in flat slabs without shear reinforcement.
The procedure adopted in this analysis was to virtually substitute the existent ribbed slabs and beams for solid flat slabs and verify then the influence of the columns and slabs configuration in the normative estimates for the punching resistance. Results are compared with the ones predicted by alternative methods that consider the behavior to bending in the estimates of the ultimate punching resistance of the slabs.
Figure 6 shows the columns' configuration for the pattern floor and the slabs' dimensions. Columns dimensions were (300mm x 1.200mm), which represents a rectangularity index of 4. Slabs have 200 mm height and are predominantly one-way slabs and this characteristic is maintained in the slabs' global analysis. As well as in most of the constructions with flat slabs, the areas with negative moments are susceptible to punching in the areas around the columns. The dis-

Figure 4 – Influence of the relation on the code estimates



position of these columns, as in the case of columns P12, P13 and P14, is fundamental for the distribution and intensity of the bending moments and shear forces that reach the columns' faces.

The slabs panel was modeled and calculated using the Finite Elements Method (F.E.M.), using the program SAP2000n. The element used was rectangular Shell with 4 joints. The resistance of the concrete adopted was 30 MPa, and the young's module was predicted in accordance with recommendations presented in NBR 6118:2003. The total load ap-

Figure 5 – Slabs' details of the pattern floor

Figure 6 – Distribution of the columns on the pattern floor

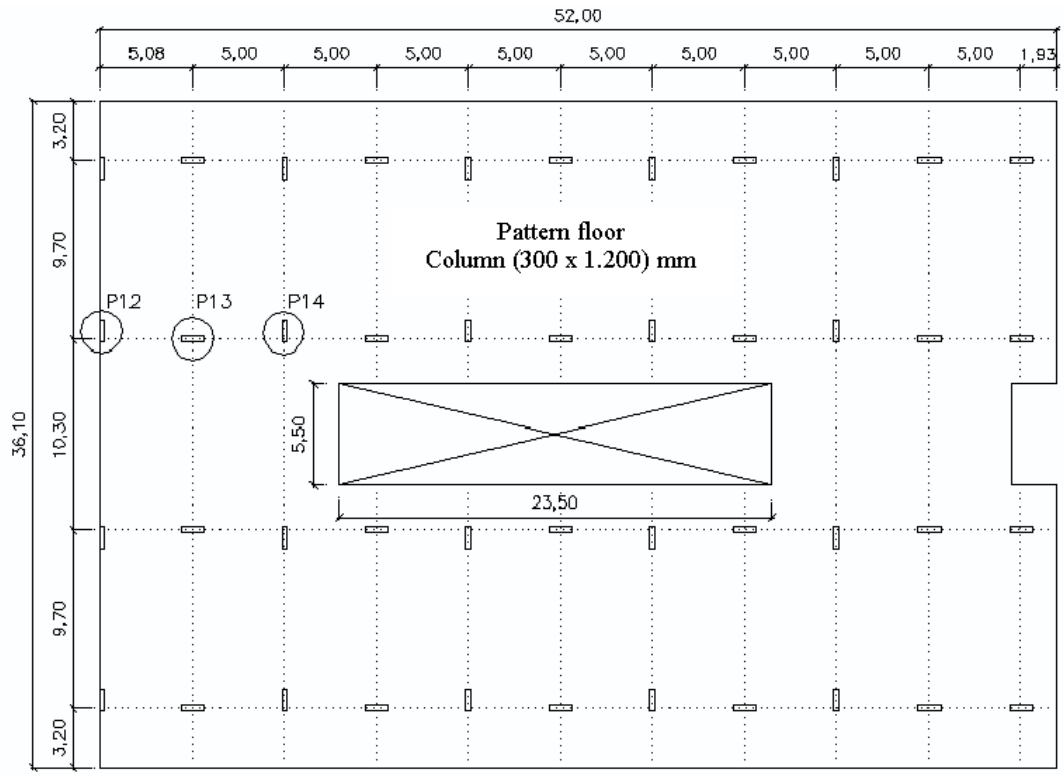
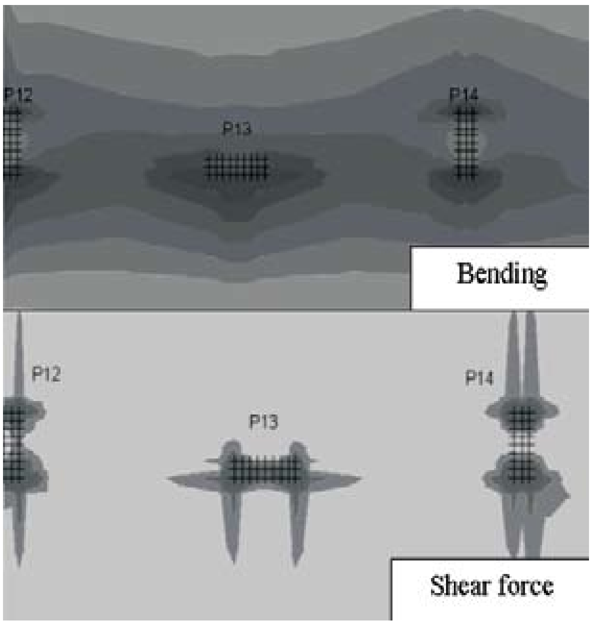


Figure 7 – Shear force and bending concentration around the columns P12, P13 e P14



plied in the elements that compose the slab was 6 kN/m², besides its self-weight.
The results for the maximum bending moments (M22) and shear forces around the columns P12, P13 and P14 are presented in figure 7. The clearest colors indicate the positive moments. The high intensity of the negative bending moments close to the ends of the columns is noticed, being more accentuated in the columns disposed longitudinally. The shear force's distribution agrees with the bending moments one, however, it is evident the shear force's polarization, indicating that, for reinforced concrete slabs, the radial cracks can concentrate around the ends of the columns, difficulting the propagation of the tangential cracks for intermediate areas of the columns, between the ends. In practice, the negative reinforcement is perpendicularly positioned to the largest sides of the columns and almost always there is no reinforcement in the other main direction. This reinforcement can not be absent because, as it will be discussed later, it influences considerably in the punching resistance around elongated columns.
The behavior of the radial and tangential cracks, and these last ones are more evident in the last load stages, it is well defined in several experimental works just involving circular and square columns. The research of OLIVEIRA [13], where 15 reinforced concrete flat slabs with dimensions of (1.680mm x 2.280mm x 130mm) were tested supported by columns with rectangularity index between 1 and 5,

Table 1 – Flexural factors

Case	λ
(1) One-way slab with span parallel to c_{max}	$(c_{max}/d)^{0,17}$
(2) One-way slab with span perpendicular to c_{max}	$0,93\cdot(c_{max}/d)^{0,14}$
(3) Two-way slab	$1,03\cdot(c^{max}/d)^{0,02}$

brings important information about the behavior of those cracks in bi-directional and one-way flat slabs. In the last case, load was applied so much at the parallel edges to the largest sides of the column as in the parallel edges to the smallest sides. This procedure aimed to evidence the shear stresses concentration around the ends of the columns. It was observed that the radial cracks, on the smallest side of the column, avoided propagation of the tangential cracks for the intermediate areas on the largest sides, leading the slab to punching with the failure surface drawing a considerably reduced perimeter. In this work are proposed bending factors (λ) to correct the tendency of the results using the recommendations of CEB-FIP MC90 (1993) and using NBR 6118:2003, in accordance with the equation 6 and the table 1. The results were compared with those of the tests of 85 flat slabs and are presented in the table 2.

$$V_{ck} = (0{,}18/\lambda)\cdot(1+\sqrt{200/d})\cdot(100\cdot\rho\cdot f_{ck})^{1/3}\cdot u\cdot d \qquad \textbf{(06)}$$

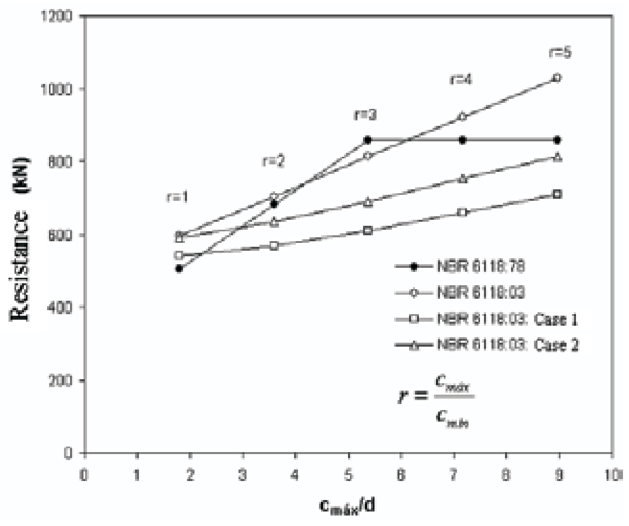
Figure 8 shows the tendency of the codes results and of the modified ones (case 1 and 2) for the use of bending factors, as well as the differences found for the case of the one-way slabs. As those factors are higher in the cases 1 and 2, and considering that the modified results are satisfactory, differences of up to 45% can be observed among those results and the predicted ones by NBR 6118:2003 (case 1) for rectangularity indexes of 5, tending to increase this difference. The tendency of NBR 6118:1978, for one-way flat slabs, is to overestimate the resistance of the slabs before the stabilization, when $r = 3$. For values above 5, approximately, the tendency is to underestimate the punching resistance.
The influence of the bending effects in the punching resistance is clear, since the bending cracks collaborate for the punching failure of the slabs, what in fact happens. In the case of elongated columns supporting one-way flat slabs, the tendency is the reduction of the punching effects as the rectangularity index increases. For the slabs with span parallel to the largest side of the column (case 1), in an extreme situation, punching failure tends to give place to shear force failure. In practice, most of the one-way flat slabs is more requested parallel to the largest sides of the column (case 2), and wide beam and bending failures can be predominant.
Figure 9 shows a typical case 2 situation, where the punching failure may not happen due to the transition of the shear stresses typical of punching for those characteristics of the shear failure in beams. This depolarization of the shear stresses is function of the relation $c_{máx}/l$, where l is the dimension of the supported or loaded edges of the square slab. This effect can also be interpreted through the reduction of the area of larger shear stresses concentration, illustra-

Table 2 – Results using the flexural factors

Code	$\frac{V_{Exp}}{V}$ (85 slabs)					
	Normal weight concrete			Lightweight concrete*		
	AV	SD	CV (%)	AV	SD	CV (%)
NBR 6118:2003	0,95	0,09	9,04	0,84	0,06	7,49
NBR 6118:1978	1,44	0,28	19,71	1,44	0,35	24,33
NBR 6118:2003 + λ	1,00	0,06	5,77	1,00	0,06	6,21

* Results using Mowrer and Vanderbilt's tests with two way slabs and square columns.
AV: Average; SD: Standard deviation; CV: Coefficient of variation.

Figure 8 – Code and modified results



tively delimited by the control perimeter, for high rectangularity indexes.

In order to evaluate the influence of the rectangularity index in the flat slabs behavior to bending, and possibly to establish a more ductile failure, the minimum dimensions on the largest sides of the columns (c_{Flex}) was determined so that punching resistance equals the ultimate bending resistance (predicted by the yield line theory). This was possible by the fact that bending resistance is considered constant while the punching resistance, for the same slab and predicted considering the bending factors incorporated to the recommendations of NBR 6118:2003, varies in accordance with the values of r and ρ (average), that takes into account the slabs traverse reinforcement. The results obtained for a square flat slab and $c_{mín} = d$, that make possible the direct analysis, are presented in the table 3. The geometric rate of the used reinforcement (longitudinal: ρ_{Flex}) for determination of the ultimate bending load was of 0,73%, reminding that the bars of this reinforcement are perpendicular to the largest sides of the column. Concrete compression resistance was adopted as 30 MPa.

The influence of the traverse reinforcement is verified (ρ_V) in the punching resistance increment through

Figure 9 – Polarization and despolarization of the shear forces on one-way slabs in case 2

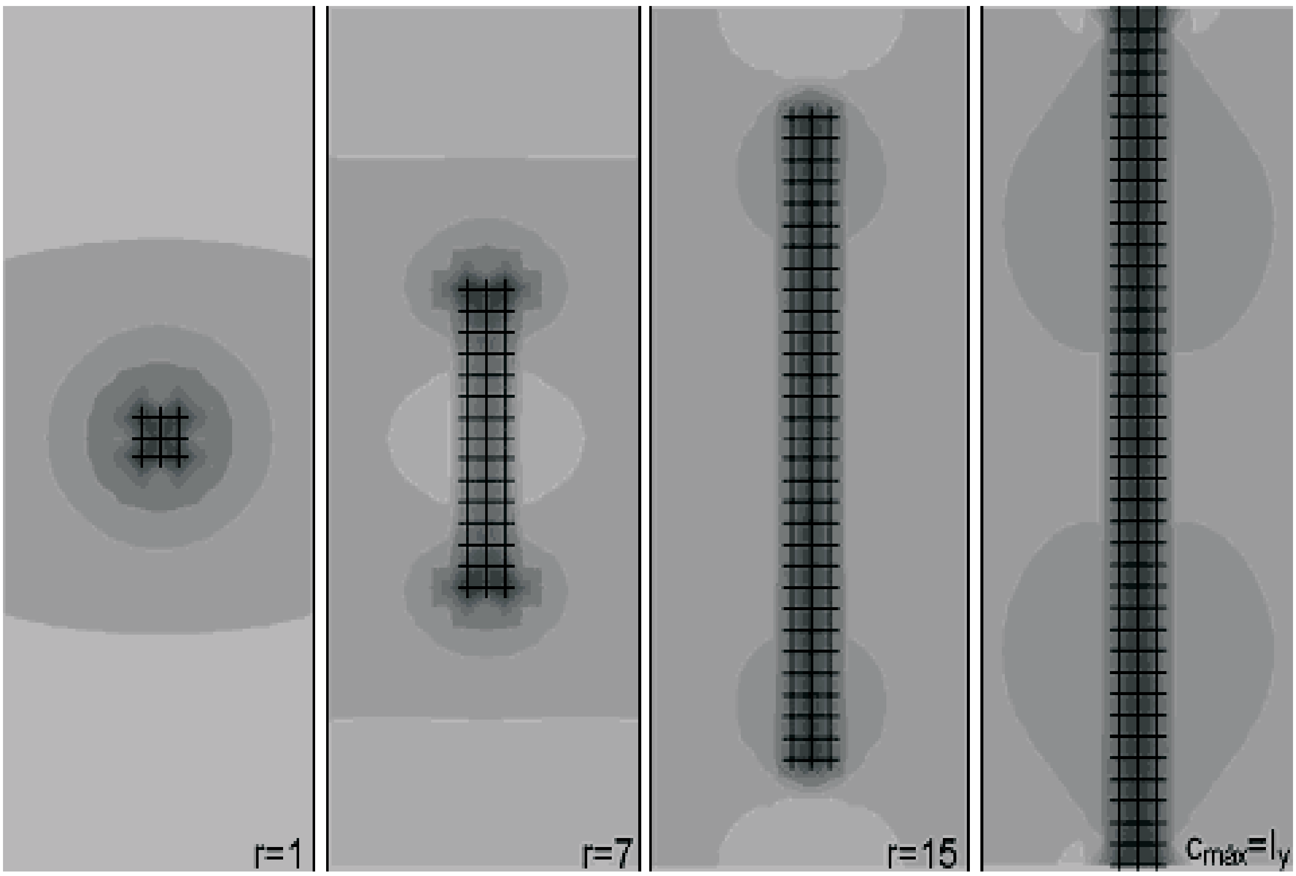


Table 3 – Rectangularity index for ductile failure

ρ_V/ρ_{Flex}	c_{Flex}/d
0,47	5,3
1,88	2,7
4,23	1,4

the reduction of c_{Flex}, once the longitudinal reinforcement rate is maintained constant. For low average geometric bending reinforcement rates it could be onerous to elevate the punching resistance of flat slabs, but for values of $\rho_V \cong \rho_{Flex}$ it would not be disadvantageous. Thus, the reinforcement perpendicular to the negative bending reinforcement should be considered in one-way flat slabs with span perpendicular to the largest side of the column. Obviously, these results need to be experimentally tested and extended for other boundary conditions.

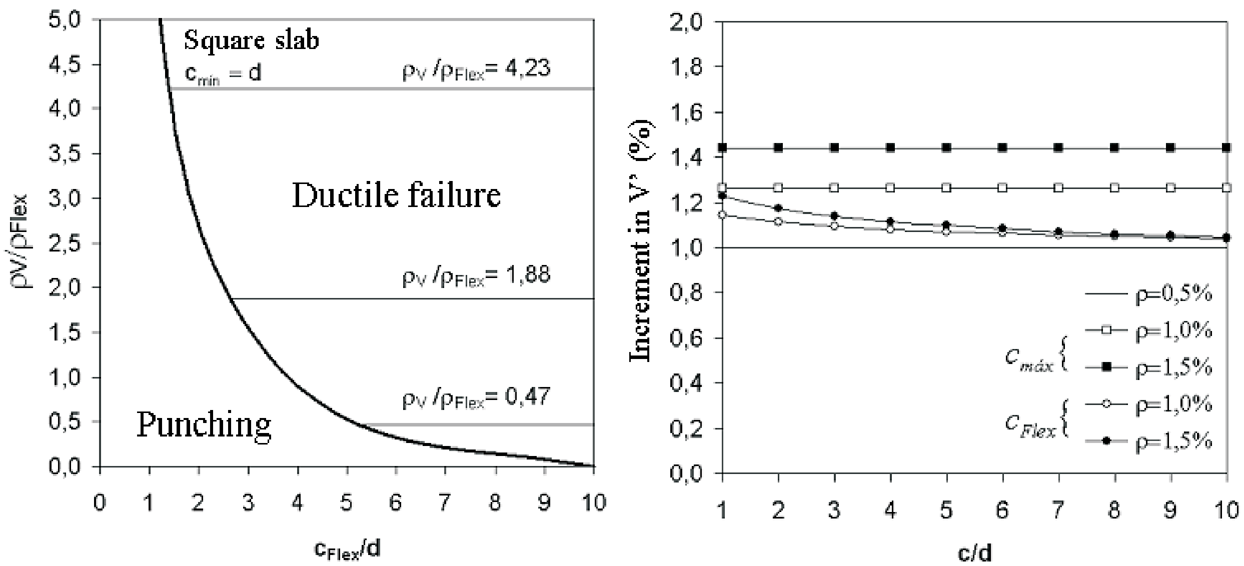
In the effective codes recommendations, any increase of ρ_V generates constant increments in the codes and modified codes predictions (V'), what makes sense due to disregard of the bending resistance in its expressions, being admitted only the punching failure. With the use of c_{Flex}, the increments in punching resistance (figure 10) would be decreasing for the growing relations between the largest side of the column and the effective depth of the slab, once the shear stresses distribution no more would be characterized as that typical for punching, with ρ_{Flex} becoming preponderant. Punching failure surface would be, then, substantially reduced. For values of c_{Flex} larger than the necessary ones, when ρ_V increases, the ductile failure theoretically prevails. In a more advanced stage of this research, it can be settle down minimum values for the largest side of the column in function of the transverse reinforcement rate, as display figure 10. In the figure, any point above the line curves could indicate the bending failure of the slab, considered square and supported in columns with $c_{mín} = d$.

3 Conclusions

The recommendations of NBR 6118:2003 are less conservative than the ones from NBR 6118:1978, in spite of the fact that in this code the effects of the columns rectangularity index and the slabs behavior to bending are not considered. The estimates of the new code tend to overestimate in up to 20% the punching resistance of one-way flat slabs with span perpendicular to the largest sides of the column and in up to 45% when the spans are parallel to the largest sides of the column. This tendency is maintained for growing values of $c_{máx}/d$. NBR 6118:1978 tends to be conservative for rectangularity indexes above 5, approximately. The bending factors eliminate the tendency of NBR 6118:2003 of overestimating the slabs punching resistance and consider the slabs behavior to bending. The resistance of the material component of the bending reinforcement continues not being considered, but the bending effects can already be incorporated to the design.

The traversal bending reinforcement shall not be despised because, even in the cases where the one-way flat slabs present spans perpendicular to the largest sides of the columns, it contributes significantly to the slabs punching resistance.

Figure 10 – Limit for flexural failure and modified results' tendency



The punching effects can be minimized with the use of the length on the column largest side equivalent to the necessary for a bending failure, c_{Flex}. The use of the bending factors made possible the theoretical determination of rectangularity indexes necessary for a ductile failure (c_{Flex}), reducing the punching resistance increments for one-way flat slabs with spans perpendicular to the largest sides of the column as the relations c_{Flex}/d increases.

4 Acknowledgments

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