

# Critical analysis of models and recommendations for designing column-base connection by socket of precast concrete structures

Análise crítica dos modelos e recomendações para o projeto da ligação pilar-fundação por meio de cálice em estruturas de concreto pré-moldado



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# Abstract

This paper presents a comparative study of the main design models found on technical literature for column-foundation connection by socket, with a critical analysis and important considerations of some aspects for designing this connection. The main variables studied were embedded length of column in socket base and friction of interfaces. The main conclusions are: a) there was a significant discrepancy in the design using the LEONHARDT & MÖNNIG model, that is normally used in designing socket bases, and by other design models, b) the friction consideration is very important in the connection design and, therefore, it should be taken into account in order to lead to a more reasonable design and, c) considering, per example, the smooth socket base with a length equal to 2 times the cross section height of the column and the friction coefficient equal to 0.6, the differences of the LEONHARDT & MÖNNIG model related to the other design models were up to 58%.

*Keywords:* connection, socket base, precast concrete, embedded length, design models.

# Resumo

Nesse trabalho, apresenta-se um estudo comparativo com os principais modelos de projeto encontrados na literatura técnica para a ligação pilar-fundação por meio de cálice, com uma análise crítica e importantes considerações sobre alguns aspectos do dimensionamento dessa ligação. As principais variáveis abordadas foram o comprimento de embutimento do pilar no cálice e o atrito mobilizado nas interfaces. As principais conclusões são: a) houve uma significativa discrepância no dimensionamento pelo modelo de LEONHARDT & MÖNNIG, mais empregado no projeto dos cálices, e pelos outros modelos de projeto, b) a consideração do atrito é de fundamental importância no projeto da ligação e, portanto, deve ser levado em conta de forma a conduzir a um dimensionamento mais racional e c) considerando-se, por exemplo, o caso de cálice liso com comprimento de embutimento equivalente a 2 vezes a altura da seção transversal do pilar e coeficiente de atrito igual a 0,6, ocorreram diferenças de até 58% do modelo de LEONHARDT & MÖNNIG em relação aos outros modelos de projeto.

Palavras-chave: ligação, cálice de fundação, concreto pré-moldado, comprimento de embutimento, modelos de projeto.

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### 1 Introduction

The column-foundation connection by socket, shown on Figure 1, is established by embedding a column portion into a cavity of the structural element of foundation in order to fit the column. The alignment and the level positioning of the column is usually done by means of a centralizing device. For the temporary support and to keep the column vertically, wooden wedges are used. After setting-up the column, the space between the two elements is filled with concrete. The socket base can be cast-in-place or precast, or only the pedestal, which constitutes the walls around the column, can be precast.

The main advantages are: a) good capacity of transmission of axial forces and bending moments and very close behavior to a monolithic structure; b) smaller sensibility to the design and erection inaccuracies makes the fittings easier to execution deviations; c) special protective cares against atmospheric agents and fire are not necessary. On the other hand, the disadvantages are: a) the foundation is very pronounced and, therefore, it is usually hidden; b) the pedestal walls must be in a certain distance between the column and the boundaries of the construction site.

Although the socket base connection is widely used in Brazil, this connection presents a very peculiar behavior and there are still doubts about it. Thus, the connection design has been conservative, and due to the influence of some parameters it is usually neglected or improperly considered.

This paper focus on the analysis of the design models of the technical literature for the socket base calculation and includes the recent model proposed by the authors in CANHA [2], which is based on theoretical and experimental results of CANHA [2]. The name *design models* is used here for the theoretical model from which structural designs are usually made of.

This research was done because the current models implicate in very different designs. Apart from main models and design recommendations, important considerations about the current researches regarding the socket base are presented. From the applications of the design models, some recommendations are prescribed.

The main focus of this paper is the calculation of the resultant of top pressures from the column onto one socket base wall (wall 1 at Figure 2) and the tension and compression forces of longitudinal walls (walls 3 and 4 at Figure 2). These forces are defined in the following section. To show the differences of these forces and, consequently, of the design, the friction mobilized on the socket interfaces is presented. This friction is one of the main parameters that govern the behavior of this connection.

Among another variables and questions regarding the design of this connection, the influence of the embedded length variation in the design of the main walls reinforcements is also undertaken.

# 2 Behavior of the socket base with pedestal walls

For a better understanding of the transfer of forces in the socket base with smooth interfaces and of the production of the friction efforts on the interfaces, Figure 2 is shown. By the cast-in-place concrete, the moment  $M_d$  and the horizontal force  $V_d$  that act in the columns are transmitted onto the traverse walls 1 and 2. On the interfaces among the column and the pedestal walls 1 and 2, the friction forces occur, that are mobilized by the pressures  $(H_{apd} \text{ and } H_{bad})$  originated by the force transfer in the connection. The direction of the friction force of wall 2  $(F_{frided})$  depends on the relation between the internal forces and the geometry. On wall 1, the friction force  $(F_{frided})$  has the same direction of the normal load  $(N_d)$ .  $N_d$  is reduced by the punching shear when the thickness of the base is





smaller. The transference of the forces from wall 1 onto walls 3 and 4 occurs by bending-tension, practically in most common cases, because walls 3 and 4 have a larger rigidity to transmit efforts for the socket base. In order to resist the force  $H_{\rm logal}$ , the reinforcement  $A_{\rm low}$  is used on the top of longitudinal wall 3 and 4. As these walls behave as corbels clamped in the foundation, the compression strut strength must be verified, and the main  $(A_{\rm low})$  and secondary  $(A_{\rm low})$  vertical reinforcements, and secondary horizontal reinforcement  $(A_{\rm low})$  are calculated according to the corbels models. Due to the small distance between the resultant of pressure on wall 2 and the base, this pressure can be considered as directly transmitted onto the base.

The roughness on the interfaces of the walls and column, as the Figure 3 shows, improves the force transmission in the connection. Besides the mobilization of friction forces, the shear transference occurs by shear keys in all walls.

# 3 Design models and recommendations

# 3.1 LEONHARDT & MÖNNIG (3) and NBR-9062/85 (4) models

The model proposed by LEONHARDT & MÖNNIG [3] is the mostly widely formulation used for socket calculation. In



this model, which is presented at Figure 3, the friction forces on interfaces are neglected and two extreme cases of interfaces are considered: smooth and rough.

The formulation of the Brazilian code NBR-9062/85 [4] is similar to the LEONHARDT & MÖNNIG [3] model, but with a different value of y (position of the resultant of top pressures  $H_{red}$  on wall 1) for rough interface that is 0.15  $\ell_{red}$ .

According to LEONHARDT & MÖNNIG [3] and NBR-9062/85 [4], the internal walls surface of the socket base should have at least the same surface characteristic of the column. The type of the elements roughness can cause the total or partial force transfer that can occur through the interface.

Table 1 shows the embedded lengths values  $\ell_{emb}$  that are recommended by LEONHARDT & MÖNNIG [3] and NBR-9062/85 [4]. Except the minimum embedded length  $\ell_{emb}$  equal to  $_{1.2b}$  for small eccentricity and rough interface, the values presented by LEONHARDT & MÖNNIG [3] are more conservative than those of NBR-9062/85 [4].

The reinforcement for transmitting the force  $H_{\rm total}$  from wall

1 and equally employed on walls 3 and 4 is calculated as:



For the design of walls 3 and 4, the corbels models and recommendations are used, according to the relation  $a_c/d_c$ , where  $a_c$  is the distance of the application point of the force  $H_{u_{pd}}$  to the clamped section and  $d_c$  is the effective depth of corbel. The calculation procedures for each corbel type are shown in detail in CANHA [2].

The experimental investigation carried out in CSTC<sup>1</sup> apid SANTOS [5] shows the conservatism of the LEONHARDT & MÖNNIG [3] model, comparing the theoretical ultimate force with the corresponding experimental value. According to the test results, for the specimens with the total casting of the joint, the rupture always occurs in

Table 1 – Minimum embedded length $\ell_{ m emb}$ of column according to LEONHARDT & MÖNNIG (3) and NBR-9062/85 (4)							
	M <sub>d</sub> / N <sub>d</sub>	,≥0,15h	M <sub>d</sub> / N <sub>d</sub> ≥ 2,00h				
Walls	LEONHARDT & MÖNNIG (3)	NBR-9062/85 (4)	LEONHARDT & MÖNNIG (3)	NBR-9062/85 (4)			
Smooth	1,68h	1,50h	2,80h	2,00h			
Rough	1,20h	1,20h	2,00h	1,60h			
Intermediary values should be interpolated $\ell_{\rm emb} \ge 40 {\rm cm}$ (NBR-9062/85 (4))							

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<sup>1</sup> CSTC. Calcul et execution de l'assamblage des structures industrialisées: Recommandations pratiques. Bruxelles, CSTC - Centre Scientifique et Technique de la Construction, 1978.



the column. Thus, as columns were designed with strength approximately equal to 3 times the theoretical strength of the socket base, the theoretical force calculated by the LEONHARDT & MÖNNIG [3] model that is used for the socket base design is at least 3 times the experimental value. The socket base rupture was only observed when the partial casting of the joint was done in the regions of the binary theoretical forces, but the difference was still large regarding this theoretical model. Although the relation between the theoretical and experimental ultimate forces has changed with the embedded length, for  $\ell_{emb} = b$ , this value was larger than 1.5, and for  $\ell_{emb} = 1.5b$ , this value was larger than 2.5 times. It means that, even for the embedded length smaller than that suggested by LEONHARDT & MÖNNIG [3] and partial casting of the joint, the experimental strength was very large.

One of the main reasons of the LEONHARDT & MÖN-NIG [3] model for socket base leading towards an overstrengthening design is due to the disregard of friction forces on walls and foundation base. However, it is the base for understanding the transfer forces from the column to the socket base. With the other models that take into account the friction to evaluate the connection strength; this model can be an important tool for the socket base design. The following main models that consider the friction are presented.

### 3.2 WILLERT & KESSER (6) model

In the WILLERT & KESSER [6] model, whose forces transfer of smooth socket base is shown on Figure 4, the friction forces are mobilized on the interfaces with walls 1 and 2 and with the foundation base. However, the displacement of the normal reaction  $F_{\rm nhd}$  regarding the column center is neglected.

For smooth interface, the friction coefficient  $\mu$  equal to 2/3 is suggested. According to these authors, this model can also be used for rough socket base fitting appropriately the friction coefficient.

The embedded length  $\,\ell_{\,{\rm emb}}$  of the column should be calculated by the following relation:



Although it is not explicit, the extreme values (1.5b and 3b) of  $\ell_{emb}$  seem to be suitable for smooth socket base with small and large eccentricity, respectively, because they are close to the corresponding values of LEONHARDT & MÖNNIG [3].

Combining the equilibrium equations of forces and of moment in relation to point O with the equations (3) to (8) of the reactions, the resultant  $R_{badd}$  on column base is determined by the equation (9).









For bending without axial force, where  $N_d = 0$ ,  $e/h = \infty$  and  $\beta_{\gamma} = 1$ , and ignoring the friction on interface, the shear force  $R_{\rm herd}$  is calculated by:

$$R_{\text{bot,d}} = H_{\text{bot,d}} = \frac{3M_{\text{d}}}{2\ell_{\text{emb}}} + \frac{V_{\text{d}}}{4}$$
(13)

The values of equations (12) and (13) coincide with the force  $H_{_{bad}}$  of the LEONHARDT & MÖNNIG [3] model for rough and smooth interfaces, respectively, that is indicated on Figure 3.

# 3.3 OLIN et al. (7) model

The OLIN *et al.* [7] model for the distribution of forces in the connection (Figure 5) takes into account the friction force  $F_{\text{printpd}}$  on interface with wall 1 and the displacement  $e_{ab}$  of the vertical reaction  $F_{abd}$  regarding the column center. These parameters contribute for the increase of the socket base strength. The value b / 6 can be used for the eccentricity  $e_{ab}$  of rigid connections.

The embedded length equal to 1.3 times the largest dimension of the cross section of the column is suggested ( $\ell_{emb} = 1.3h$ ).

The design is done for two extreme conditions: smooth interface ( $\mu = 0.3$ ) and rough interface ( $\mu = 0.6$ ). EUROCODE 2 [8] also indicates the maximum value of the friction force  $\mu = 0.3$  for smooth interface. However, the friction force for smooth interface is related with the type of mold used in building of socket base and column. This value of  $\mu = 0.3$ that is suggested by OLIN *et al.* [7] is more coherent for steel molds. The value of  $\mu = 0.6$  is conservative for rough interface, compared with the usual value of  $\mu = 1.0$ , where this last value seems be more reasonable for the experi ments and leads to a more economical design. From the equilibrium of moments at point A, the force  $H_{npd}$  for smooth socket base is given by:

$$H_{topd} = \frac{M_d - 0.17hN_d + 0.9\ell_{emb}V_d}{0.8\ell_{emb} + 0.33\mu h}$$
(14)

The bond strength is preserved with a reasonably superficial roughness, although small cracks caused by shrink of the joint concrete appear on the connection. Thus, for rough interface, the contribution of vertical bond stresses  $\tau_{imy}$  is taken into account on half of internal longitudinal interfaces of the connection (Figure 6). With the equilibrium of moments in relation to the point A in Figure 5, the following equation for the reaction  $H_{inp,d}$  of the rough socket base is obtained:

$$H_{topd} = \frac{M_d - 0.17hN_d + 0.9\ell_{emb}V_d - 0.083hF_{bu,y}}{0.8\ell_{emb} + 0.33\mu.h}$$
(15)

Where the resultant  $F_{buy}$  is given by:

$$F_{bu,y} = 0.5 \tau_{bu,y} \ell_{emb} h$$
(16)

and the vertical bond stress  $\tau_{_{\text{int},\text{i}}}$  is calculated according to:

where:

$$\tau_{bu,y} = \tau_{bu} = 0.3. f_{ctd}$$
 (17)

In order to avoid the concrete split and assure the bond stresses of rough socket base, horizontal stirrups are distributed along the height of walls. The minimum area of these stirrups is:



#### where:

*b*: is the cross section height of column

- s: is the spacing among stirrups
- $f_{dk}$ : is the characteristic tensile strength of concrete
- $f_{vk}$ : is the characteristic yield strength of steel

Other reinforcement which depends on the force  $H_{bacd}$  and the friction force that acts in the bottom of the connection is still used. The area of this reinforcement is given by:

$$A_{s,bhot} = \frac{H_{botd} - \mu . F_{nb,d}}{2 f_{yd}}$$
(19)

$$F_{nb,d} = N_d - \mu H_{top,d} - F_{bu,y}$$
 (20)

If the friction force  $\mu$ . $F_{\tiny nbd}$  on the foundation base is larger than the resultant of bottom pressures  $H_{\tiny badd}$  the reinforcement  $A_{\tiny char}$  in the bottom of the walls is not necessary.

According to OLIN *et al.* [7], this design model is suggested when the eccentricity of axial load is large enough to produce a positive reaction  $H_{bad}$  although the friction force  $F_{fi,hd}$  acts on the base of column. Thus, the friction force  $F_{fi,hd}$ presents the opposite sense of that of  $H_{bad}$ . Therefore, this model differs of the WILLERT & KESSER [6] and OSANAI *et al.* [9] models, where this last model is shown later.

The vertical reinforcement of socket base should be calculated for the total transfer of forces in the connection between walls and foundation base, i.e., as a monolithic connection.

# 3.4 ELLIOTT (10) model

In ELLIOTT [10], two theoretical models are presented: one only considers the eccentric vertical load (Figure 7(a)), while the other also considers the shear force (Figure 7(b)).

For the model with the eccentric load without shear force, the contact pressures on the lateral faces and the base of



column generate, respectively, the vertical friction forces  $\mu$ . $H_{np,d} \in \mu$ . $H_{hot,d}$  on the lateral faces and the horizontal friction force  $\mu$ . $F_{nb,d}$  on the base. However, this friction force on the column base was not taken into account in the first design model, that means that the resultant of bottom pressures  $H_{hot,d}$  is equal to the resultant of top pressures  $H_{hot,d}$  and, consequently, the vertical friction forces are equal to  $\mu$ . $H_{np,d}$ . The force  $H_{np,d}$  acts on the opposite faces as a binary with distance  $z_i$  that is given by the largest value of the following equations:

$$z = (\ell_{emb} \ 0.1\ell_{emb})/2 = 0.45\ell_{emb}$$
(21)

or



where c is the cover of the reinforcement  $A_{\rm s,hm}$  regarding the top of pedestal walls.

As the distance z between the resultants of pressures  $H_{_{hptd}}$  and  $H_{_{hutd}}$  is small, the procedure proposed by ELLIOTT [10] seems to be more conservative than the other models that taken into account the friction.

From the equilibrium of moments in relation to the point A of Figure 7(a), the following equation is obtained:



where  $f_d$  ' is the design compressive strength of concrete obtained with cubic specimens. According to comments in MEHTA & MONTEIRO [11], this strength is equivalent to the strength obtained with cylindrical specimens  $15 \text{ cm} \times 30 \text{ cm}$  increased by 10% to 15%.

15 cm x 30 cm increased by 10% to 15%. ELLIOTT [10] suggests  $\ell_{emb}=1.5\,h$ , for any type of interface and any eccentricity. The value  $\mu=0.7$  is recommended for the friction coefficient of smooth interface.

In the second model, the effects of the shear force are included. A compression stress equal to  $0.4 f_d$  ' acts in the column width *b*. The distance  $\ell_v$  of Figure 7(b) is calculated by the following equation:

$$\ell_v = \frac{V_d}{0.4 f_{cd}!b}$$
(25)

From the equilibrium of moments in point A of Figure 7(b), the following expression is obtained:

$$N_{d}e+V_{d}.(0.1\ell_{emb}+0.5\ell_{v})=(\mu.0.4f_{cd}'bh\ell_{h})+[0.4f_{cd}'b\ell_{h}(0.9\ell_{emb}-\ell_{v}-\ell_{h})]$$
(26)

Thus,  $\ell_{\rm b}$  is calculated with the second degree equation shown below:

$$\ell_{\rm h}^{2} - (\mu.{\rm h} + 0.9\ell_{\rm emb} - \ell_{\rm v}).\ell_{\rm h} + \frac{M_{\rm d} + V_{\rm d}.(0.1\ell_{\rm emb} + 0.5\ell_{\rm v})}{0.4f_{\rm cd}!{\rm b}} = 0$$
(27)

Therefore, the resultant of top pressures in socket base is calculated according to the following expression:



The value  $0.1\ell_{emb}$  is substituted by the reinforcement cover  $\iota$  if  $\iota$  is the larger than  $0.1\ell_{emb}$ . The vertical friction force is just taken into account in the length  $\ell_b$ , because this force is mobilized with the bending and the effect of the horizontal shear force is considered in  $\ell_v$ . Unlike the comments in ELLIOTT [10], this horizontal shear force can generate friction on the interface of the compressed side, even because it causes the bending of wall 1, although with a small eccentricity.

The stresses on the opposite interfaces can not overlap so that  $\ell_{v} + 2\ell_{b} < 0.9(0.9\ell_{emb})$ . The top reinforcement setting around the column should

The top reinforcement setting around the column should be calculated to resist half of the total horizontal force of the top region of the connection  $((V_d + H_{lep,d}) / 2)$  plus half of the lateral force due to the inclination of the cavity  $(N_d \tan 5^\circ)$ , according to the equation (29). This reinforcement should be distributed in the top half of the embedded length.





### 3.5 OSANAI et al. (9) model

OSANAI *et al.* [9] presented a design model for the socket base connection of structures subjected to seismic actions. This model considers a normal reaction on the column base with an eccentricity and the friction forces among the column and the internal faces of the socket base.

When the axial and horizontal forces act at the column, a moment and a shear force appear on the column base.

These forces result in horizontal reactions, friction reactions on the interfaces and the vertical base reaction. Figure 8(a) shows the equilibrium of the connection forces. In order to facilitate the resolution of equilibrium forces, the total design model (Figure 8(a)) was split into two other models (Figures 8(b) e 8(c)) that take into account the mentioned forces.

For the formulation of the equilibrium equations, some hypotheses were adopted:



- The tensile strength of the foundation concrete is ignored;
- The tensile forces in the foundation are taken only by foundation reinforcement, ignoring the auxiliary reinforcement in the foundations, such as the hoops;
- The friction forces on interfaces are considered;
- The vertical reaction acts on the bottom face of the column.

For the calculation of the position of the concrete compressive resultant  $\xi'.\varkappa$ , the hypothesis of the materials strength in which the plane sections remain plane is considered. Figure 9 shows the ultimate stresses distribution with the following notation:

- R<sub>a</sub>: Compression force of concrete due to column axial force and moment in column section;
- R<sub>a</sub>: Compression force of column reinforcement due to column axial force and moment in column section;
- *b*: Cross section height of column;
- *d*<sup>\*</sup>: Distance from extreme compression and tensile fibers to centre of compressive stresses and tensile reinforcements, respectively;

- R<sub>a</sub>: Tensile force of column reinforcement due to column axial force and moment in column section;
- x: Distance from extreme compression fiber to neutral axis;
- $\xi'_{.x}$ : Distance from extreme compression fiber to point where  $R_{\omega}$  is acting.

Therefore, the horizontal force  $V_{Id}$  of model 1 and the reaction  $H_{iopl,d}$  are calculated by the equilibrium conditions and are given by:

$$H_{top1,d} = V_{1d} = \frac{M_{1d}}{e_v + y} \cong \frac{M_{1d}}{e_v} = (0.5h - \xi' \cdot x) \cdot \frac{N_d}{e_v}$$
(30)

where  $N_d = F_{nhd}$  and y is negligible compared with  $e_r$ .

The connection stresses distribution of model 2 is shown in Figure 10.

The force  $R_{botd}$ , acting in the position y", is the sum of the horizontal reaction  $H_{botd}$  and the friction force  $F_{iri,bd}$ .



 $R_{bot,d} = H_{bot,d} + F_{fit,b,d}$ (31)

$$y'' = (\ell_{emb} - 2.y)/6$$
 (32)

From the equilibrium of forces and moment of model 2, the following equation for the reaction  $H_{_{Inp2,d}}$  is obtained:



The reaction  $H_{\rm hpd}$  is transmitted to the foundation reinforcement and is given by the sum of equations (30) and (33). Denominating the eccentricity of the vertical base reaction of column related to its gravity center of  $e_{\rm nb} = 0.5h - \xi'.x$ , the following equation for  $H_{\rm hord}$  is obtained:



The OSANAI *et al.* [9] model considers the three friction forces ( $F_{jri,top,d}$ ,  $F_{jri,top,d}$ , e,  $F_{jri,tod}$ ,  $F_{bold}$ ,  $F_{$ 



$$H_{topd} = \frac{M_{d} - \left[e_{nb} - \frac{\mu^{2}(0.5h + e_{nb})}{1 + \mu^{2}}\right] N_{d} + \left[\frac{5\ell_{emb}}{6} + \frac{y}{3} + \frac{\mu(0.5h + e_{nb})}{1 + \mu^{2}}\right] N_{d}}{\frac{5\ell_{emb}}{6} - \frac{2y}{3} + \mu.h}$$
(35)

This model is appointed in the theoretical application of the following item as OSANAI *et al.* [9] modified model.

In OSANAI *et al.* [9], specific recommendations about the embedded length  $\ell_{emb}$  are not presented, although the experimental results of the rough specimens with  $\ell_{emb} > h$  were similar to those of the smooth specimens with  $\ell_{emb} > 1.5 h$  and these connections behaved as rigid connections.

### 3.6 CANHA (2) model

Due to the need of experimental results to fundament a more consistent theoretical model, CANHA [2] carried out an experimental investigation in specimens of socket base connection. A design model based on these experimental results is proposed for the socket base with smooth interface. This proposed model considers the contribution of the friction forces  $F_{jri,log,d}$ ,  $F_{jri,log,d}$ , and the eccentricity  $e_{ab}$  of the vertical reaction on the column base  $F_{ahd}$ . The scheme of the forces in the connection is shown in Figure 11.

The top, bottom and foundation base friction forces are defined by the friction coefficient times the corresponding normal force according to the equations below:



From the equilibrium conditions, the following equations are obtained:

• Equilibrium of vertical forces:



### Equilibrium of horizontal forces:

$$V_{d} + F_{fribid} - H_{topid} + H_{boxid} = 0$$
 (40)

Equilibrium of moments at point O:

$$M_{d} - N_{d}e_{nb} + V_{d}\ell_{emb} - H_{topd}(\ell_{emb} - y) + H_{bot,d}y' - F_{frijtopd}(0.5h - e_{nb}) + -F_{frijtopd}(0.5h + e_{nb}) = 0$$
(41)

Combining the equations (39) and (40) and replacing the values of equations (36) to (38),  $F_{j_{fi,h,d}} e H_{bad}$  are:



Then, values of  $H_{bot,d'}$   $F_{fri,bot,d}$  and  $F_{fri,bot,d}$  are substituted in equation (41), that results in the following expression for calculation of  $H_{bot,d'}$ .



The main difference of these proposed model in relation to the OSANAI *et al.* [2] adapted model, presented in the former item, is that the first model considers the friction force  $F_{_{frihd}}$  acting on column base for the assembly of moment in expression (41), and the last model is simplified, that is, bottom horizontal force  $H_{_{held}}$  and friction force  $F_{_{frihd}}$  act on height y" equivalent to half of y'.

With the known forces  $M_a$ ,  $N_a$  and  $V_a$  on column, the expression (44) is recommended for designing of socket base with smooth interface, using the parameters  $e_{ab} = h / 4$ ,  $y = \ell_{emb} / 6$ ,  $y' = \ell_{emb} / 10$  and  $\mu$  according to the form material of the connection elements. These values are appropriate for cases in which the embedded length is not smaller than the value suggested by NBR-9062/85 [4], which was 2h for this case.

This model should be applied for cases of large eccentricity, in which the predominant action of moment about axial



force tends to generate friction force  $F_{frihd}$  on the foundation base with the same direction of  $H_{badd}$  and friction force  $F_{frihdd}$ at transverse wall 2 with upward direction and at the column with downward direction, as showed in Figure 11. For small eccentricity, the proposed equation could be used after an experimental investigation, and the correct directions of friction forces  $F_{frihdd}$  and  $F_{frihd}$  should be analyzed, which can be influenced by the relation among the forces  $M_{dt}$ ,  $V_{d}$  and  $N_{d}$  and by the geometry.

As the two specimens with rough interface presented a behavior very close to a monolithic connection, in other words, the total transfer of moment and normal force from column to socket base was verified, for socket base with rough interface by shear keys, the design of vertical reinforcements by bending theory is suggested. However, this model should be applied for rough socket bases with embedded length not smaller than *1.6b*, that is suggested by NBR-9062/85 [4].

# 4 Comparison and analysis of results of the design models and recommendations

In order to present the main differences among the models, these were applied in the pedestal walls design of a socket base, whose forces and dimensions are shown in Figure 12 and design characteristics of materials are indicated in Table 2.

Initially, the analyses with smooth and rough interfaces were done, adopting the embedded length suggested by NBR-9062/85 [4] for the case of large eccentricity ( $M_d$  /  $N_d \geq 2b$ ).

In the LEONHARDT & MÖNNIG [3] model, some recommendations presented by NBR-9062/85 [4] and EL DEBS [1] were considered. The WILLERT & KESSER [6], ELLIOTT [10] and OSANAI et al. [9] models do not present recommendations for the calculation of the main and secondary vertical reinforcements. According to OLIN et al. [7], these reinforcements should be calculated by the bending theory, considering the total transfer of forces in the connection between pedestal walls and foundation base. The proposed model of CANHA [2] adopt the recommendations of the LEONHARDT & MÖNNIG [3] model, in which the longitudinal wall (walls 3 and 4 of Figure 2) should be calculated as a corbel. For unifying the results, the main vertical reinforcement was calculated using the LEON-HARDT & MÖNNIG [3] model, and the secondary vertical reinforcement was calculated by the recommendation for short corbel presented in EL DEBS [1], so that  $A_{cor}$  is equal to 0.4A

Table  $3^{5}$  shows the main results of the application of the design models for socket base with smooth interface.

The LEONHARDT & MÖNNIG [3] model, that ignores the

Table 2 – Materials design characteristics of calculation example						
Concrete	Socket base	Column/Joint				
Classification	C25	C35				
Characteristic compression strength ( $f_{\mbox{\tiny ck}}$ )	25 MPa	35 MPa				
Lower characteristic tensile strength* (f <sub>ctk.low</sub> )	1.8 MPa	2.3 MPa				
Characteristic compression strength related to cubic specimens $(f_{ck}'=1, 1f_{ck})$	2.5 MPa	38.5 MPa				
Reduction coefficient of strength ( $\gamma_{c}$ )		1.4				
Modulus of elasticity $*(E_{cs})$	23.8 MPa	28.2 MPa				
Steel	С	A-50				
Characteristic yield strength ( $f_{yk}$ )	500	) MPa				
Reduction coefficient of strength ( $\gamma_s$ )	1	.15				
Modulus of elasticity (E <sub>s</sub> )	210	) GPa				
Interface	Frict	ti <b>on (</b> μ)				
Smooth		0.6				
Rough		1.0				
* calculated according to NBR-6118/03 (12)						

Table 3 – Results obtained by the design models for socket base with smooth interface							
Design model		LEONHARDT & MÖNNIG (3)	WILLERT & KESSER (6)	OLIN et al.(7)	ELLIOTT (10)	OSANAI <i>et al.</i> (9) modified	CANHA (2)
	$H_{top,d}$	437.50	316.37	309.26	277.77 <sup>b</sup>	291.10	281.25
Main	$H_{bot,d}$	387,50	216.06	259.26	227.77	166.10	156.25
(kN)	$F_{vm,d}$	231.85	167.66	163.89	147.20	154.23	149.05
	$R_{\scriptscriptstyle cd}$	318,76	230.51	225.32	202.38	212.09	204.92
	A <sub>s,hm</sub>	5,03	3.64	3.56	3.19	3.35	3.23
	$A_{s,hbot}$	-	-	2.88	-	-	-
	A <sub>s,vm</sub>	5.33	3.86	3.77 8.06°	3.39	3.55	3.43
Concrete (kN/cm²)	f <sub>ck,mín</sub> c	1.68	1.22	1.19	1.07	1.12	1.08

Note 1:  $\ell_{emb}$ =80cm

Note 2: µ=0.6

Note 3: Forces and reinforcements indicated on Figure 2

 $^{\rm a}$  sum of 2A\_{\rm svm} and A\_{\rm svs} (design of the socket base as monolithic)

 $^{\rm b}$  sum of H\_{top,d} + V\_d

 $^\circ$  related to the corbel verification

Table 4 – Results obtained by the design models for socket base with rough interface							
Design model		LEONHARDT & MÖNNIG (3)	WILLERT & KESSER (6)	OLIN et al.(7)	ELLIOTT (10)	OSANAI <i>et al.</i> (9) modified	CANHA (2)
	$H_{\rm top,d}$	435.00	303.14	329.34	272.62 <sup>5</sup>	282.71	274.43
Main	H <sub>bot,d</sub>	385.00	205.33	279.34	222.62	157.71	149.43
(kN)	$F_{vm,d}$	188.11	131.09	142.42	117.89	122.25	118.67
	$R_{\scriptscriptstyle cd}$	287.56	200.39	217.71	180.22	186.89	181.42
	A <sub>s,hm</sub>	5.00	3.49	3.79	3.14	3.25	3.16
	$A_{s,hbot}$	-	-	0.71	-	-	-
(,	A <sub>s,vm</sub>	4.33	3.02	3.28 7.73°	2.71	2.81	2.73 7.73°
Concrete (kN/cm²)	f <sub>ck,mín</sub> c	1.69	1.18	1.26	1.06	1.10	1.07

Note. 1:  $\ell_{\text{emb}}$ =64cm

Note. 2: µ=1

Note 3: Forces and reinforcements indicated on Figure 2

 $^{\circ}$  sum of 2A\_{s,vm} and A\_{s,vs} (design of the socket base as monolithic)

<sup>b</sup> sum of  $H_{top,d} + V_d$ 

 $^{\circ}$  related to the corbel verification

friction, was the most conservative one. Among other models, the ELLIOTT [10] model was the least conservative, although it considers that the main horizontal reinforcement  $A_{s,bar}$ , in this case, is calculated to resist to the force  $(H_{upd} + V_d)/2$ . Although the proposed model of CANHA [2] differs a little of the ELLIOTT [10] model, the difference between the results of these models was close to 1%. The largest and smallest percentile differences related to the LEONHARDT & MÖNNIG [3] model were -37% and -28%, respectively, that refer to the ELLIOTT [10] and WILLERT & KESSER [6] models. In this case, the negative percentile differences indicate a reduction of the reinforcement area or force of the corresponding model related to the LEON-HARDT & MÖNNIG [3] model.

Other two calculations were done in the OLIN *et al.* [7] model, according to their recommendations, i.e., the total stressed vertical reinforcement  $(2A_{sym} + A_{sys})$ , considering the monolithic connection, and the bottom horizontal reinforcement  $A_{show}$  to transmit the force  $H_{bad}$  / 2 reduced by half of the friction force of foundation base  $F_{gi,kd}$ . This vertical reinforcement  $2A_{sym} + A_{sys}$  for the monolithic connection. However, unless the proximity of behavior of the smooth socket base with that of a monolithic connection is experimentally proven, the largest reinforcement should be used, i.e., that calculated according to the working of longitudinal walls 3 and 4 as corbel. The reinforcement  $A_{show}$ 

seems not to have a meaning, because due to the small distance between the resultant of pressure  $H_{\rm bold}$  on wall 2 and the base and as the bottom region of the stressed side is much more rigid than the top region of the opposite side, this pressure can be directly transmitted to the foundation base.

The main results of the analysis by the design models of the socket base with rough interface are shown in Table 4. In case of OLIN *et al.* [7] and CANHA [2] models, the main  $A_{\rm srm}$  and secondary  $A_{\rm sc}$  vertical reinforcements were calculated according to the corbel recommendations, and the total vertical reinforcement of traverse wall 2 ( $2A_{\rm srm} + A_{\rm srs}$ ) was calculated for the monolithic connection.

Considering the main forces on pedestal walls, the LEON-HARDT & MÖNNIG [3] model was the more conservative. Among the models that really consider the friction, the ELLIOTT [10] and CANHA [2] models were the least conservatives and OLIN *et al.* [7] model presented the largest forces and reinforcements. The difference between the CANHA [2] and ELLIOTT [10] model was smaller than *1%*. The differences among the results of the CANHA [2] and OLIN *et al.* [7] models related to those of the LEONHARDT & MÖNNIG [3] model were, respectively, *-37%* and *-24%*.

Some observations should be done regarding the OLIN *et al.* [7] and CANHA [2] models. First, the difference between the total vertical reinforcement of the stressed side  $(2A_{s,rm} + A_{s,rs})$  for the calculation of the monolithic connection and this



reinforcement considering the corbel design, for the OLIN *et al.* [7], was practically negligible model, close to 2%, and for the CANHA [2], was 18%. The reinforcement  $A_{sheat}$  in the bottom region of the pedestal walls, according to the OLIN *et al.* [7] model, was very small and is dispensable for smooth and rough interfaces.

Comparing the two cases of interface, for the LEONHARDT & MÖNNIG [3] model, the resultant of top pressures  $H_{\mu\nu}$ with the corresponding reinforcement  $\mathcal{A}_{\mbox{\tiny s,hm}}$  was  $\mbox{ practically}$ equivalent, while the reinforcement area A of the smooth socket base was reduced to 19% related to the rough socket base. Among the models that take into account the friction, only the OLIN et al. [7] model presented an increase of 6% for the reinforcement  $A_{s,bm}$ , while, for the other models, the reduction of the rough socket base related to the smooth socket base was insignificant, between 2% and 4%. Considering yet that the main vertical reinforcement  $A_{max}$ was also reduced for up to 22%, the volume decrease of concrete and if there is facility of roughness building, the increase of the friction coefficient of 0.6 of smooth socket for 1 of rough socket in the design models that consider the friction show that the rough socket is very economical. As CANHA [2] proved the behavior of the rough socket base with shear keys close to a monolithic connection, the vertical reinforcement for this socket base should be calculated according to the bending theory.

Other comparative analysis among the design models was done, changing the embedded length to the values equal to 1b (40 cm), 1.5b (60 cm), 2b (80 cm) and 2.5b (100 cm) and the friction coefficient to the values equal to 0, 0.6 and 1. Although the embedded length equal to 40 cm is lightly below the minimum value recommended in the literature equal to 1.2b, this value was just used in this work for effect of this analysis.

The results of the calculation regarding the main horizontal reinforcement  $A_{s,lm}$  and the main vertical reinforcement  $A_{s,rm}$  are presented. For very short corbel, according to EL DEBS [1], the largest main vertical reinforcement between very short corbel and short corbel was used. The same procedure was employed for long corbel, which was calculated as beam and as short corbel.

Figure 13 shows the area of the main horizontal reinforcement  $\mathcal{A}_{\tiny adm}$  with the variation of the embedded length  $\ell_{\tiny emb}$  and Figure 14 shows this area according to the change of the friction coefficient  $\mu$ .

As the embedded length  $\ell_{emb} = 40 \text{ cm}$  for  $\ell_{emb} = 100 \text{ cm}$ , the calculation of this main horizontal reinforcement is more economical, with reduction of up to 64%, 49% and 46%, re-



spectively, for  $\mu = 0$ ,  $\mu = 0.6$  and  $\mu = 1$ . For these two last coefficients, only the models that consider the friction were used. The reduction of the steel area for the specimens with  $\ell_{emb} = 100 \text{ cm}$  for  $\ell_{emb} = 80 \text{ cm}$  was smaller than that of the specimens with  $\ell_{emb} = 80 \text{ cm}$  and  $\ell_{emb} = 60 \text{ cm}$ , considering the constant increase of concrete volume.

 $\ell_{emb} = 1h = 40 \text{ cm}$  was also used in case of smooth interface just for comparison effect. This embedded length can cause large strains in the connection and the connection can not remain perfectly rigid. Thus, this embedded length should be avoided.

Despising the friction, the LEONHARDT & MÖNNIG [3] model was usually the more conservative one, where this reinforcement area was very close to that of the WILLERT & KESSER [6] model, with differences about 4%. As this percentile difference is positive, it represents the increase of reinforcement area or force of the LEONHARDT & MÖN-

NIG [3] model regarding to the referred model. Only for  $\ell_{emb} = 40 \text{ cm}$ , the area  $A_{_{abm}}$  according to the ELLIOTT [10] model presented a value larger than that of LEONHARDT & MÖNNIG [3] model, but very close and with a despicable difference to the LEONHARDT & MÖNNIG [3] model. In reality, as there was not a real root in equation (27) for the length  $\ell_{b}$  of the ELLIOTT [10] model in which the vertical friction force acts (see Figure 7(b)), for  $\ell_{emb} = 40 \text{ cm}$ ,  $\ell_{b}$  was approximated for half of  $0.9\ell_{emb} - \ell_{v}^{0}$ . Except for this case, all models provided areas of this reinforcement smaller than that of the LEONHARDT & MÖNNIG [3] model, with differences between -4% and -21%. However, as the friction coefficient was increased, the disparity in the design between these models and the LEONHARDT & MÖNNIG [3] model was increased, with differences, per example, of -28% up to -37% for  $\ell_{emb} = 80 \text{ cm}$  and  $\mu = 0.6$ .

For  $\mu = 0.6$  and  $\mu = 1$ , the models that take into account the



friction presented results relatively close to  $A_{shu}$  for the embedded lengths equal to 1.5*h*, 2*b* and 2.5*h*, and the WIL-LERT & KESSER [6] and OLIN *et al.* [7] models were the more conservative among these models. In the ELLIOTT [10] model, for  $\mu = 0.6$ , the area of reinforcement  $A_{shu}$  was decreased, regarding that of  $\mu = 0$ , for 37%, 25% and 19%, respectively, for  $\ell_{emb} = 60 \text{ cm}$ ,  $\ell_{emb} = 80 \text{ cm}$  and  $\ell_{emb} = 100 \text{ cm}$ . For the OSANAI *et al.* [9] modified and CANHA [2] models, this reduction was a little smaller, but the differences were always smaller for larger embedded length. That is, as  $\ell_{emb}$  is increased, the forces  $H_{spd}$  and  $H_{bad}$  decrease, and, consequently, the friction forces caused by these resultants of pressure are reduced.

Although the LEONHARDT & MÖNNIG [3] model do not consider the friction, the resultants of pressure ( $H_{_{hp,d}}$  and  $H_{_{hu,d}}$ ) on traverse walls are different for the cases of smooth and rough interface ( $\mu = t$ ). Thus, using this model and considering the same embedded length,  $A_{_{a,hm}}$  of rough socket base was reduced in relation to that of smooth socket base.

Figures 15 and 16 show, respectively, the area of the main vertical reinforcement  $A_{_{\rm AMM}}$  with the variation of the embedded length and of the friction coefficient.

For  $\ell_{emb} = 60 \text{ cm}$  and  $\ell_{emb} = 80 \text{ cm}$ , the longitudinal wall was designed as short corbel ( $0.5 \le a_c / d_c \le 1$ ). In case of  $\ell_{emb} = 40 \text{ cm}$ , where  $a_c / d_c \le 0.5$ , the reinforcement was

calculated for a very short corbel, that was larger than that of designing as a short corbel. For  $\ell_{emb} = 100 \text{ cm}$ , the relation  $a_c/d_c$  was larger than the unit and the vertical reinforcement was calculated for a beam clamped in foundation base and submitted to a force  $H_{top,d}$  / 2 at the free extremity. As the three corbel models foresaw, as larger the embedded length, larger the main vertical reinforcement. Although the increase has not been so expressive, the increase of this reinforcement area in case of  $\ell_{\rm emb}$  = 80 cm regarding  $\ell_{mh} = 60 \ cm$ , considering  $\mu = 0.6$ , per example, varied between 8% and 14%. For rough interface ( $\mu = 1$ ), this increase in case of  $\ell_{\rm \tiny emb}=60~{\rm cm}$  regarding  $\ell_{\rm \tiny emb}=40~{\rm cm}$  was between 9% and 21% for the design models that take into account the friction. These embedded lengths were compared, because an experimental verification can make possible an embedded length close to 1.5h for smooth interface and 1.2h for rough interface. The embedded length 100 cm is anti-economical, because besides the increase of concrete volume, the sum of the areas  $A_{ij}$  and  $A_{ij}$  was larger than that of  $\ell_{emb} = 80 \text{ cm}$ , although the differences have been smaller than 10%.

Considering the LEONHARDT & MÖNNIG [3] model, the values of the area  $A_{xxxx}$  were also different between the smooth and rough sockets, due to the same reasons previously presented, that is, due to the different resultants

of pressure ( $H_{_{hp,d}}$  and  $H_{_{hu,d}}$ ) for the two cases of interface. As it was waited, the consideration of friction in the connection design increased the discrepancy of the main vertical reinforcement  $A_{_{LTM}}$  calculated according to the LEON-HARDT & MÖNNIG [3] model and those of the other design models. For  $\mu = 0.6$  and  $\mu = 1$ , these reinforcement areas calculated by the models that consider the friction were relatively close to each other, where the ELLIOTT [10] and CANHA [2] models were the least conservative models.

### 5 Final remarks and conclusions

Considering the results of this analysis of the design models of literature, the main aspects were observed:

• There was a discrepancy in the design by the LEON-

HARDT & MÖNNIG [3] and by the other design models. Even among the models that consider the friction, there were differences due to the variation of forces position and the friction forces are not taken into account. For the smooth socket with  $\ell_{emb} = 2h$ , per example, the difference of the LEONHARDT & MÖNNIG [3] model regarding the other design models with  $\mu = 0.6$  varied from 38% to 58%.

• The friction is very important in the connection design and, thus, should be taken into account in order to induce to a more rational calculation.

• As expected, for the socket with rough interface ( $\mu = 1$ ), the concrete volume and de reinforcements areas regarding the socket with smooth interface ( $\mu = 0$ ) are reduced, even in the case of the LEONHARDT & MÖNNIG [3] model, where the resultants of pressure are different for the two



types of interface. However, for the choice of the type of interface, the difficulties of building the two rough interfaces, of the socket base and the column, should be evaluated, that is, if they compensate the materials economy.
Among the analyzed design models, the CANHA [2] model, proposed by the authors, is more suitable for using, besides it do not present results oscillations due to the change of variables.

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