COMPUTATIONAL MODELING OF CRACKING OF CONCRETE: PAST, PRESENT AND FUTURE TRENDS

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MOTIVATION

(htttp://en.wikipedia.org/wiki/Reinforced_concrete)

- ….. on a human time-scale, small usages of concrete go back for thousands of years…. the Romans used concrete extensively from 300 BC to 476 AD,…..
- …the use of reinforced concrete is usually dated to 1848 when Jean-Luis Lambot became the first to use it. Joseph Monier, a French gardener, patented a design for reinforced garden tubs in 1868, and later patented reinforced concrete beam and posts for railway and road guardrails….
- …..as of 2005 over six billion tons of concrete are made each year, amounting to the equivalent of one ton for every person on Earth,…..

COMPUTATIONAL MODELING OF CONCRETE
… still a challenge in Computational Mechanics ?

- ☞ Composite material (complex component interaction)
- ☞ Failure dominated by (non-linear) material instability (complex mechanics)
- ☞ Unsymmetrical (tension-compression) behavior
- ☞ Multiple cracks (computational cost, robustness)
- ☞ Time-dependent effects (creep, shrinkage, aging .. etc..)

OUTLINE

I. CONCRETE: A COMPLEX MATERIAL

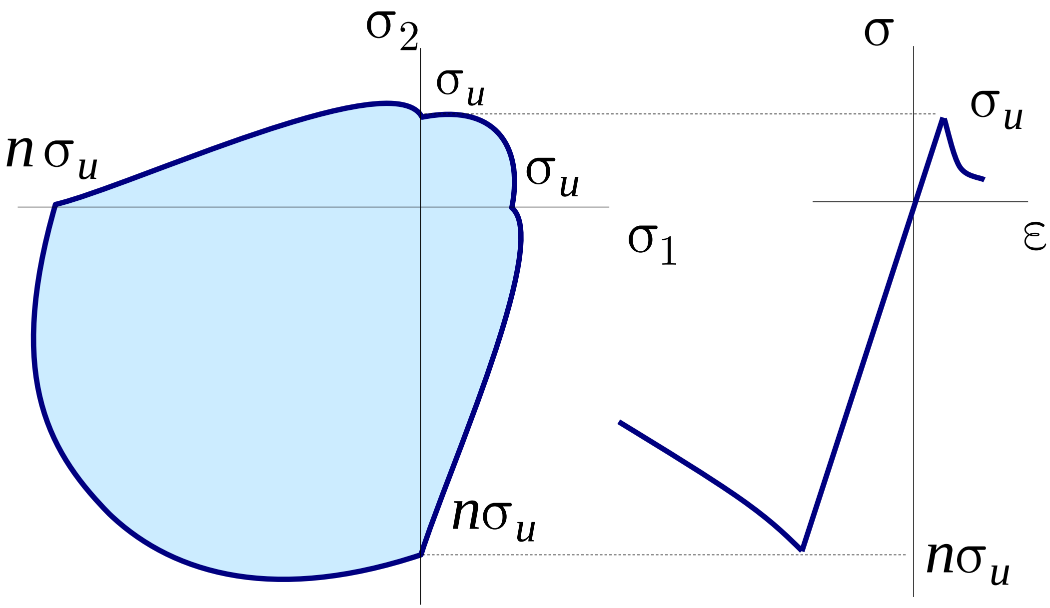
II. NUMERICAL SIMULATION OF CRACKING OF CONCRETE

III. MECHANICAL APPROACHES TO CONCRETE MODELING

IV. ABOUT THE FUTURE

CONCRETE: A COMPLEX MATERIAL

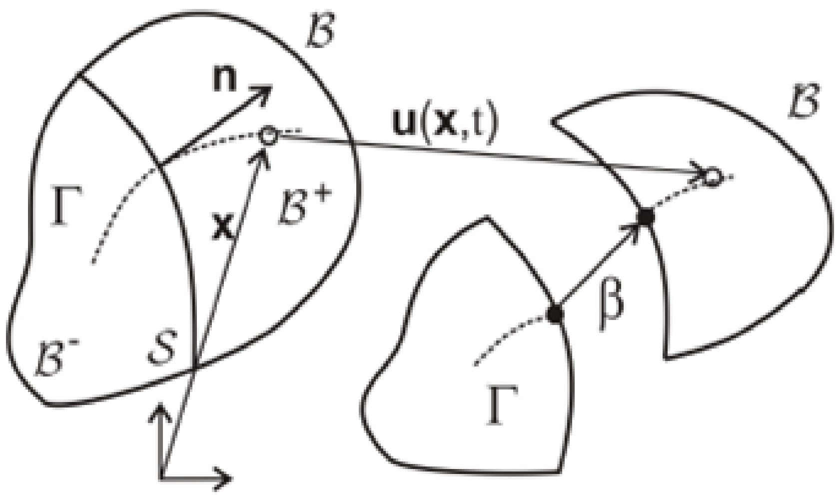
- Unsymmetrical (tension/compression) material

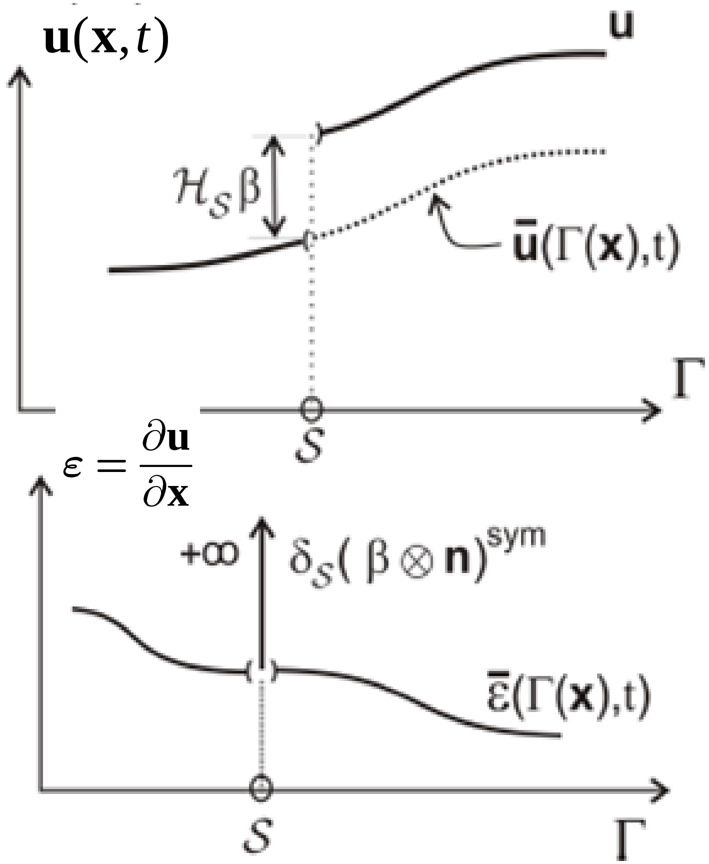


- Low tensile strength → **CRACKING**

CONCRETE: A COMPLEX MATERIAL

- Discontinuous displacement field (strong discontinuities)

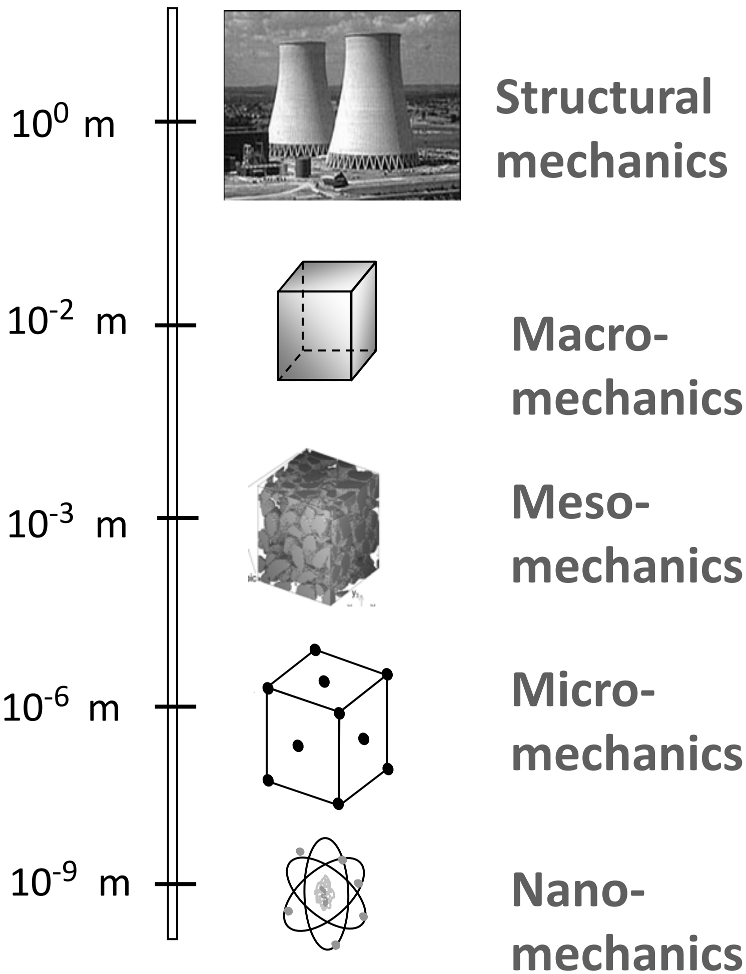




- ☞ Continuum mechanics fails !!!!
- ☞ Crack path is not known in advance (crack onset and propagation)

CONCRETE: A COMPLEX MATERIAL

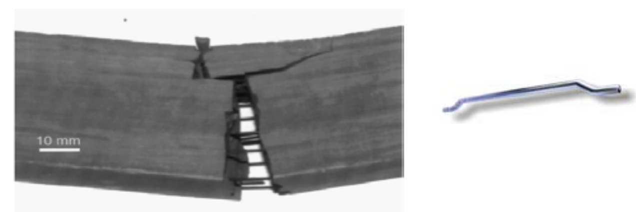
- Inhomogeneous material at multiple scales



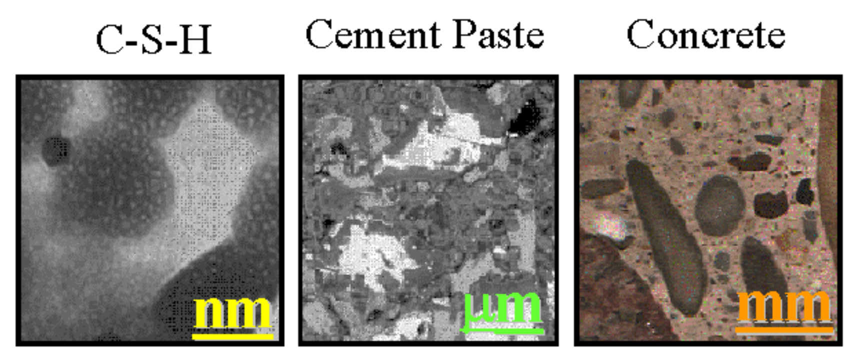
Multiscale material mechanics, Willam K. (2000).

- Reinforced concrete (mortar + aggregates+ rebars)

- Fiber-reinforced concrete (mortar + fibers)



- Plain concrete (mortar + aggregates)



NUMERICAL MODELING OF CRACKING OF CONCRETE

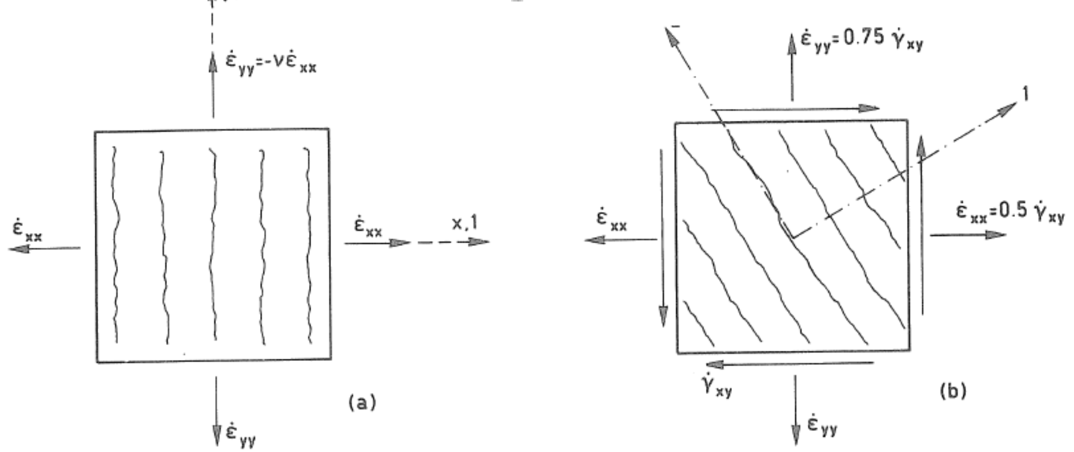
- Computational approaches to concrete fracture
- Crack path modeling strategies

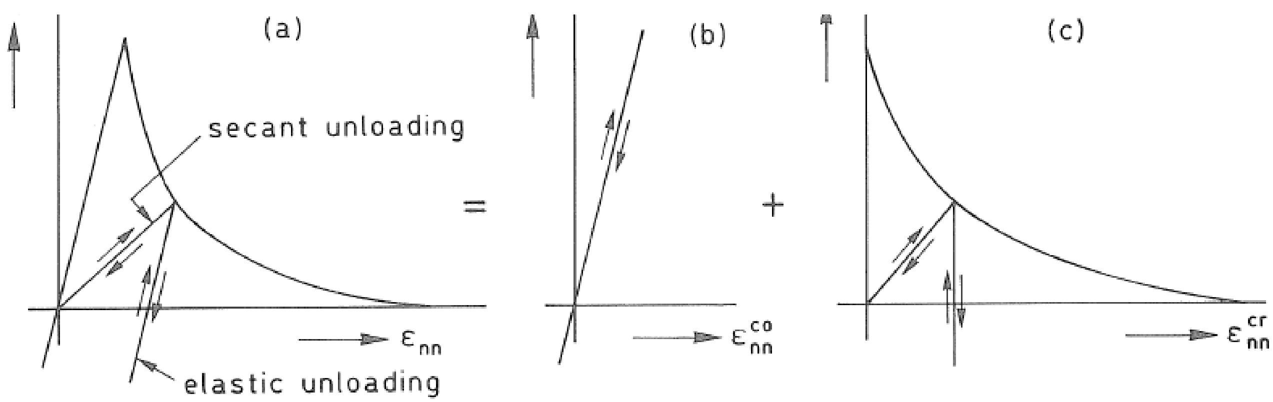
COMPUTATIONAL APPROACHES TO CONCRETE FRACTURE

1. SMEARED CRACK APPROACH Rashid (1968), Rots (1988)

- "Fuzzy" description of the crack geometry

$$\Delta\varepsilon = \Delta\varepsilon^{cr} + \Delta\varepsilon^{co}$$

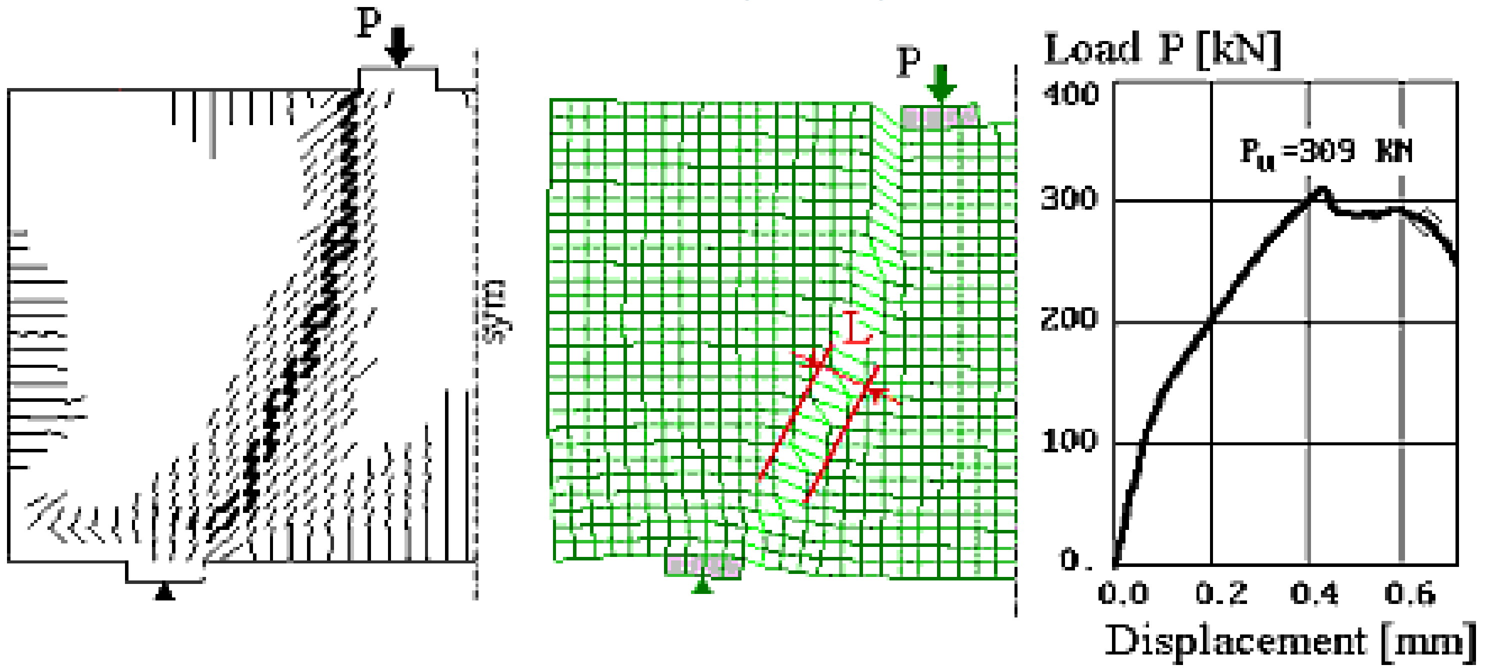




$$\Delta\sigma = [D^{co} - D^{co}N[D^{cr} + N^T D^{co} N]^{-1} N^T D^{co}]\Delta\varepsilon$$

COMPUTATIONAL APPROACHES TO CONCRETE FRACTURE

1. SMEARED CRACK APPROACH (cont.)



- Standard finite elements are used
- Results dependent om the mesh bias (**lack of mesh-objectivity**)
- Good results for pre-peak responses and heavily reinforced concrete structures

☞ Industrialized !!!! (used in commercial codes)

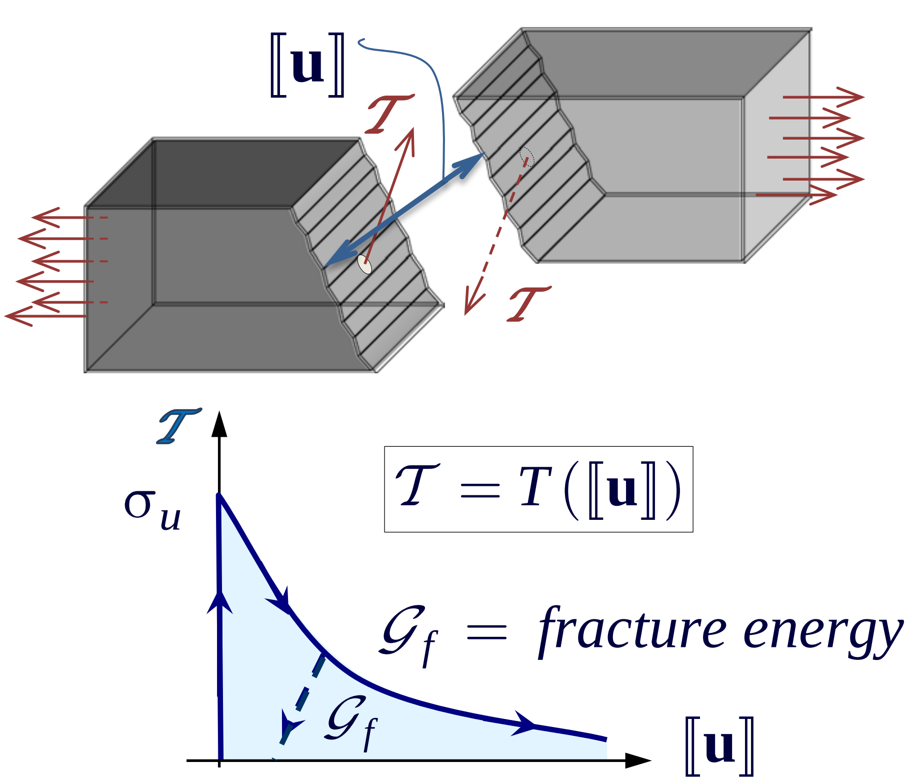
COMPUTATIONAL APPROACHES TO CONCRETE FRACTURE

2. DISCRETE CRACK CRACK APPROACH

❑ Crack: individual jump in the displacement field (Strong discontinuity)

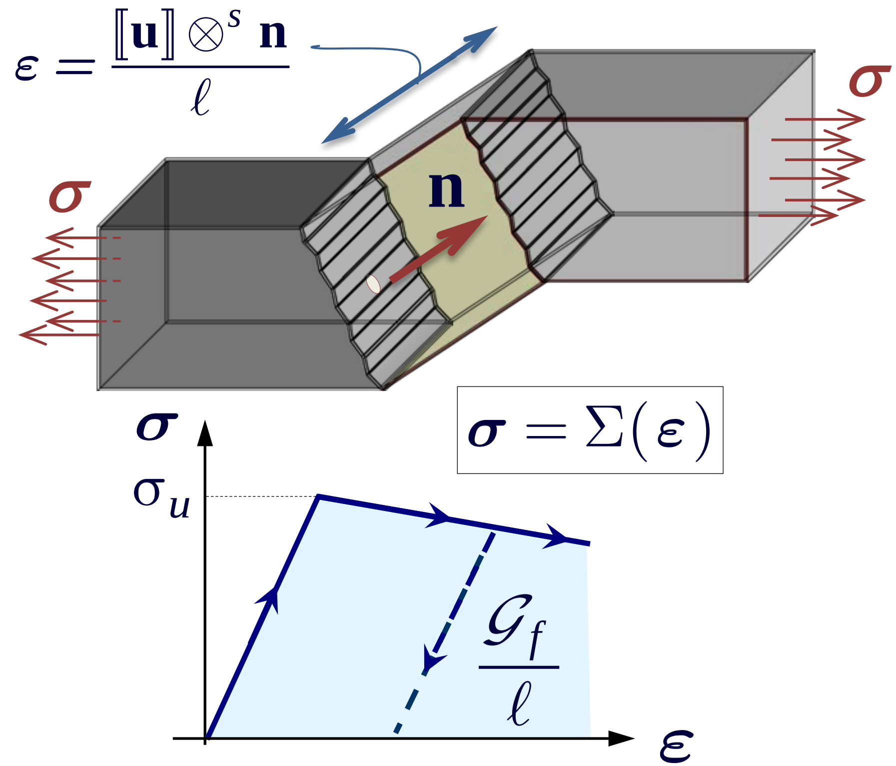
1. COHESIVE FRACTURE MECHANICS (Hillerborg 1976)

- de-cohesive traction-separation law



2. CONTINUUM-STRONG-DISCONTINUITY APPROACH (CSDA) (O. Manzoli, 1988)

- stress-strain softening law



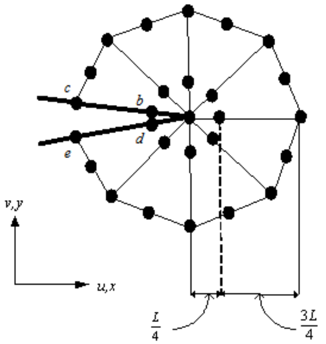
NUMERICAL MODELING OF CRACKING OF CONCRETE

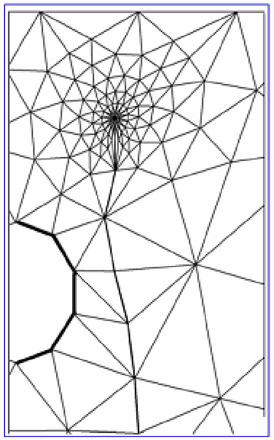
- ❑ Computational approaches to concrete fracture
- ❑ Crack path modeling strategies

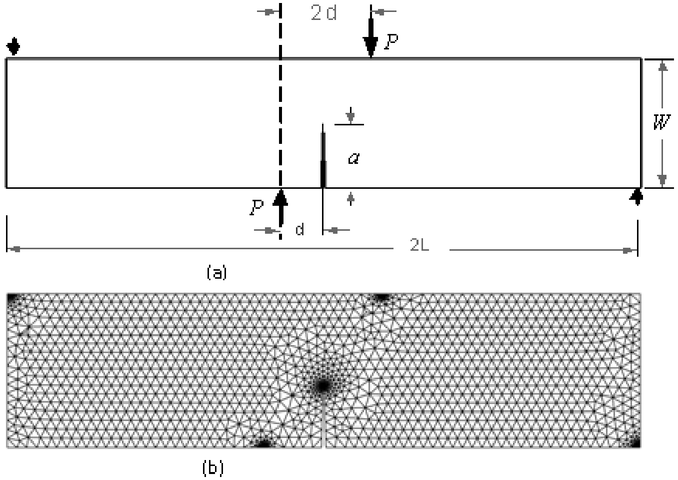
CRACK PATH MODELING STRATEGIES

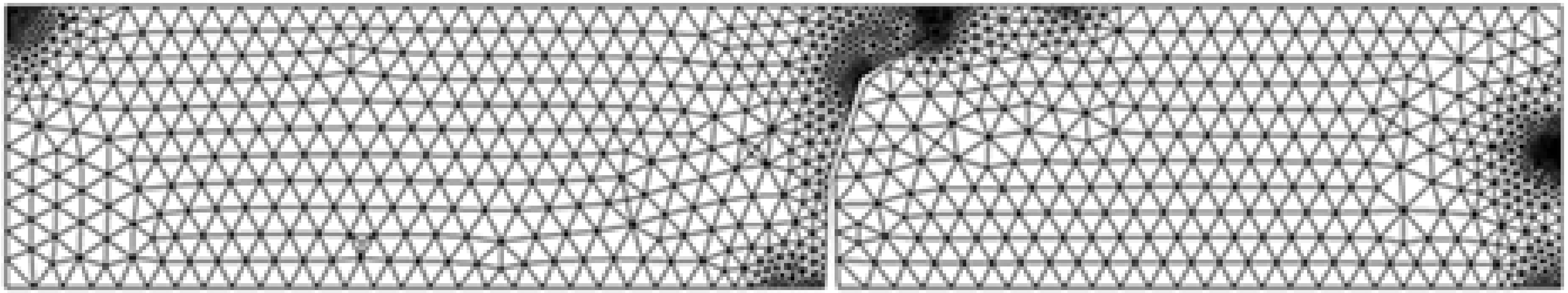
1. REMESHING STRATEGIES (linear fracture)

- Crack tip remeshing (Wawrzynek and Ingraffea 1987)







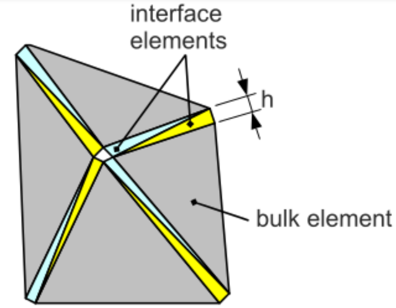


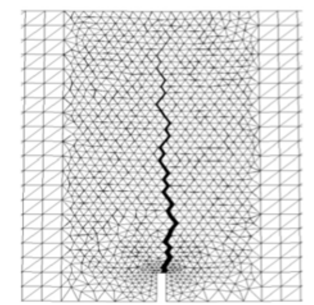
Souiyah Miloud et. al. ,Int. J. Mat. Eng. 2012

CRACK PATH MODELING STRATEGIES

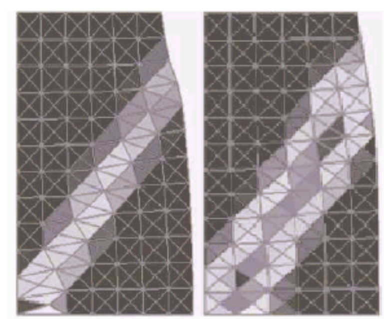
2- FIXED MESH STRATEGIES (2D-3D)

- **Inter-elemental** crack-capturing
  - o Cohesive interface elements
    (M.Ortiz/A.Pandolfi 1999, O. Manzoli 2012)

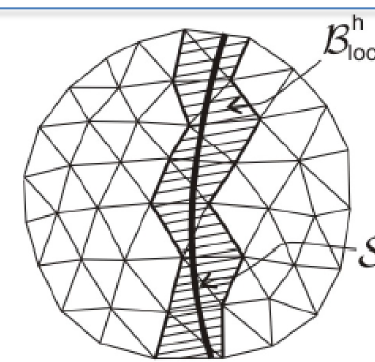




- **Extra-elemental** crack-capturing
  - o Non-local stress-strain approaches
  - o Gradient-based approaches
  - o Phase-field-based models



- **Intra-elemental** crack-capturing
  - o E-FEM techniques
    (Simo, Oliver, Armero 1993)
  - o X-FEM techniques
    (N. Möes, J. Dolbow, T. Belytschko 1998)

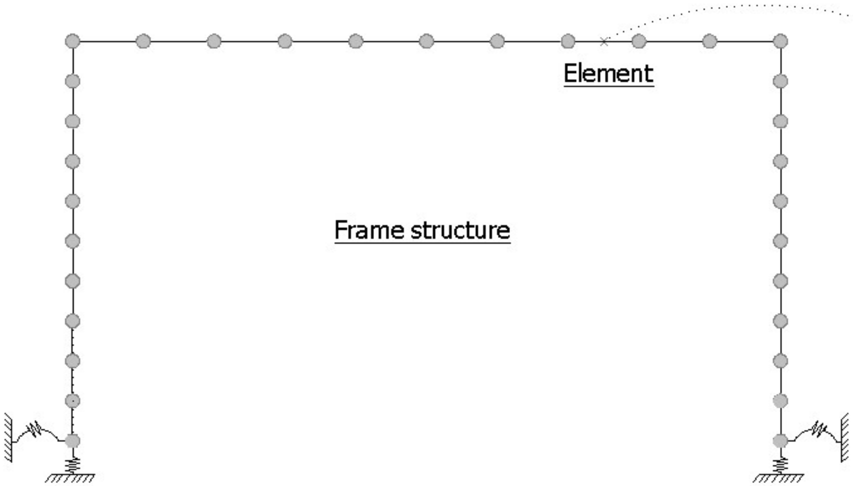


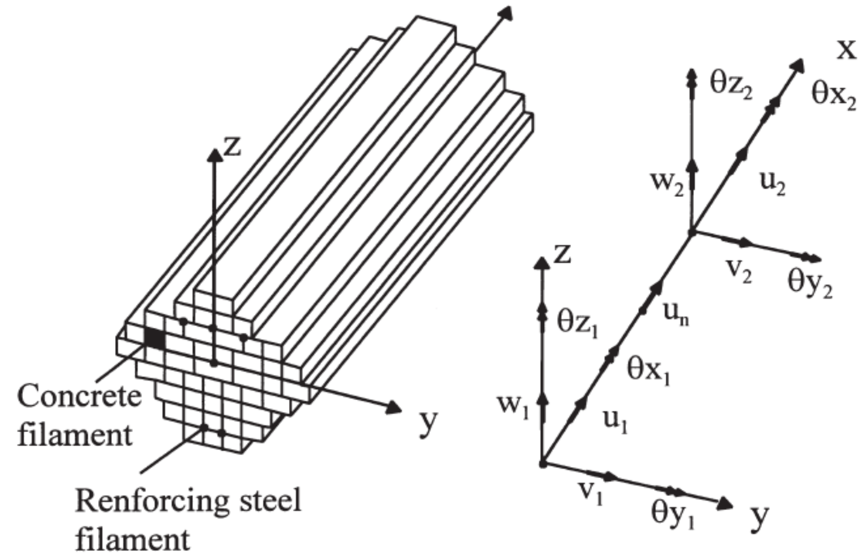
MECHANICAL APPROACHES TO CONCRETE CRACKING

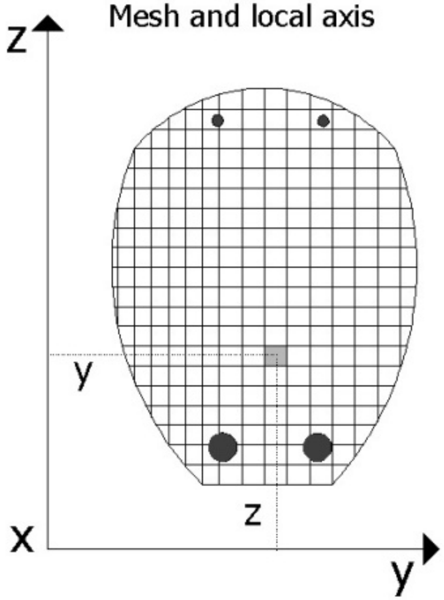
- Fiber/filament beam models
- Concrete as a composite material
- Micro-structure endowed material
- Computational homogenization

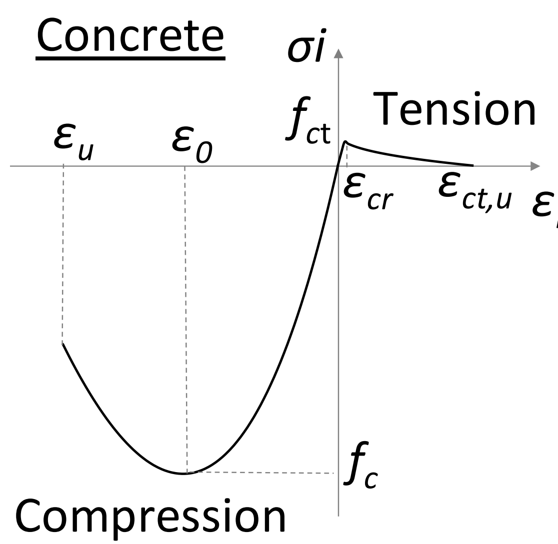
FIBER / FILAMENT BEAM MODELS

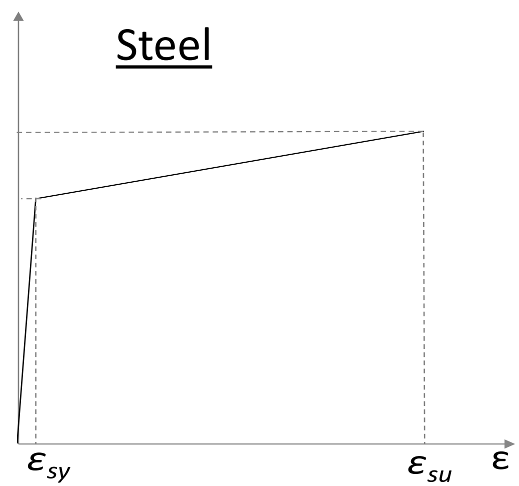
- Scordelis and Chan 1987, Marí 1987, Marí and Bairán 2014



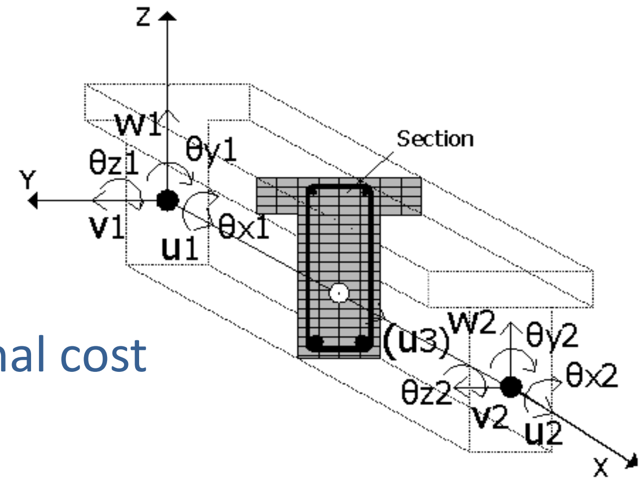






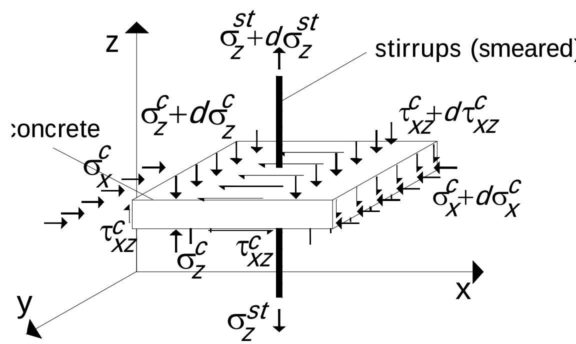


FIBER / FILAMENT BEAM MODELS



Advantages:

- Good balance simplicity/accuracy
- Complex non-linear modelling at low computational cost
- Suitable for framed structures



N-M interaction. V decoupled

$$\begin{pmatrix} N \\ M_y \\ V_z \end{pmatrix} = \begin{pmatrix} D_{11}A & D_{11}S_y & 0 \\ D_{11}S_y & D_{11}I_y & 0 \\ 0 & 0 & GA_z^* \end{pmatrix} \begin{pmatrix} \varepsilon_0 \\ \phi_y \\ \gamma_{xz} \end{pmatrix}$$

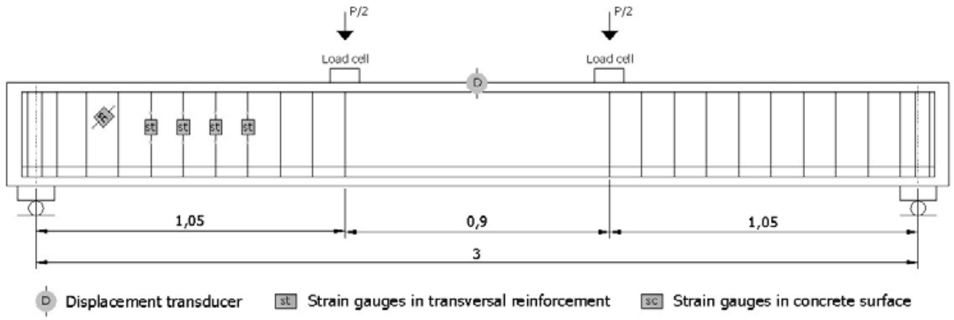
N-M-V interaction

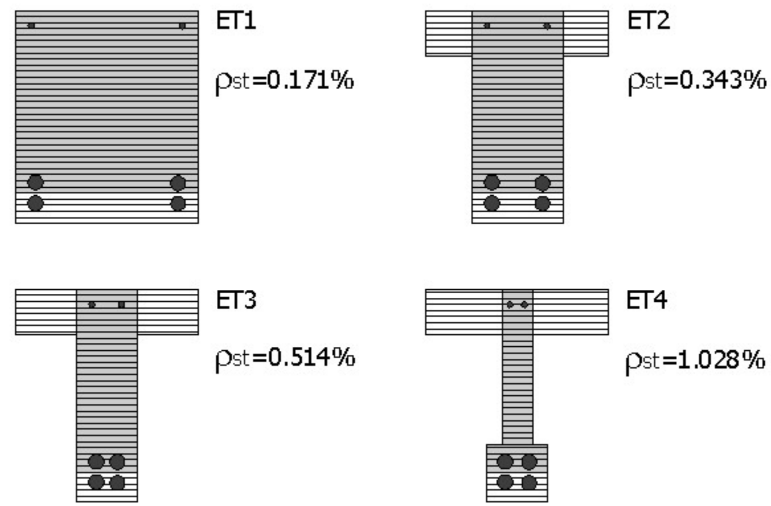
$$\begin{pmatrix} N \\ M_y \\ V_z \end{pmatrix} = \begin{pmatrix} D_{11}A & D_{11}S_y & D_{13}A^* \\ D_{11}S_y & D_{11}I_y & D_{13}S_y^* \\ D_{31}A^* & D_{31}S_y^* & GA_z^* \end{pmatrix} \begin{pmatrix} \varepsilon_0 \\ \phi_y \\ \gamma_{xz} \end{pmatrix}$$

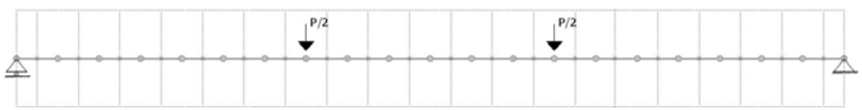
☞ D. Ferreira, J. Bairán & A. Marí (2014)

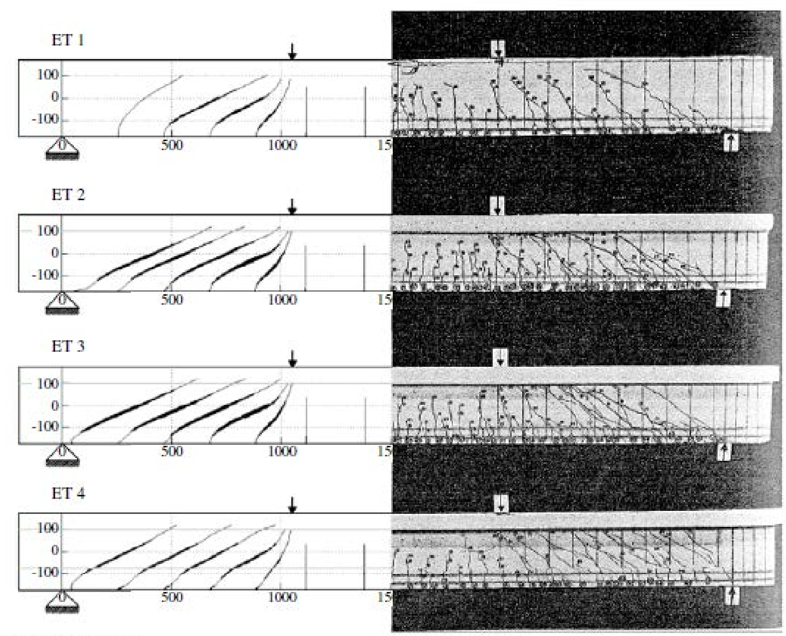
FIBER / FILAMENT BEAM MODELS

Stuttgart Shear Tests (Leonhard. 1965)

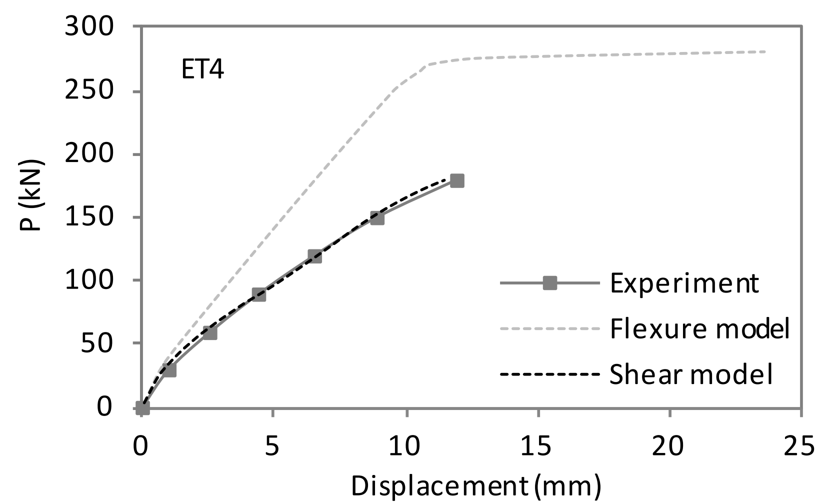








(a) Simulation (b) Experiment



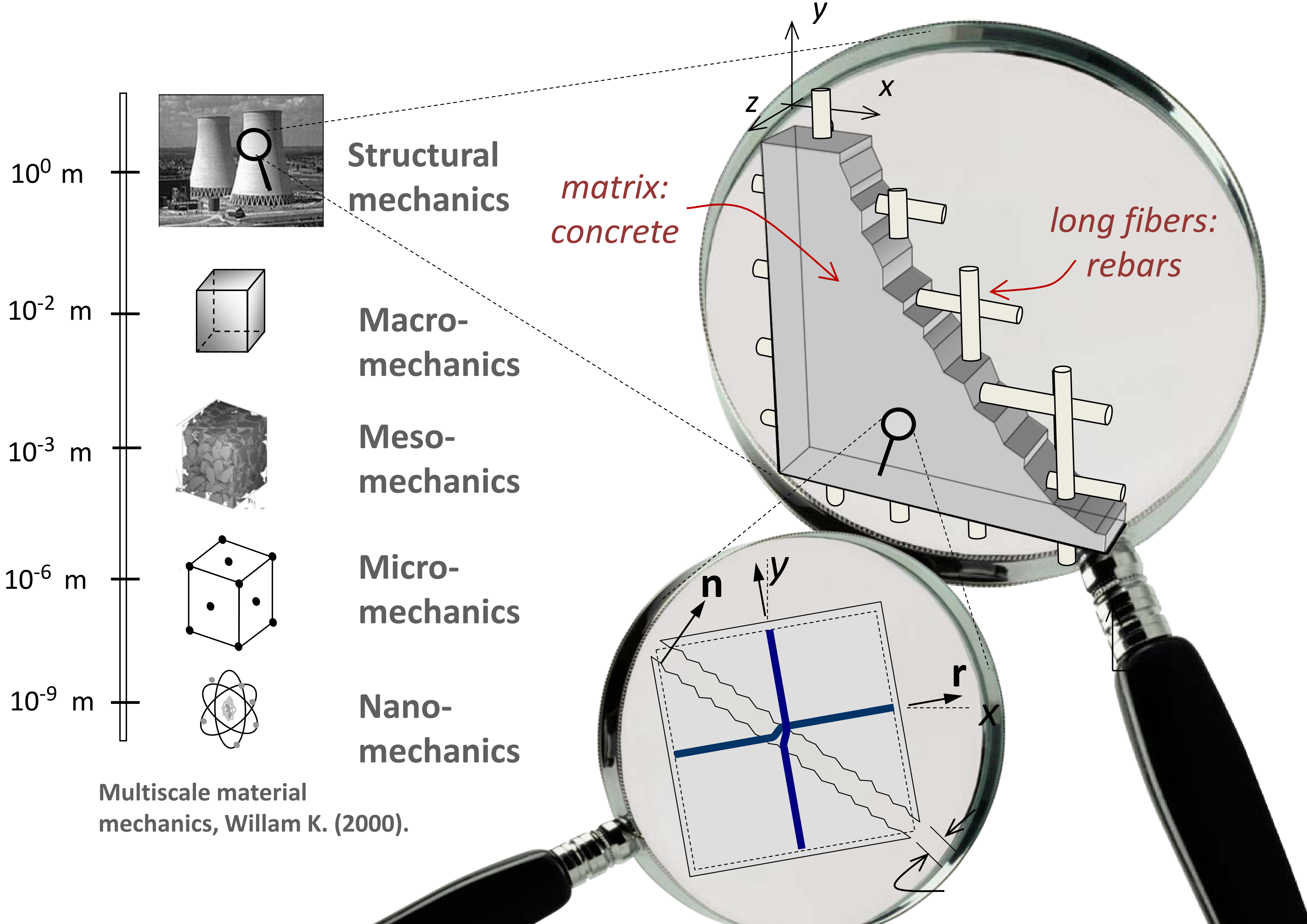
Crack pattern:

Obtained by post-process of local stress-strain in concrete fibers

MECHANICAL APPROACHES TO CONCRETE CRACKING

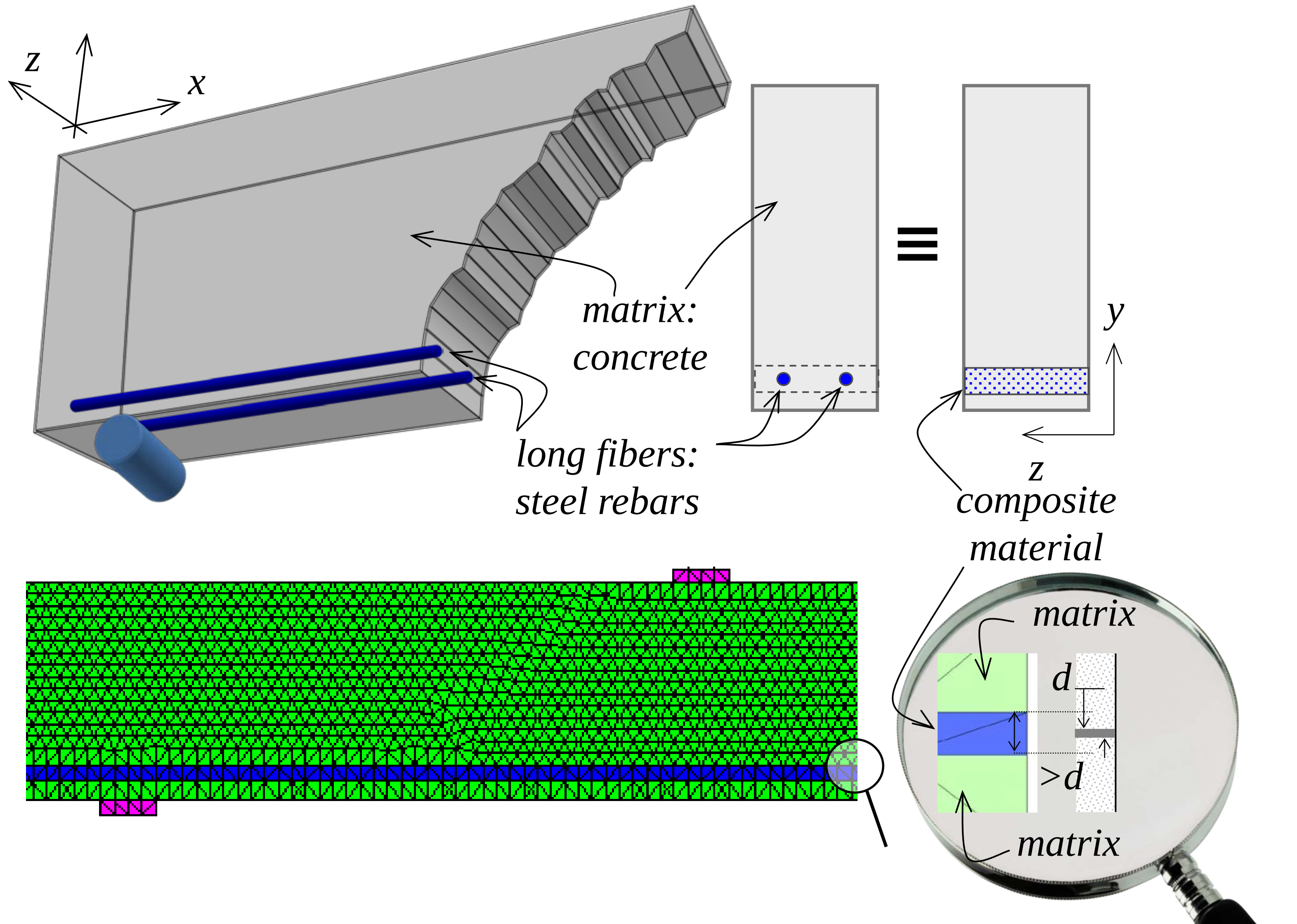
- ❑ Fiber/filament beam models
- ❑ Concrete as a composite material
- ❑ Micro-structure endowed material
- ❑ Computational multiscale modeling of concrete

REINFORCED CONCRETE AS A COMPOSITE MATERIAL



Multiscale material mechanics, Willam K. (2000).

REINFORCED CONCRETE AS A COMPOSITE MATERIAL

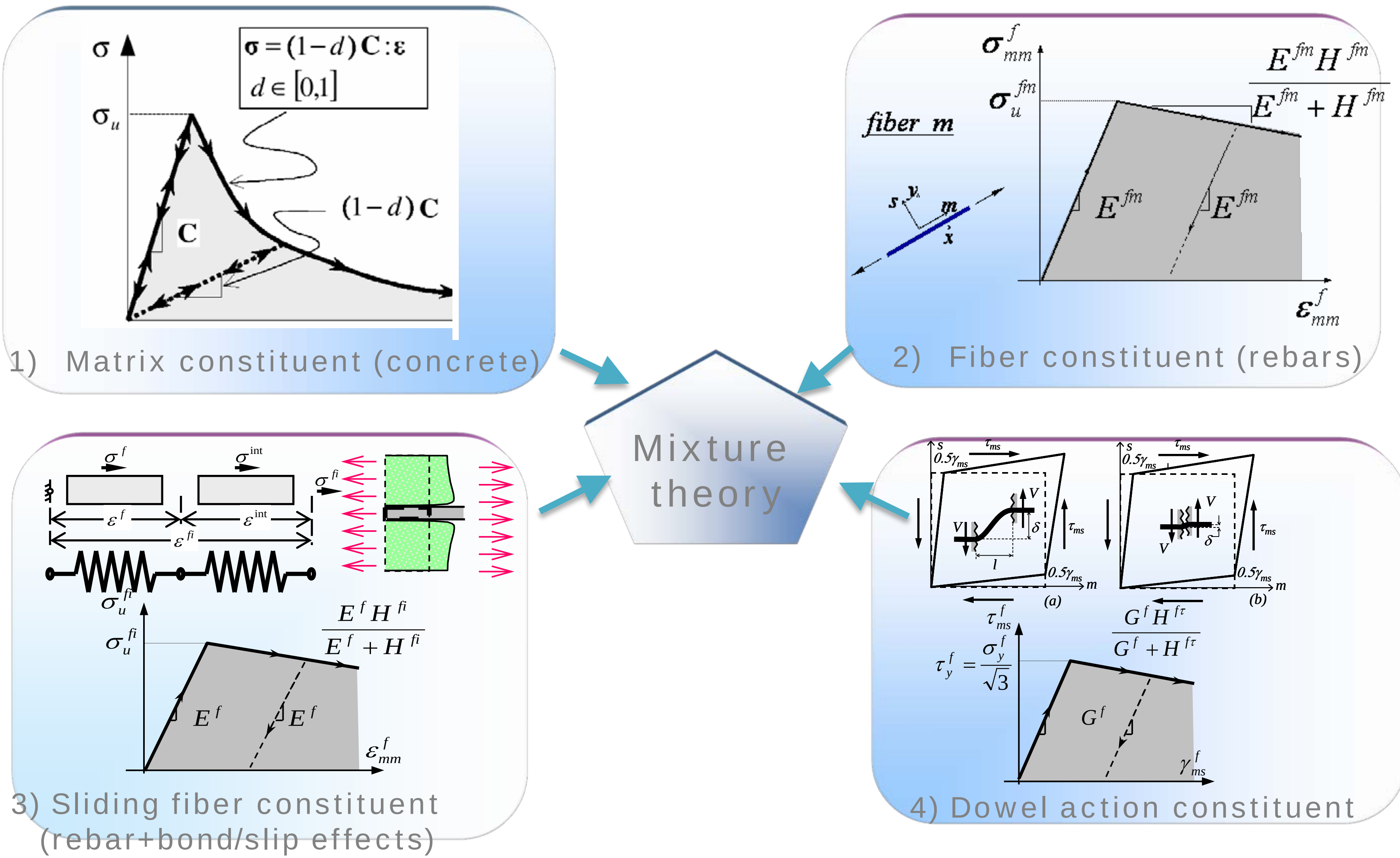


☞ **Ph. D. Thesis**: D.L. Linero 2006

REINFORCED CONCRETE AS A COMPOSITE MATERIAL

- Mixture theory (Hill, 1963)

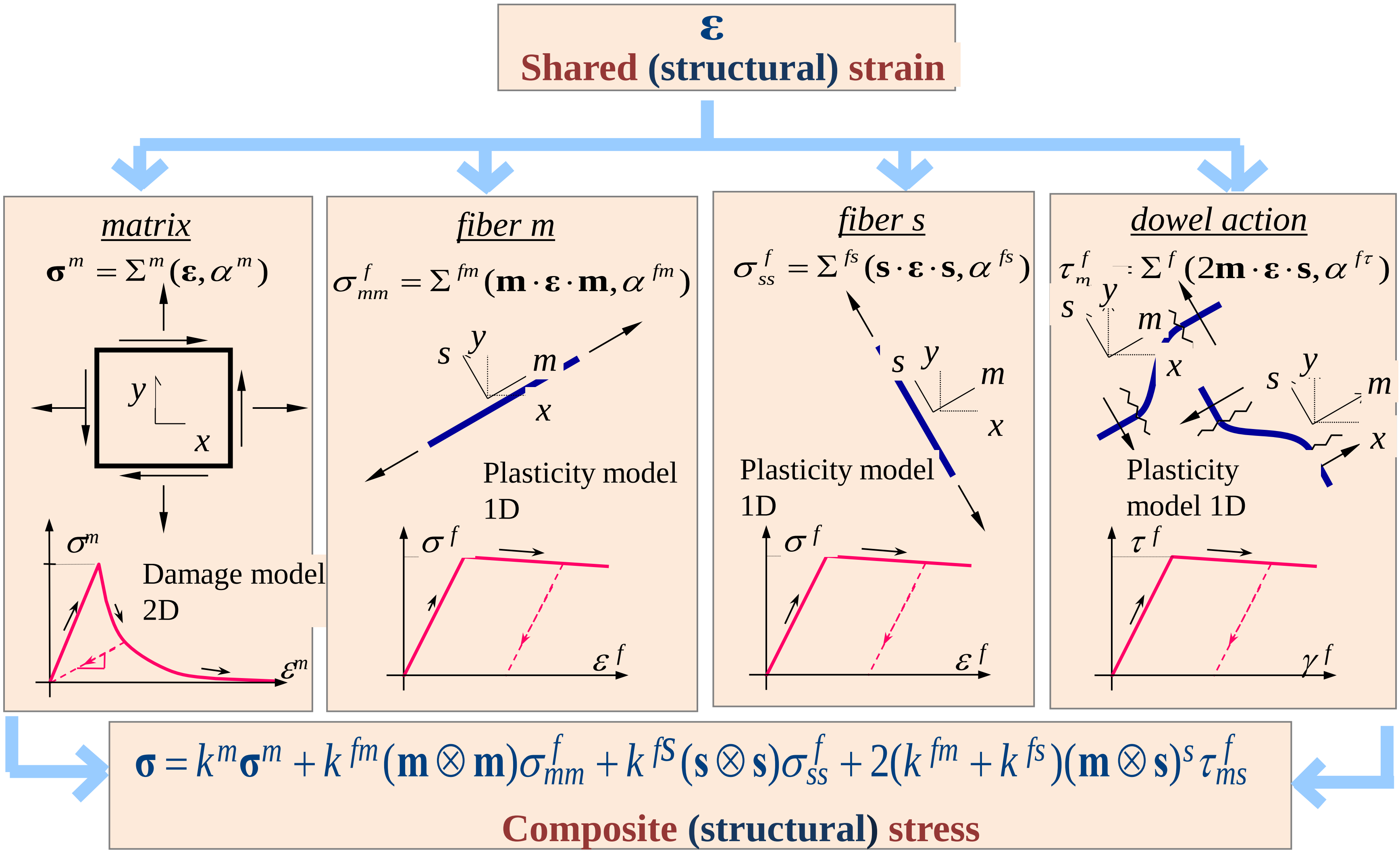
1. PHENOMENOLOGICAL MODELING OF INDIVIDUAL CONSTITUENTS



REINFORCED CONCRETE AS A COMPOSITE MATERIAL

- Mixture theory (cont.)

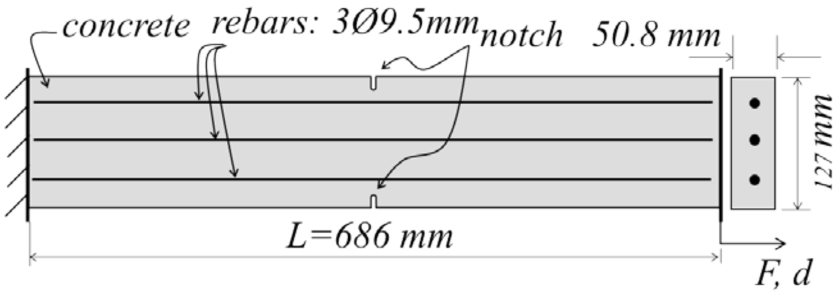
2. LINEAR HOMOGENIZATION (rule of mixtures)

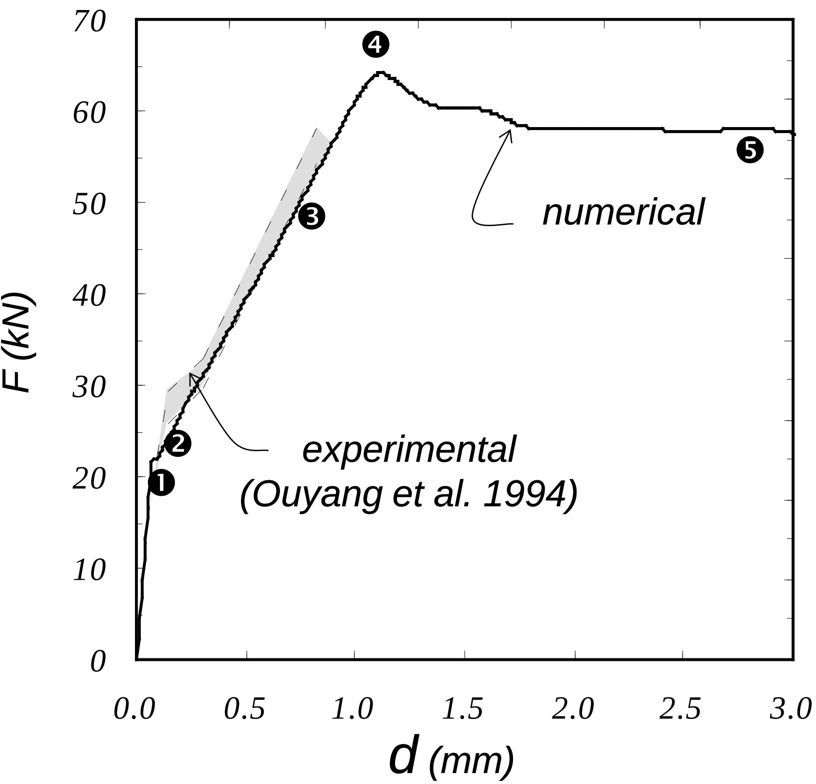


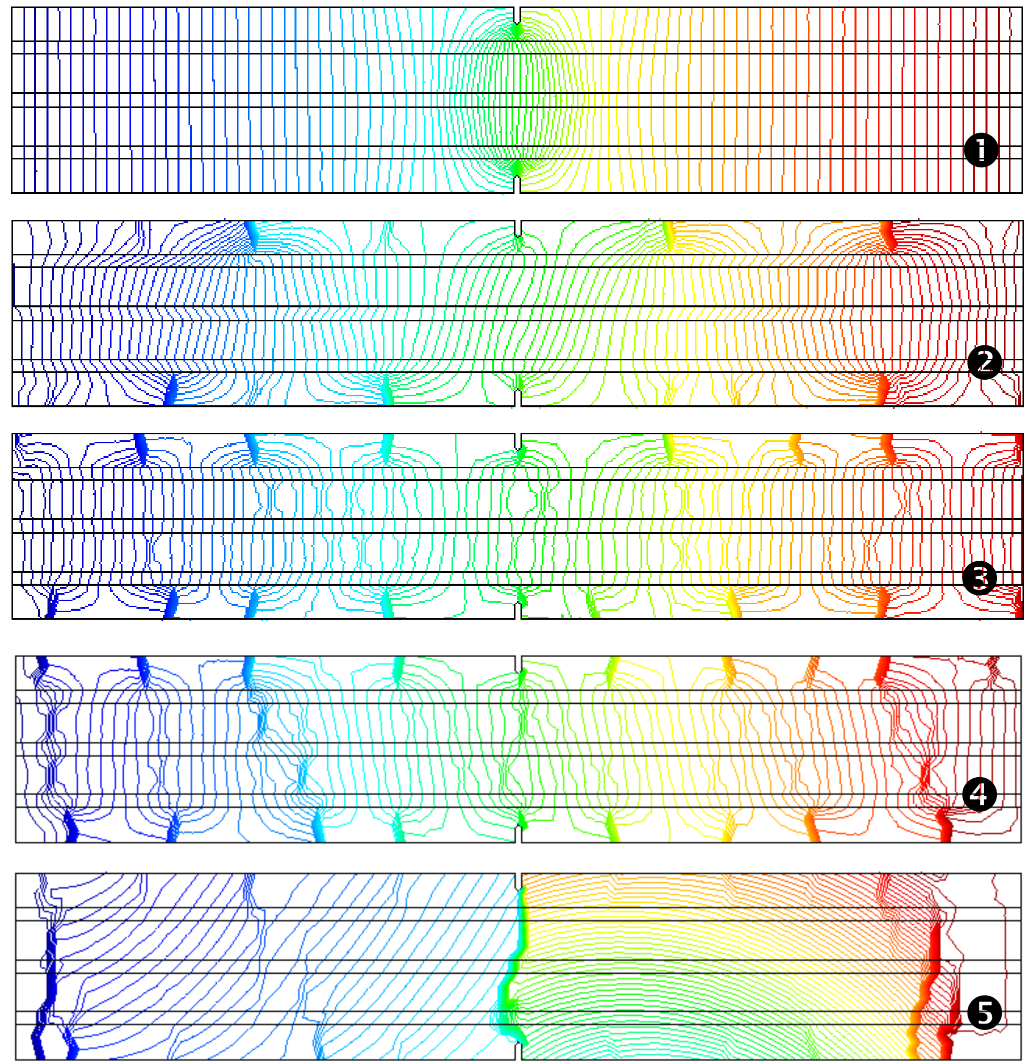
REINFORCED CONCRETE AS A COMPOSITE MATERIAL

CRACK SPACING

Reinforced concrete panel in tension





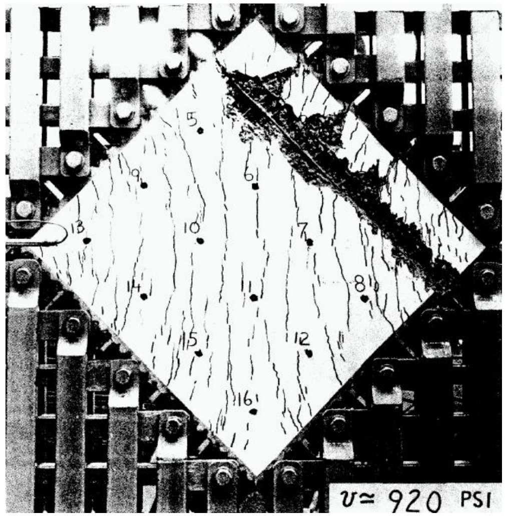


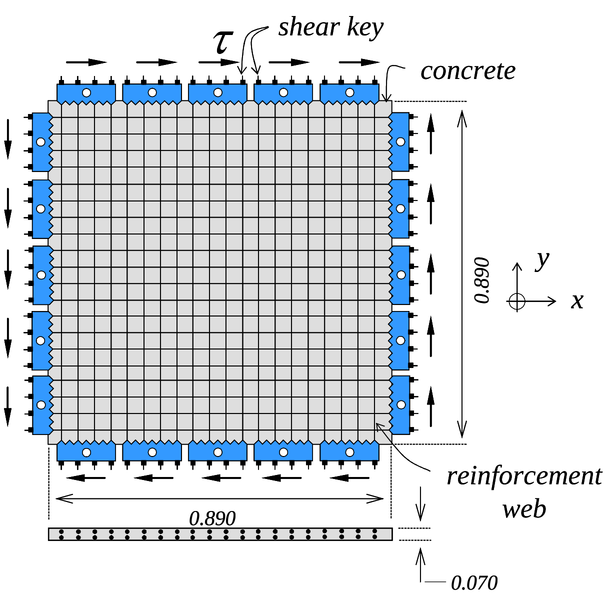
REINFORCED CONCRETE AS A COMPOSITE MATERIAL

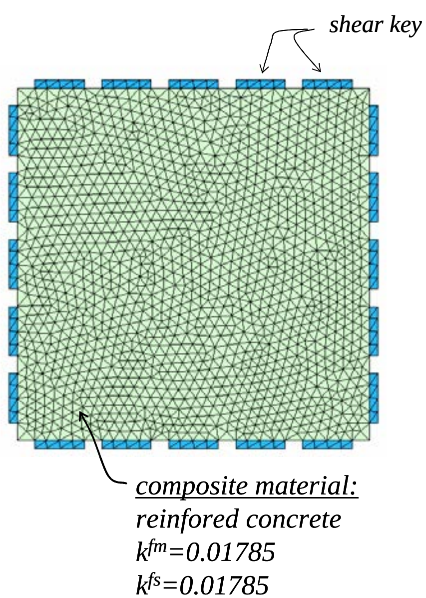
CRACK COALESCENCE

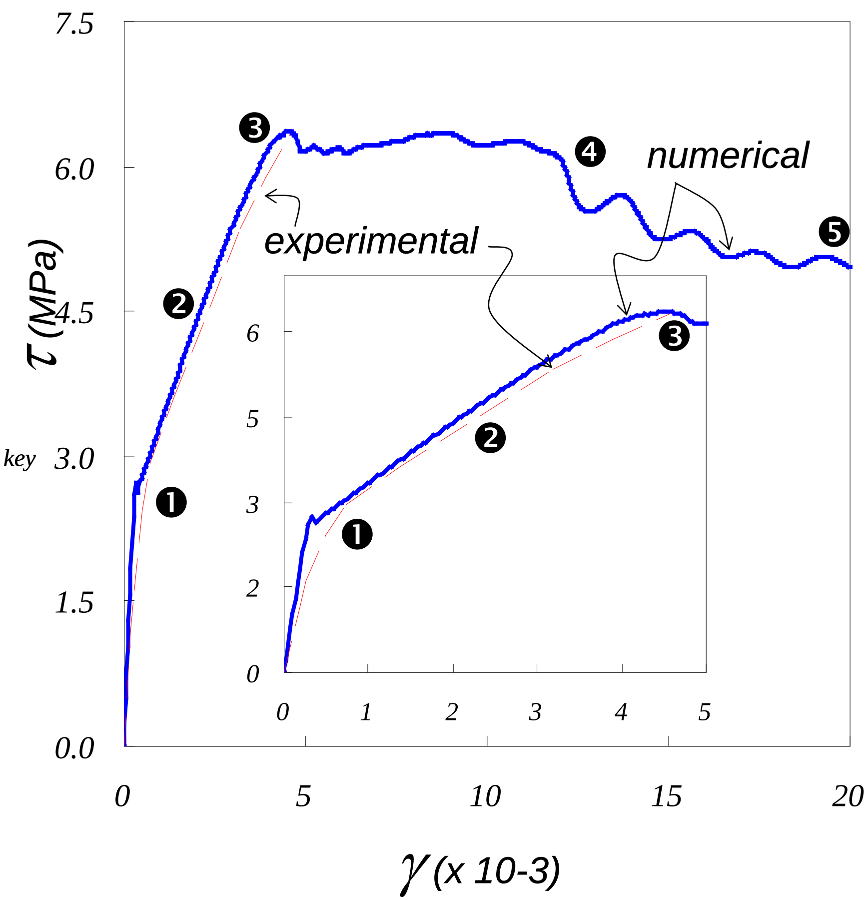
Double reinforced panel in shear

Experimental result (Collins et al. 1985).





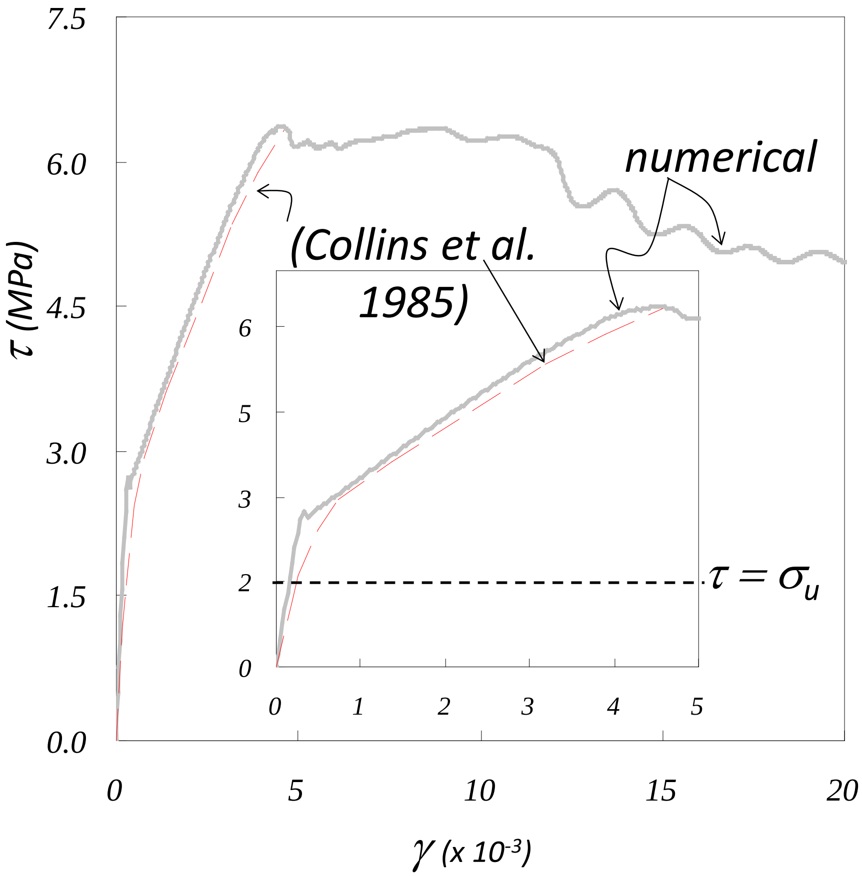


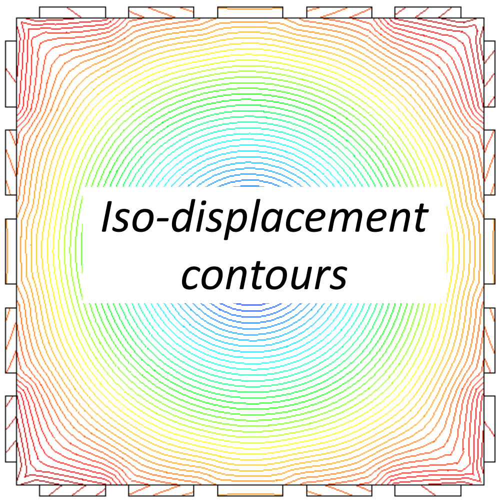


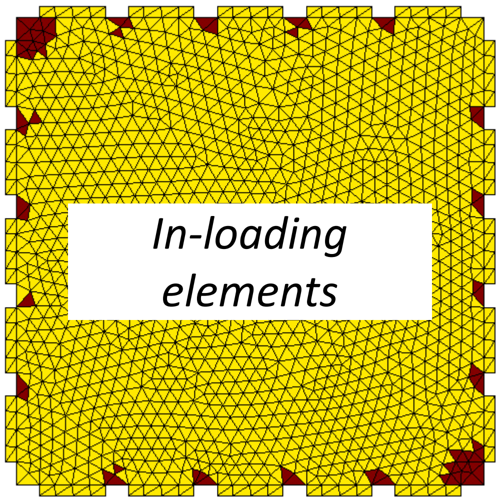
REINFORCED CONCRETE AS A COMPOSITE MATERIAL

CRACK COALESCENCE

Double reinforced panel in shear



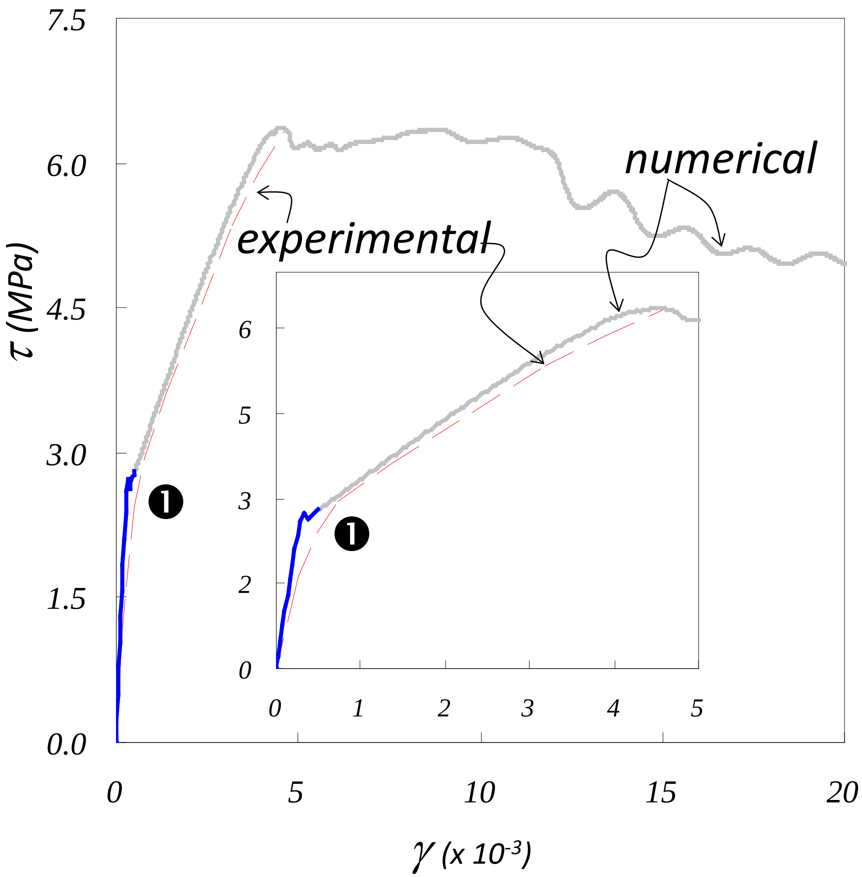


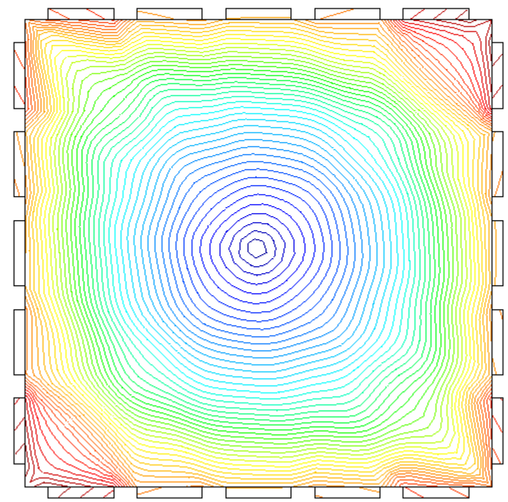


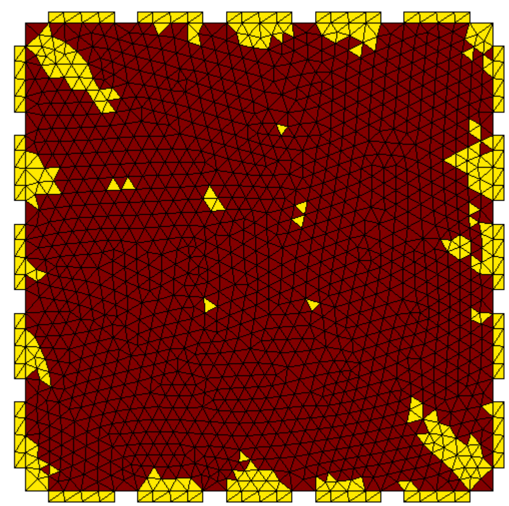
REINFORCED CONCRETE AS A COMPOSITE MATERIAL

CRACK COALESCENCE

Double reinforced panel in shear

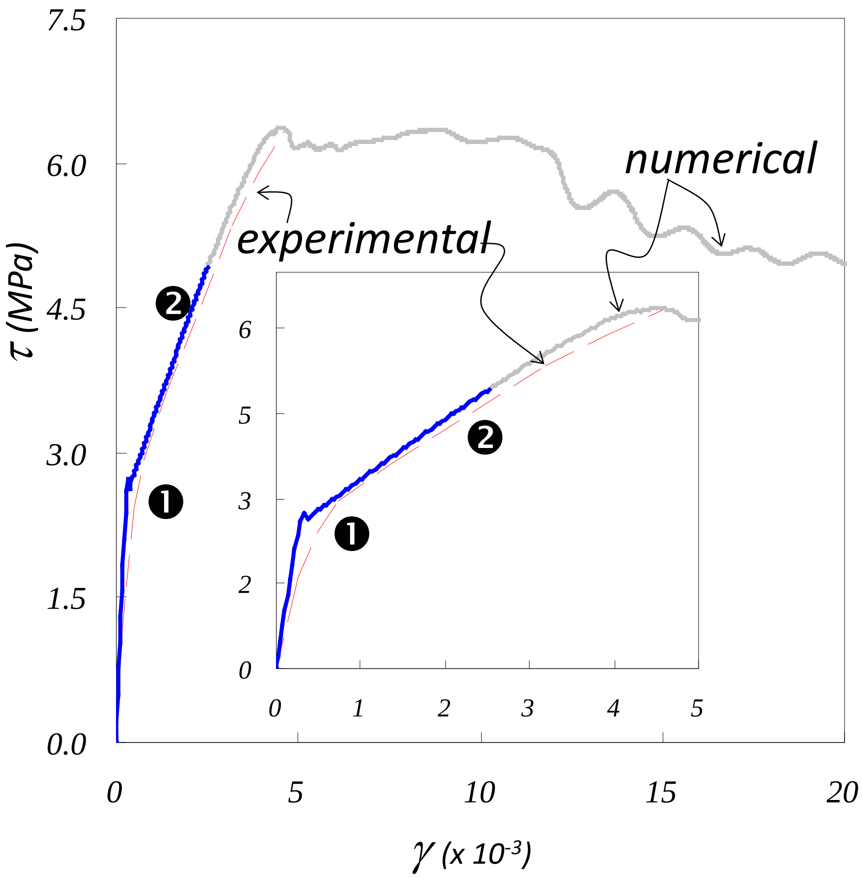


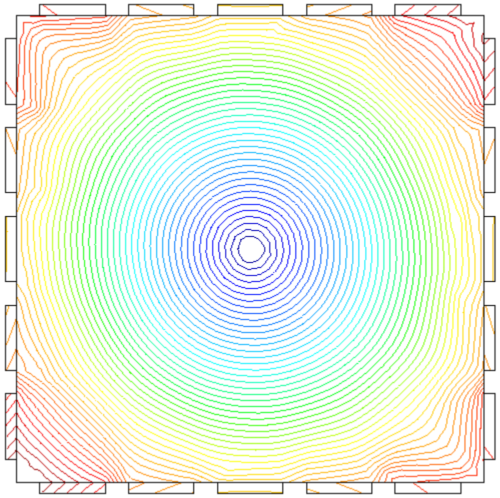


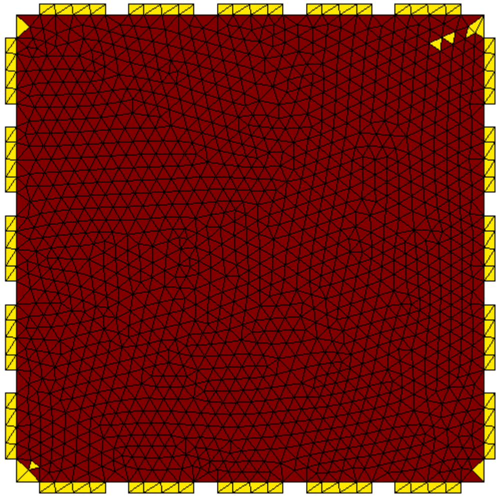


CRACK COALESCENCE

Double reinforced panel in shear



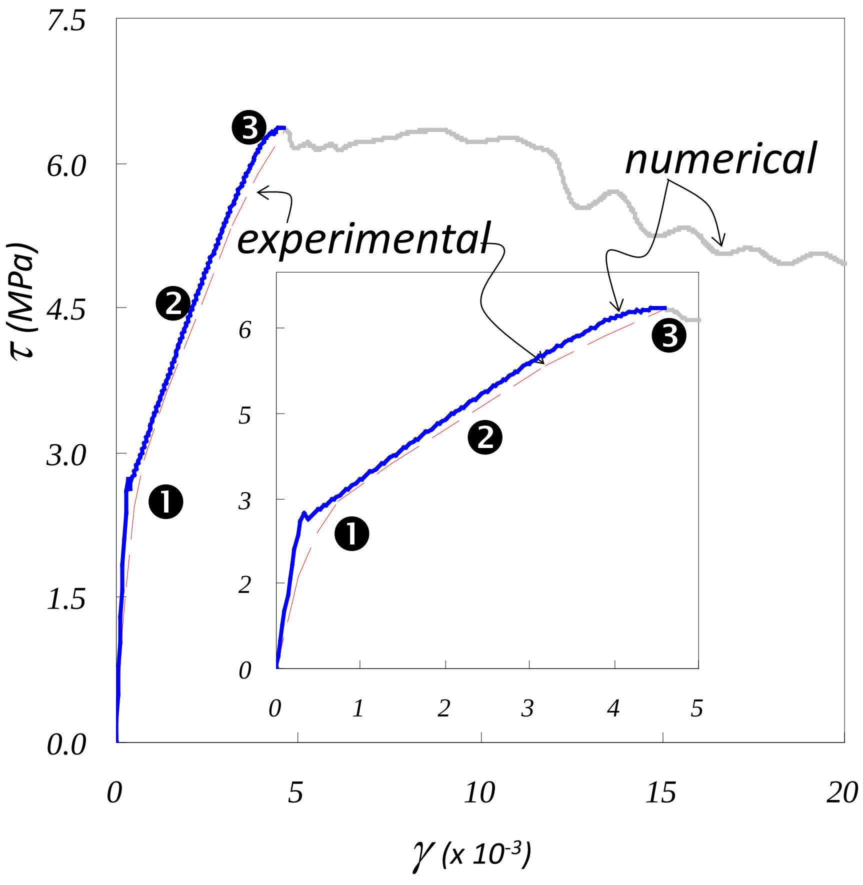


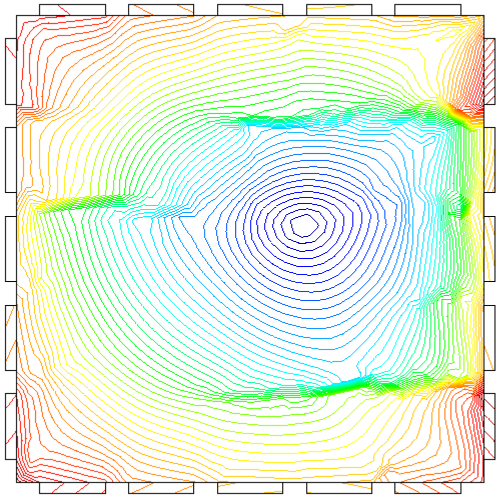


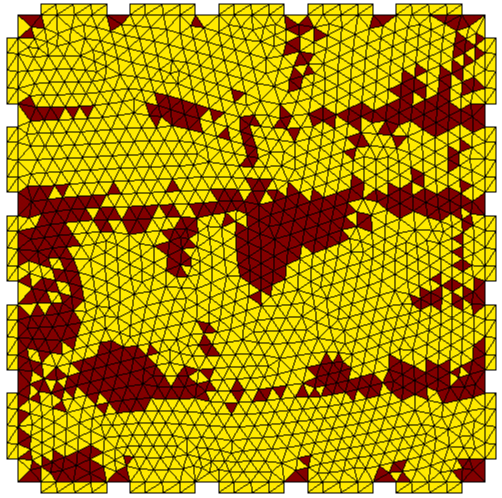
REINFORCED CONCRETE AS A COMPOSITE MATERIAL

CRACK COALESCENCE

Double reinforced panel in shear







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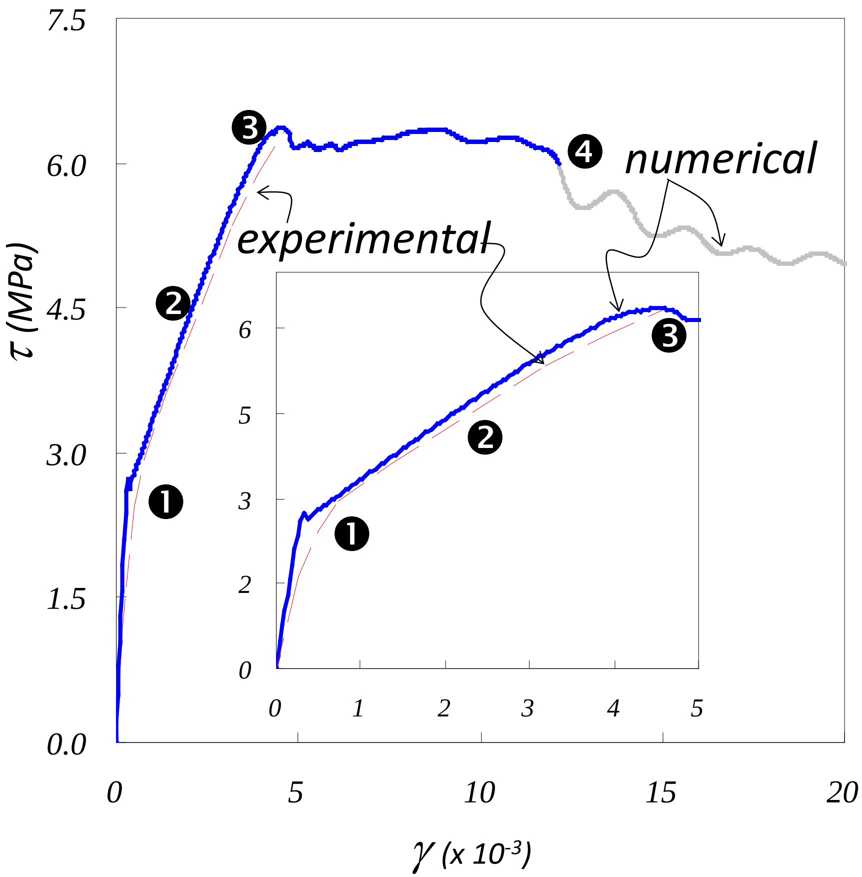
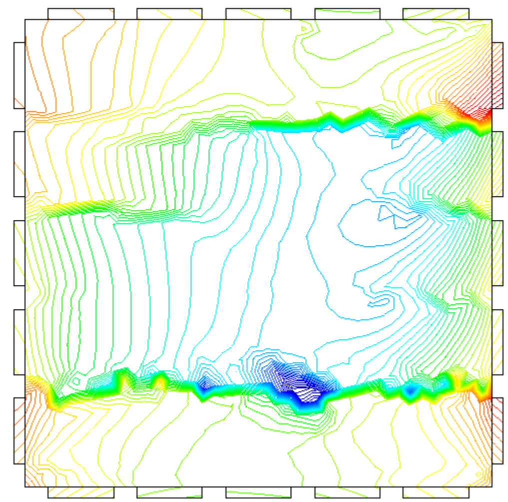
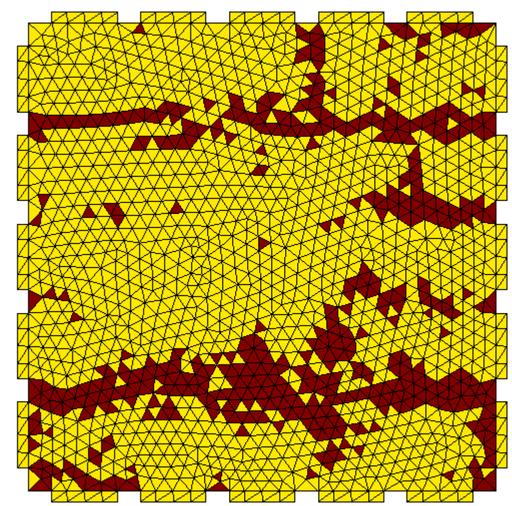
CRACK COALESCENCE

Double reinforced panel in shear



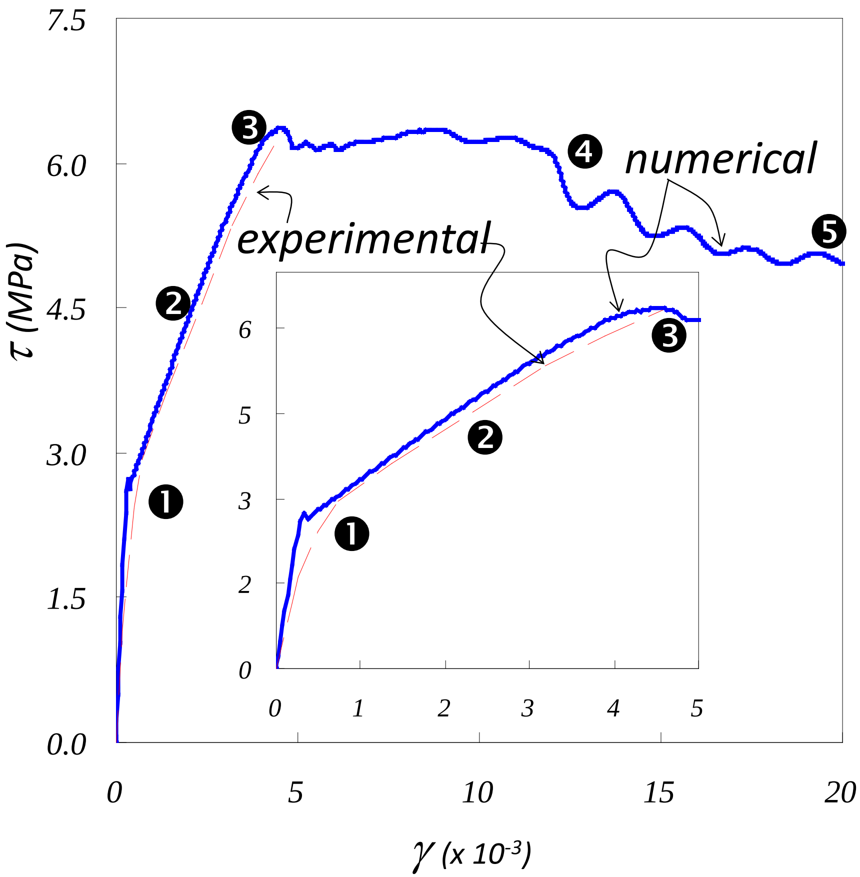


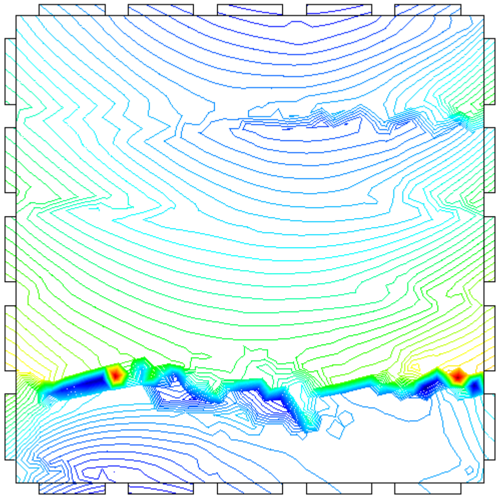


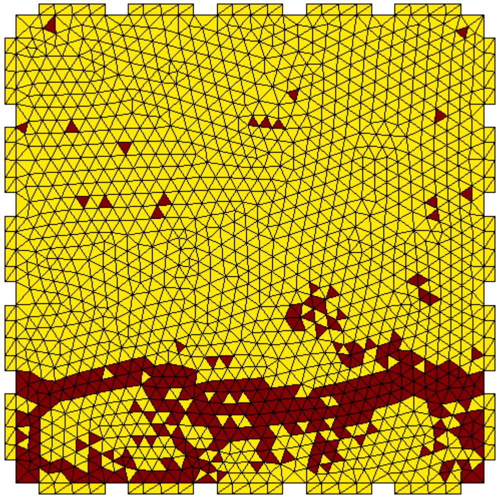
REINFORCED CONCRETE AS A COMPOSITE MATERIAL

CRACK COALESCENCE

Double reinforced panel in shear



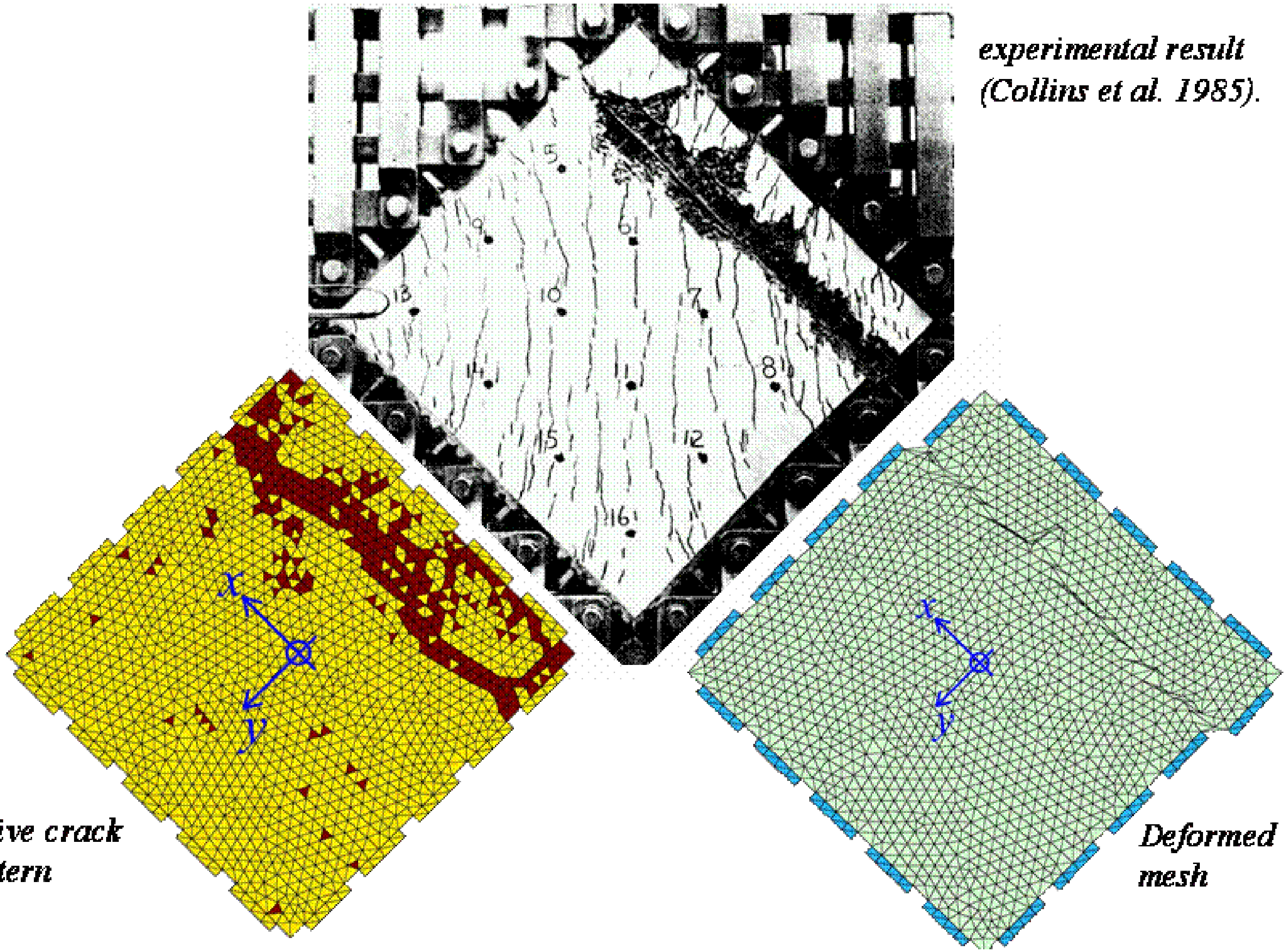




REINFORCED CONCRETE AS A COMPOSITE MATERIAL

CRACK COALESCENCE

Double reinforced panel in shear



experimental result (Collins et al. 1985).

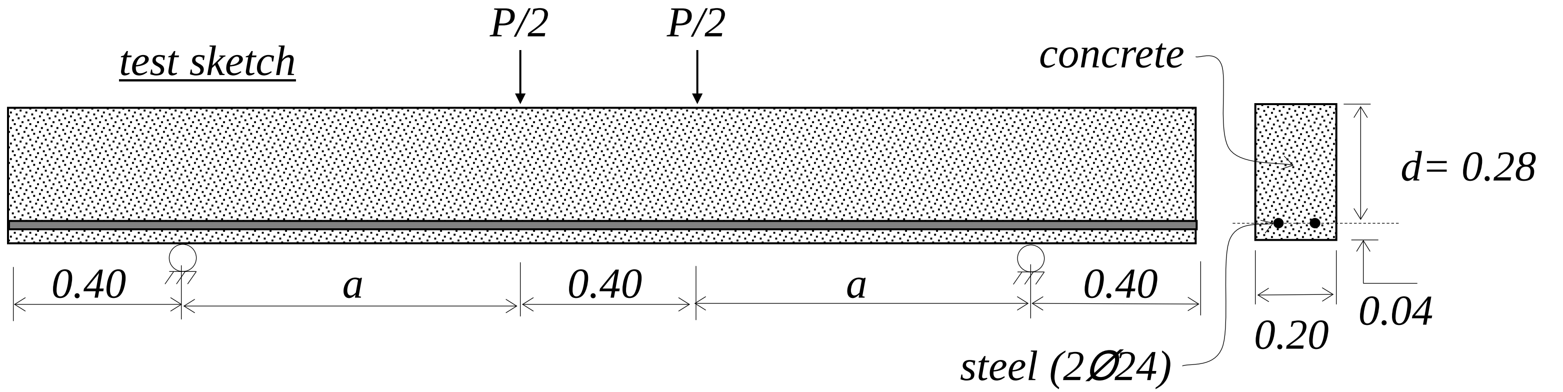
Active crack pattern

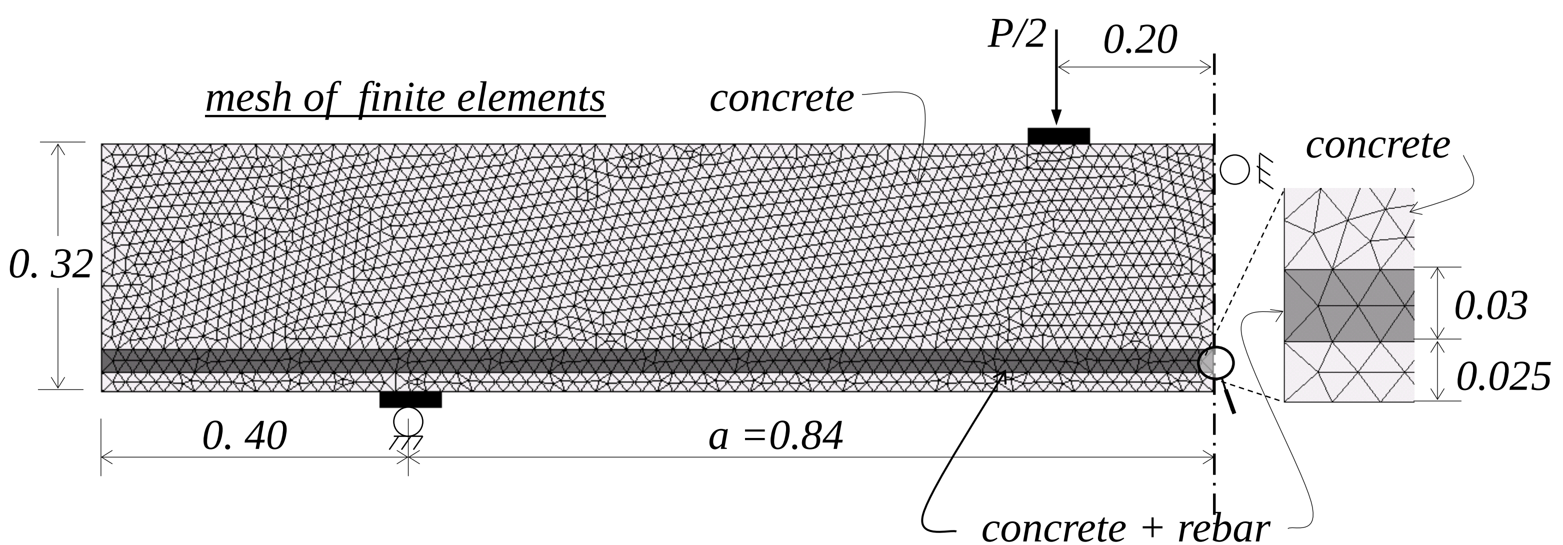
Deformed mesh

REINFORCED CONCRETE AS A COMPOSITE MATERIAL

CRACK PATTERN EVOLUTION

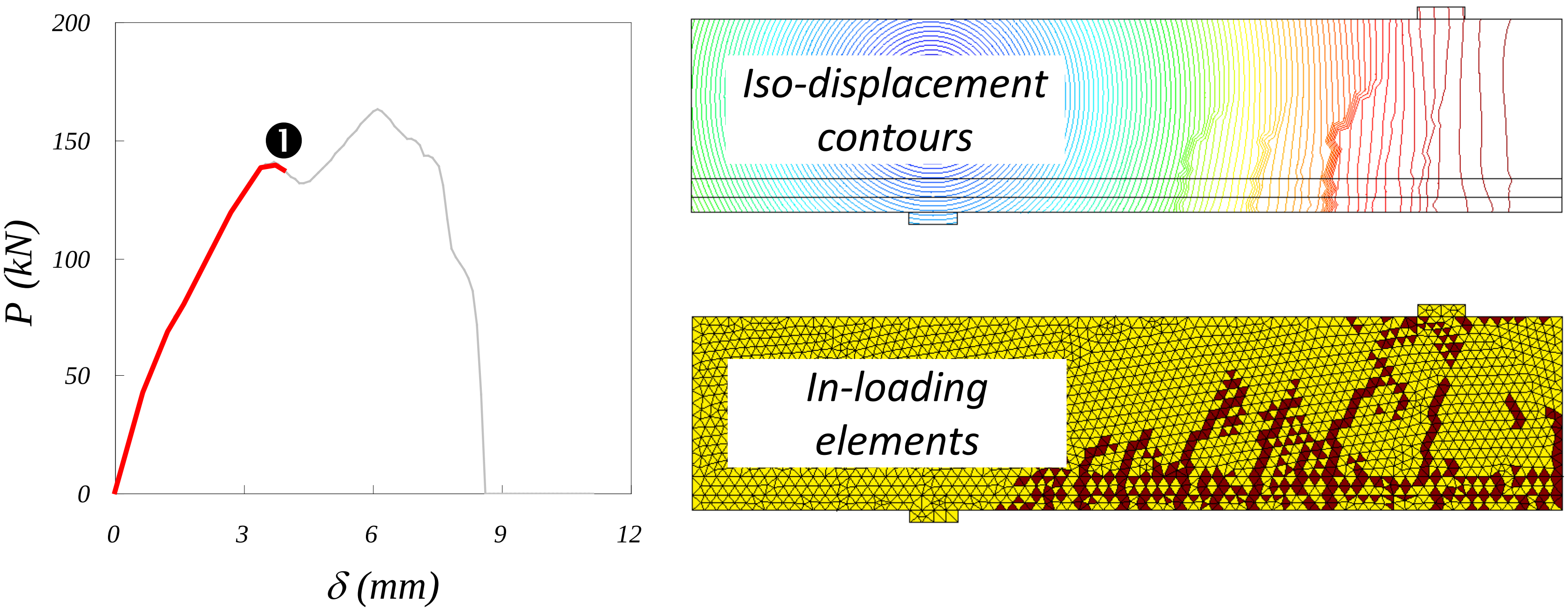
bending + shear beams (Leonard and Walter, 1965)





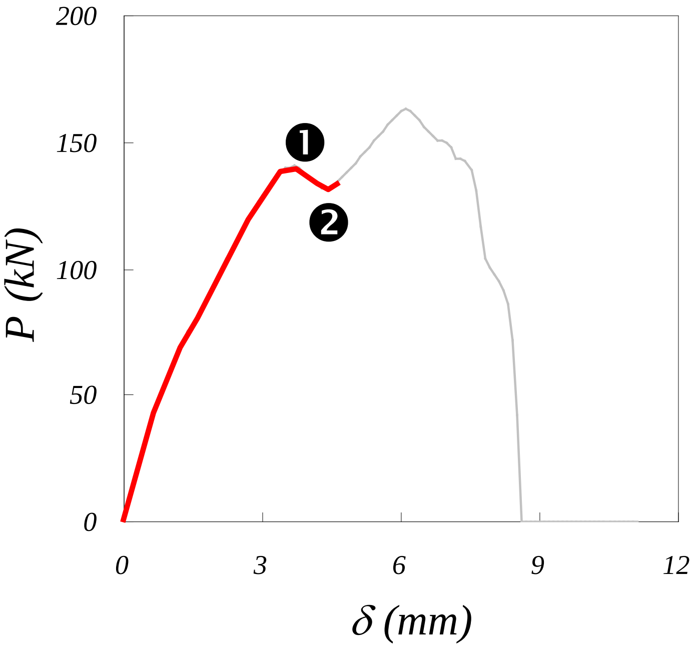
REINFORCED CONCRETE AS A COMPOSITE MATERIAL

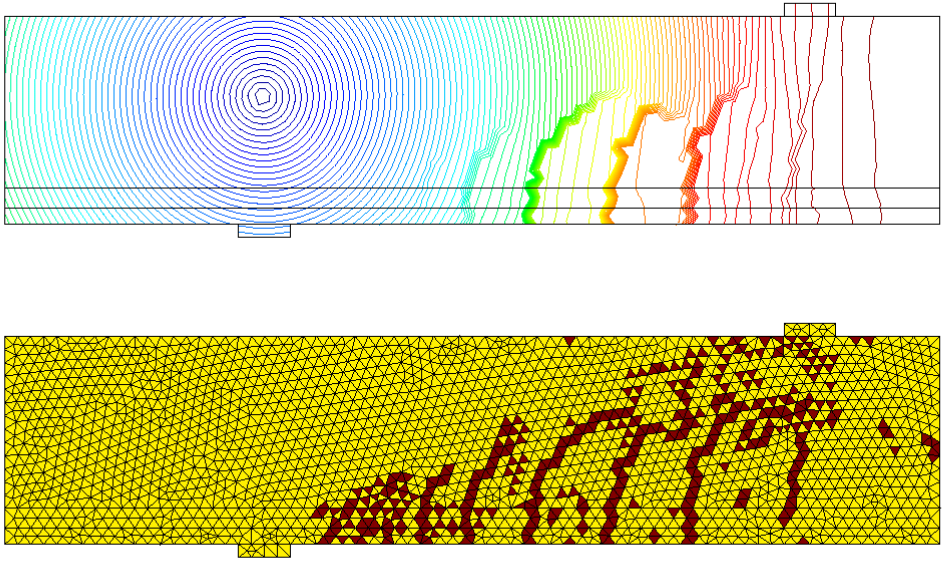
CRACK PATTERN EVOLUTION & ACTION-RESPONSE CURVE



REINFORCED CONCRETE AS A COMPOSITE MATERIAL

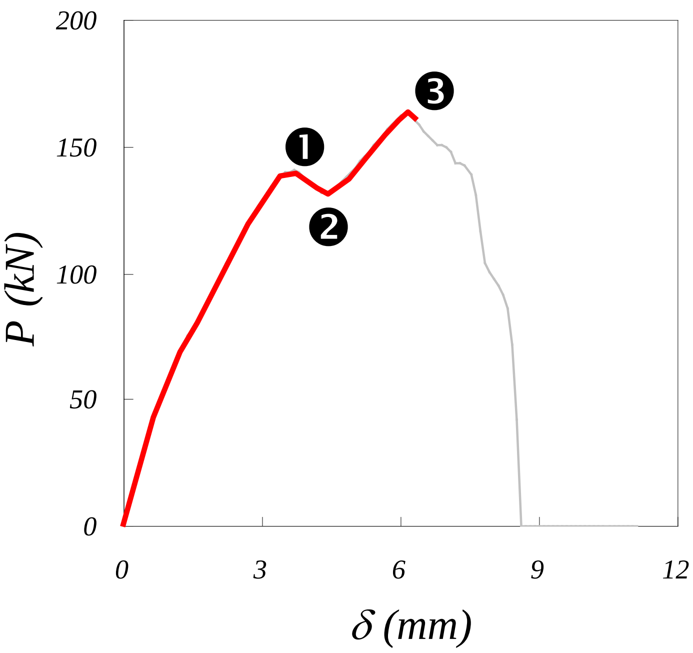
CRACK PATTERN EVOLUTION & ACTION-RESPONSE CURVE

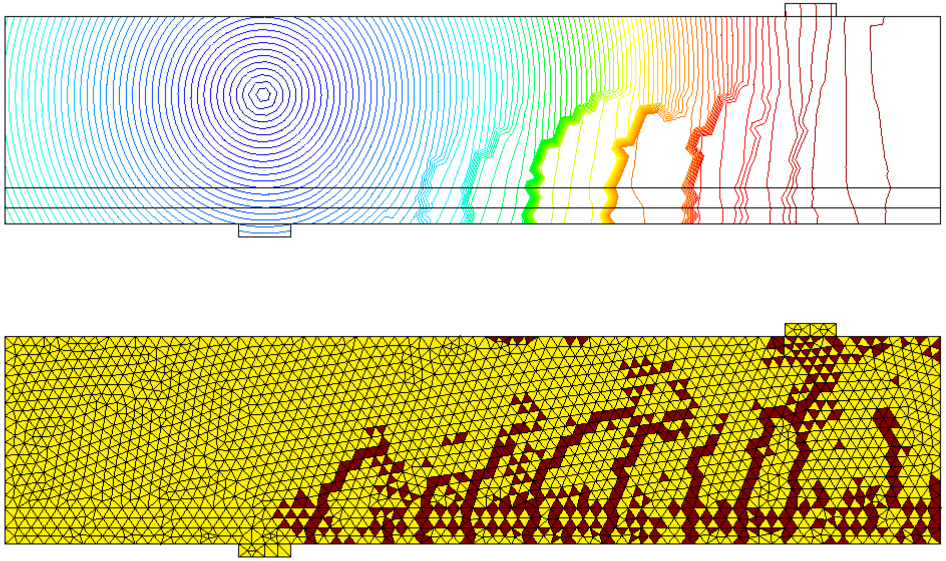




REINFORCED CONCRETE AS A COMPOSITE MATERIAL

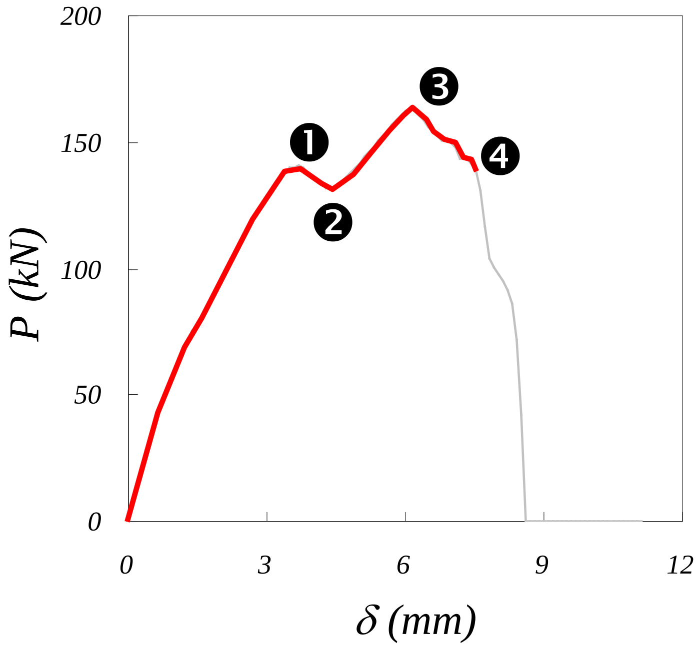
CRACK PATTERN EVOLUTION & ACTION-RESPONSE CURVE

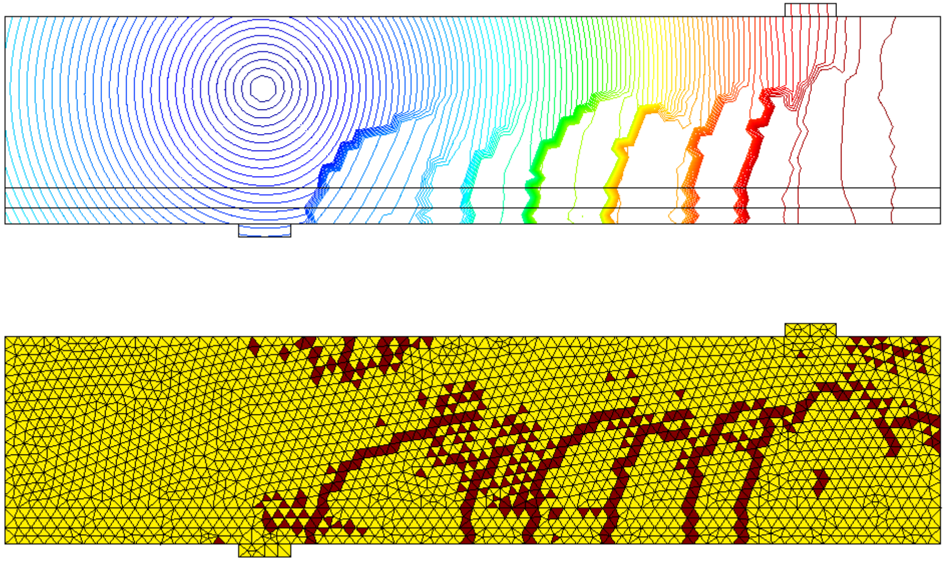




REINFORCED CONCRETE AS A COMPOSITE MATERIAL

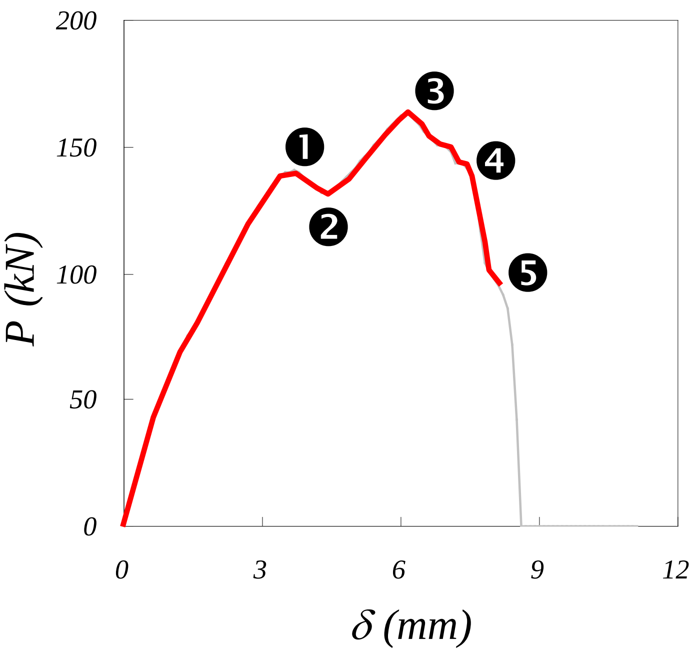
CRACK PATTERN EVOLUTION & ACTION-RESPONSE CURVE

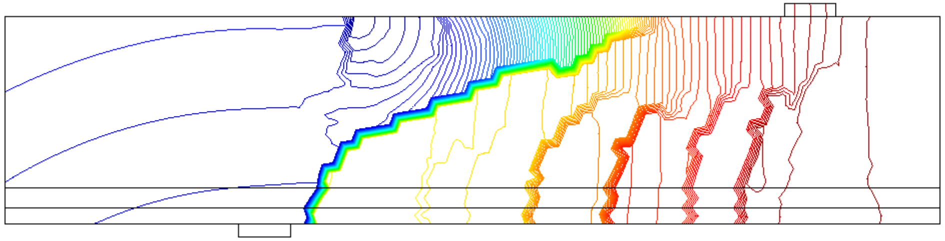


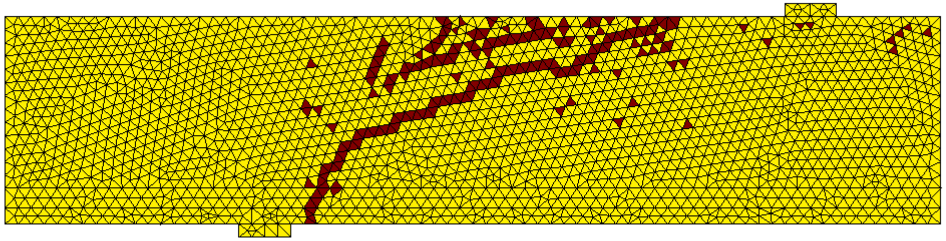


REINFORCED CONCRETE AS A COMPOSITE MATERIAL

CRACK PATTERN EVOLUTION & ACTION-RESPONSE CURVE

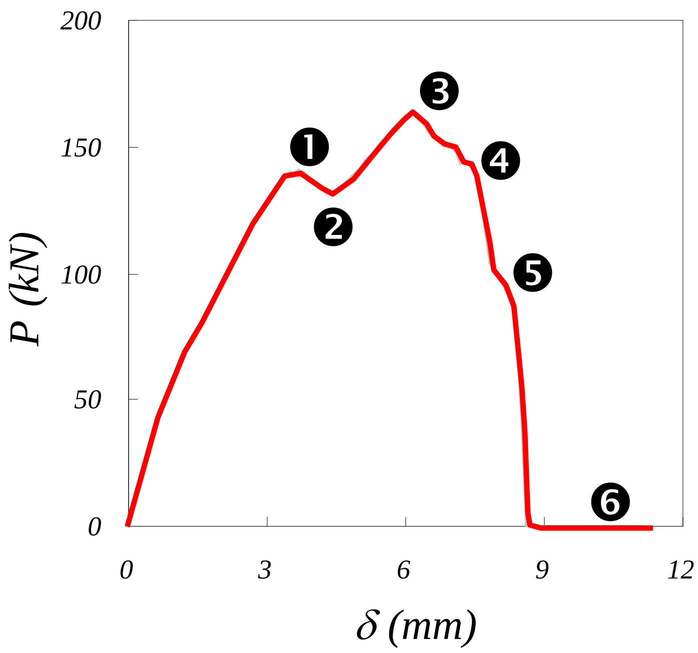


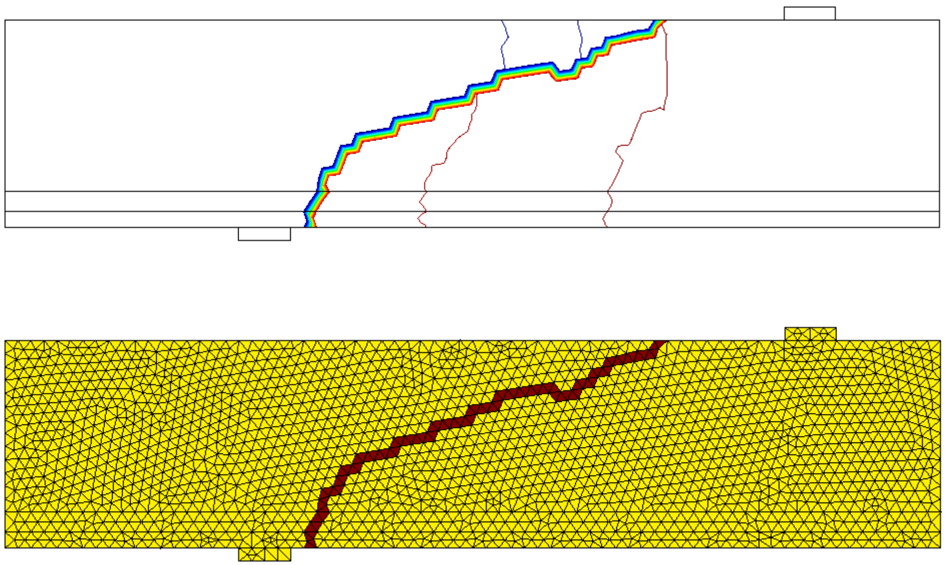




REINFORCED CONCRETE AS A COMPOSITE MATERIAL

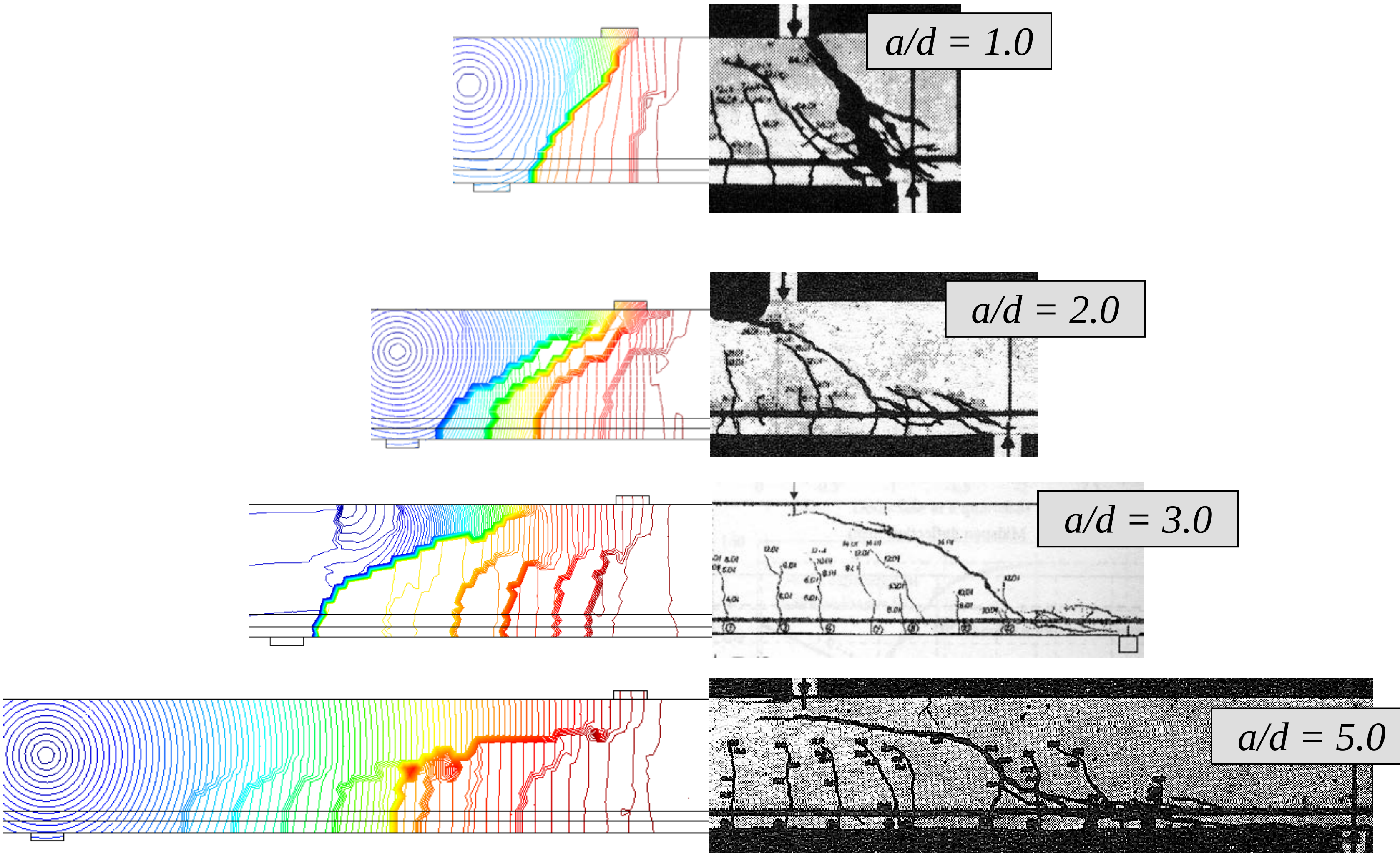
CRACK PATTERN EVOLUTION & ACTION-RESPONSE CURVE





REINFORCED CONCRETE AS A COMPOSITE MATERIAL

ASPECT RATIO EFFECTS ON CRACK PATTERN



MECHANICAL APPROACHES TO CONCRETE CRACKING

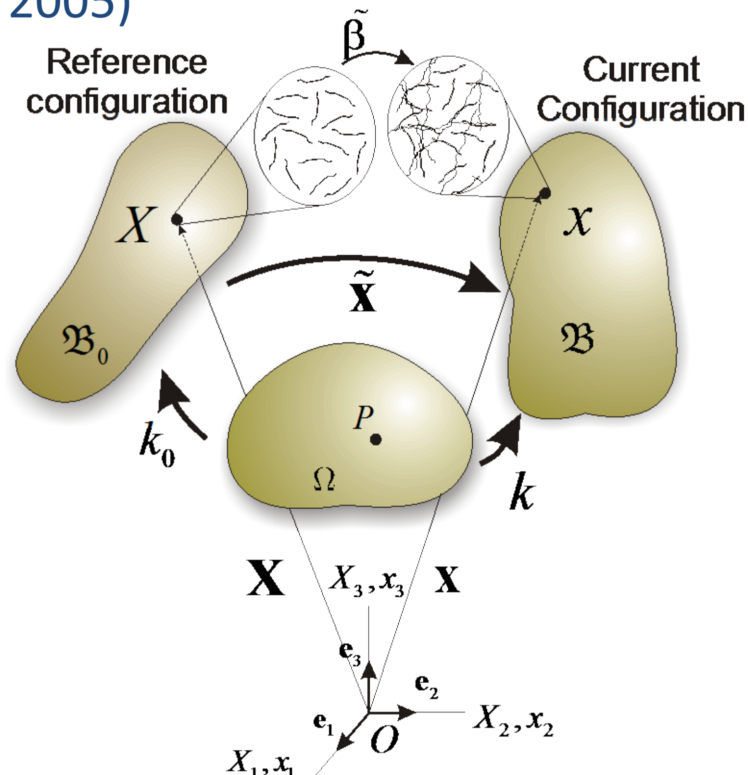
- ❑ Fiber/filament beam models
- ❑ Concrete as a composite material
- ❑ **Micro-structure endowed material**
- ❑ Computational multiscale modeling of concrete

FIBER-REINFORCED CONCRETE
(AS A CONTINUUM WITH MICROSTRUCTURE)

MACRO SCALE

MESO SCALE

Complex bodies theory (Capriz 1989, Mariano 2005)



$\tilde{\mathbf{u}}(\mathbf{X},t),\ \forall \mathbf{X}\in\mathfrak{B}_0 \rightarrow$ Motion descriptor

$\tilde{\beta}(\mathbf{X},t),\ \forall \mathbf{X}\in\mathfrak{B}_0 \rightarrow$ Morphological descriptor

☞ **Ph.D. Thesis**: D.F. Mora 2012

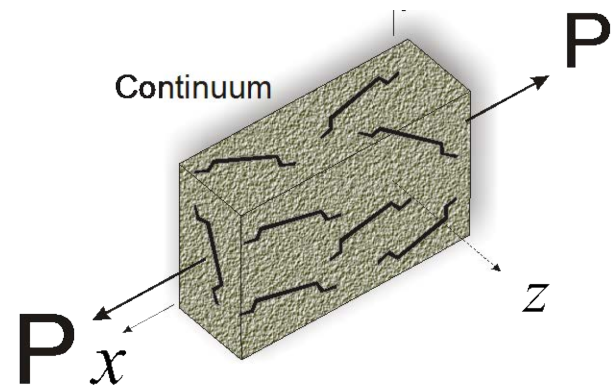
FIBER-REINFORCED CONCRETE
(AS A CONTINUUM WITH MICROSTRUCTURE)

$$\text{CONTINUUM} \rightarrow \mathbf{u}(\mathbf{x},t)\quad \mathbf{x} \in \Omega$$

$$\boxed{\nabla \cdot \boldsymbol{\sigma} + \rho \mathbf{b} = \mathbf{0}\ \forall \mathbf{x} \in \Omega \rightarrow \text{Equilibrium}}$$

$$\left.\begin{array}{ll} \boldsymbol{\sigma}\cdot\mathbf{n} = \mathbf{t}^* & \forall \mathbf{x} \in \Gamma_\sigma \\ \mathbf{u} = \mathbf{u}^* & \forall \mathbf{x} \in \Gamma_u \end{array}\right\} \rightarrow \text{Boundary conditions}$$

$$\boldsymbol{\sigma} = \sum(\mathbf{u},\beta)\quad \forall \mathbf{x} \in \Omega \rightarrow \text{Constitutive equation}$$



$$\text{MICROSTRUCTURE} \rightarrow \beta(\mathbf{x},t) = \left\{\beta^{(\theta_i)}\right\}\quad \mathbf{x} \in \Omega$$

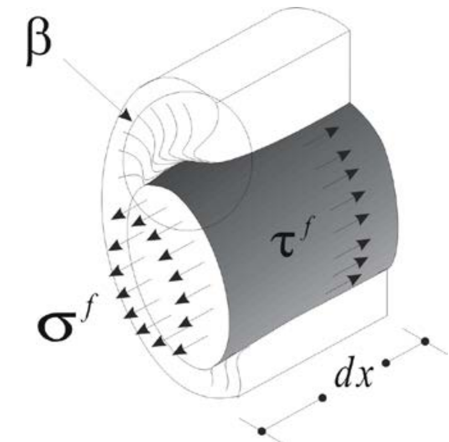
$$(i) \rightarrow \text{i-oriented fiber bundle}\ \ (i \in \{1,\ldots,n_{bundle}\})$$

$$\boxed{\tau^f - \frac{A^f}{\Pi^f}\frac{\partial \sigma^f}{\partial x'} = 0\ \forall \mathbf{x} \in \Omega \rightarrow \text{Capriz balance (fiber axial equilibrium)}}$$

$$\left.\begin{array}{l} \sigma^f = \Sigma^f(\mathbf{u},\beta) \\ \tau^f = \mathrm{T}^f(\beta) \end{array}\right\} \rightarrow \text{Fiber constitutive model}$$

Morphological descriptor

β= oriented-fiber-bundle slips

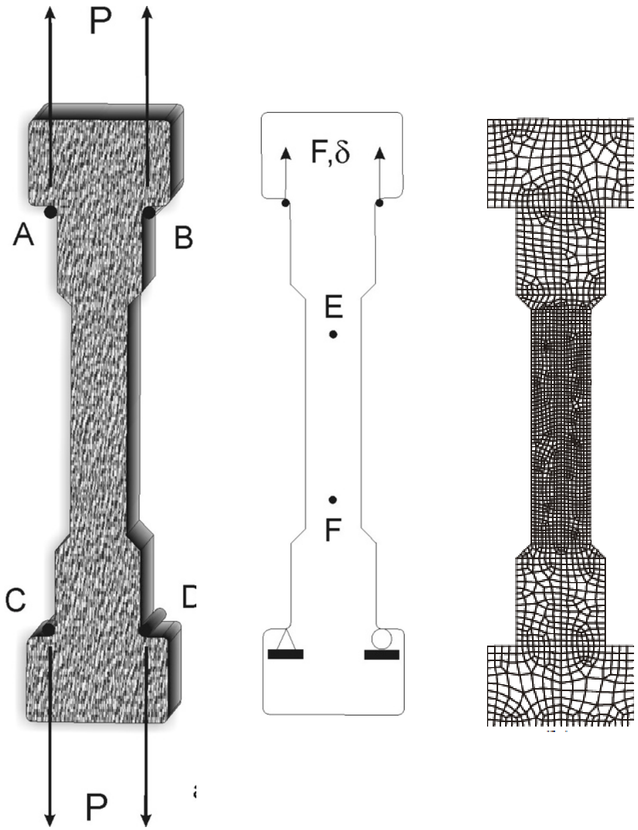


FIBER-REINFORCED CONCRETE
(AS A CONTINUUM WITH MICROSTRUCTURE)

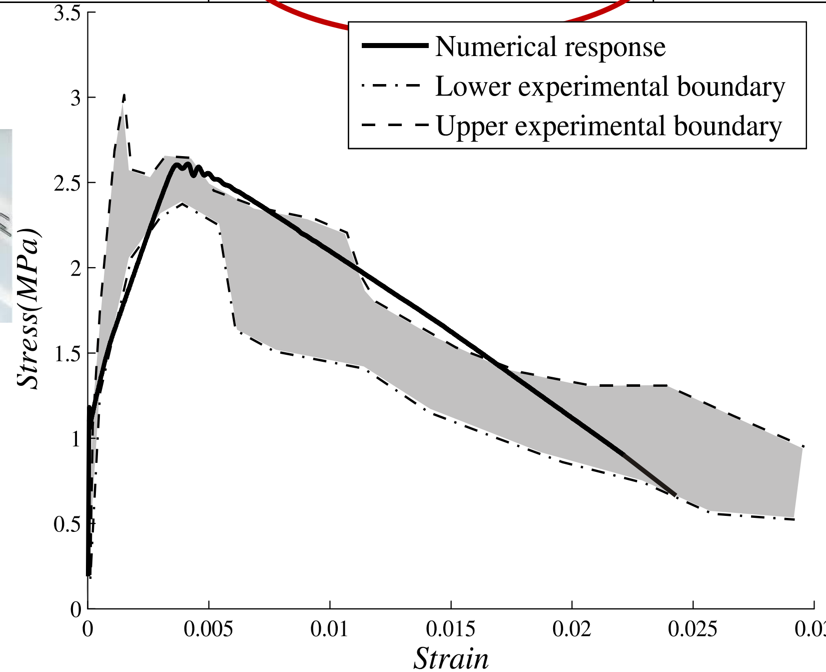
FIBER REINFORCEMENT EFFECTS

Dog-bone shaped specimen with randomly oriented fibers (Suwannakarn. 2009)

Matrix	Fiber	Debonding
$f_c' = 1.25MPa$	$\sigma_u^f = 2100MPa$	$\tau_u^f = 5.1MPa$
$E^m = 13.89GPa$	$E^f = 210GPa$	$E^d = 1e+8GPa$
$\upsilon^m = 0.2$	$H^f = 100MPa$	$H^d = 100MPa$
$G_f = 100N/m$	$\theta = \begin{bmatrix} 0^\circ, 10^\circ, 20^\circ, 30^\circ, \ldots \\ 45^\circ, 60^\circ, 70^\circ, 80^\circ, 90^\circ \end{bmatrix}$	$V_f = 0.75\%$



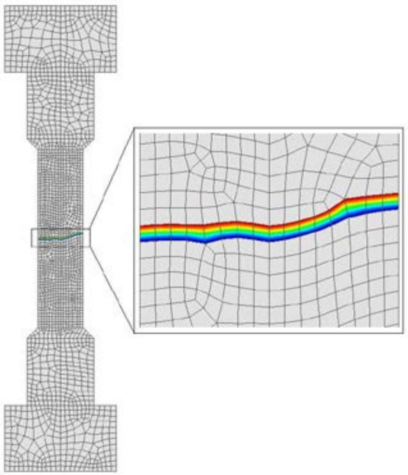
Geometry, experimental setup and F.E. mesh



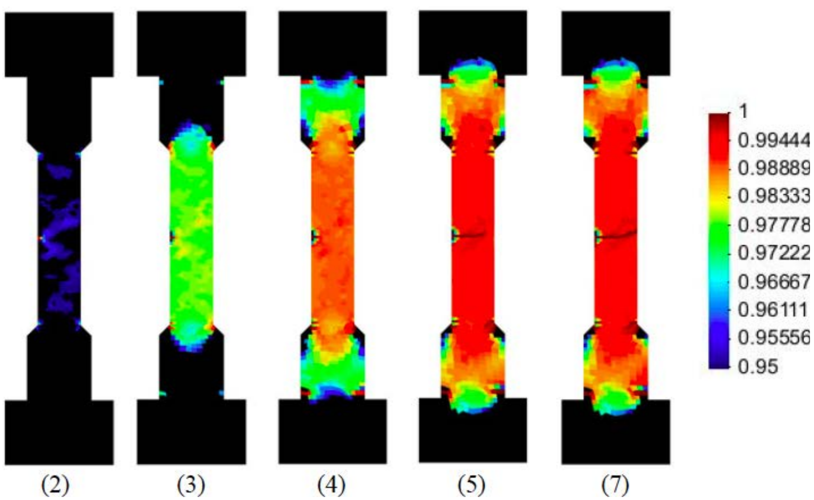
Load vs. displacement curves

FIBER-REINFORCED CONCRETE
(AS A CONTINUUM WITH MICROSTRUCTURE)

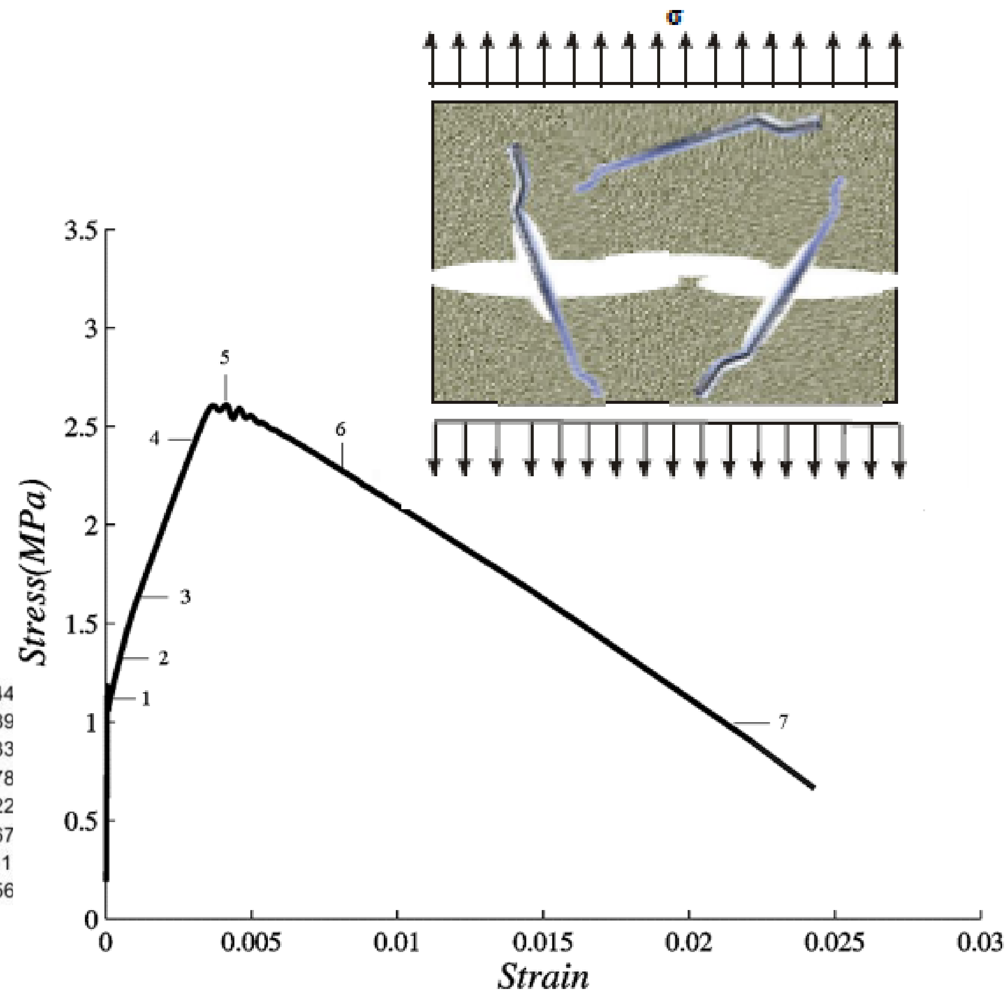
CRACK BRIDGING EFFECTS



Numerical crack pattern



Damage evolution



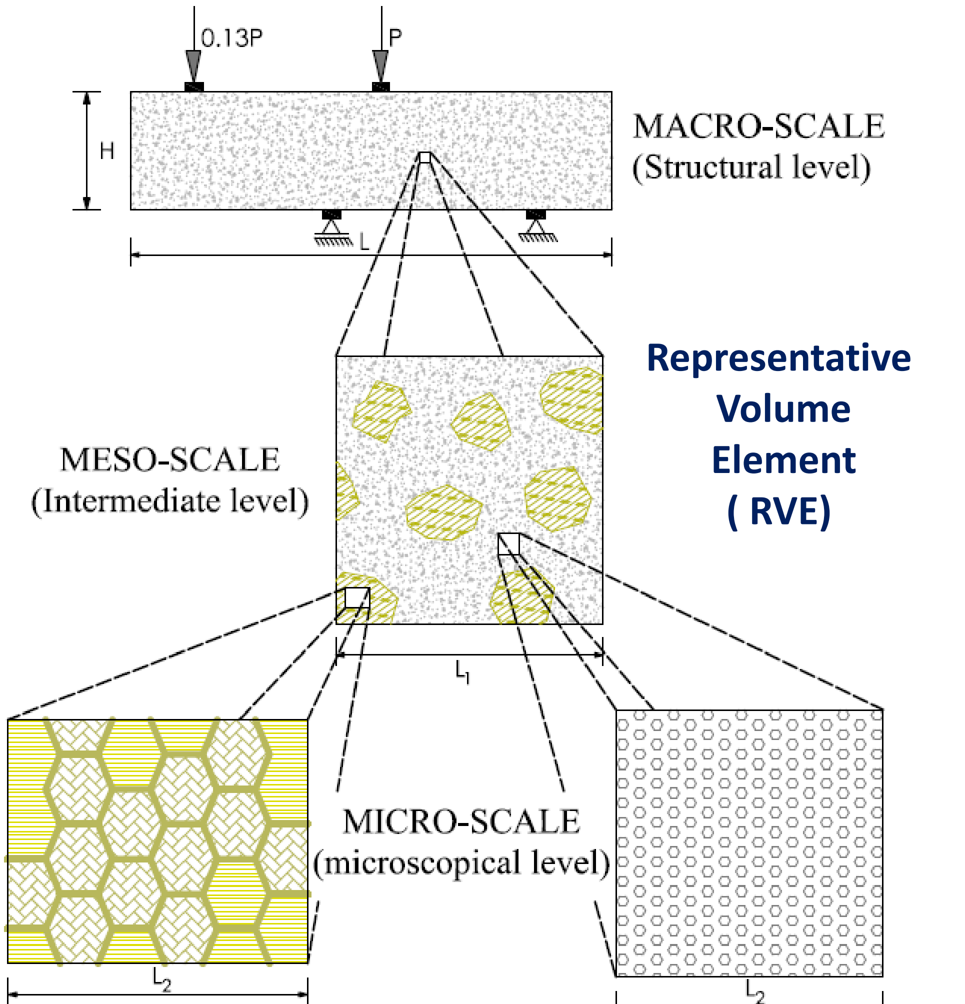
Stress-strain equivalent law

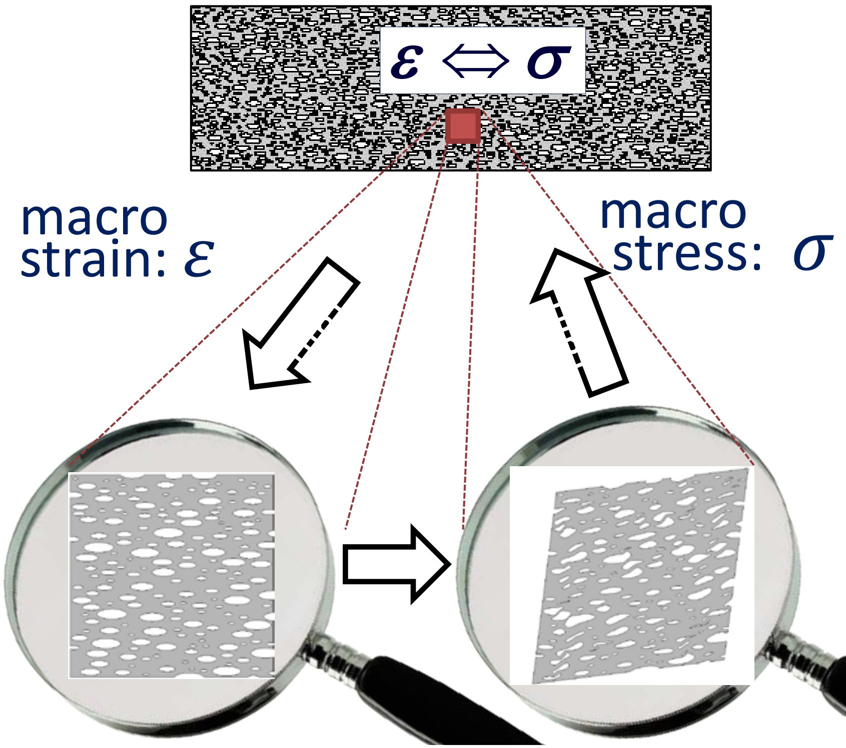
MECHANICAL APPROACHES TO CONCRETE CRACKING

- ❑ Fiber/filament beam models
- ❑ Concrete as a composite material
- ❑ Micro-structure endowed material
- ❑ **Computational multiscale modeling of concrete**

COMPUTATIONAL MULTISCALE MODELING OF CONCRETE

- **Stress-strain law obtained from low-level physics**





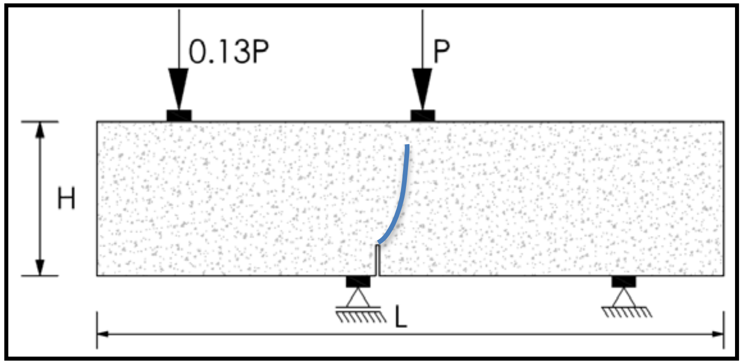
Non-linear RVE equilibrium problem

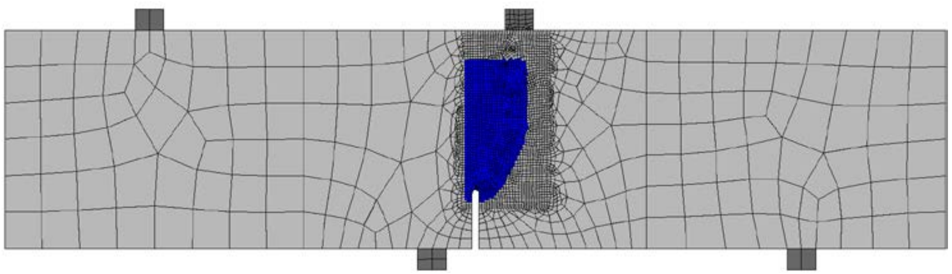
$$\boldsymbol{\sigma}(\mathbf{x},t)=\frac{1}{V_\mu}\int_{\Omega_\mu}\boldsymbol{\sigma}_\mu(\mathbf{y},t)\,\mathrm{dV}_\mu$$

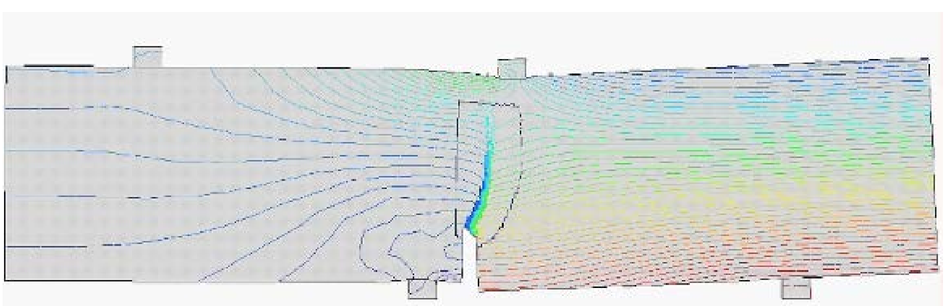
☞ **Every scale is solved by finite elements !!!!**
(brute-force approach)

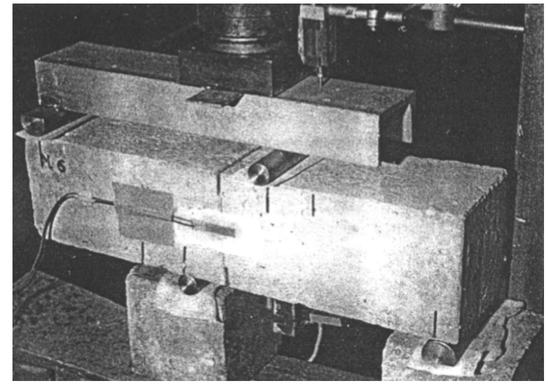
COMPUTATIONAL MULTISCALE MODELING OF CONCRETE

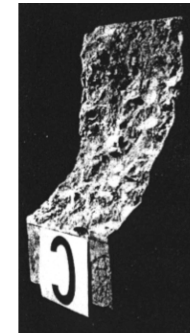
TWO-SCALE FOUR POINT BENDING TEST OF A NOTCHED BEAM

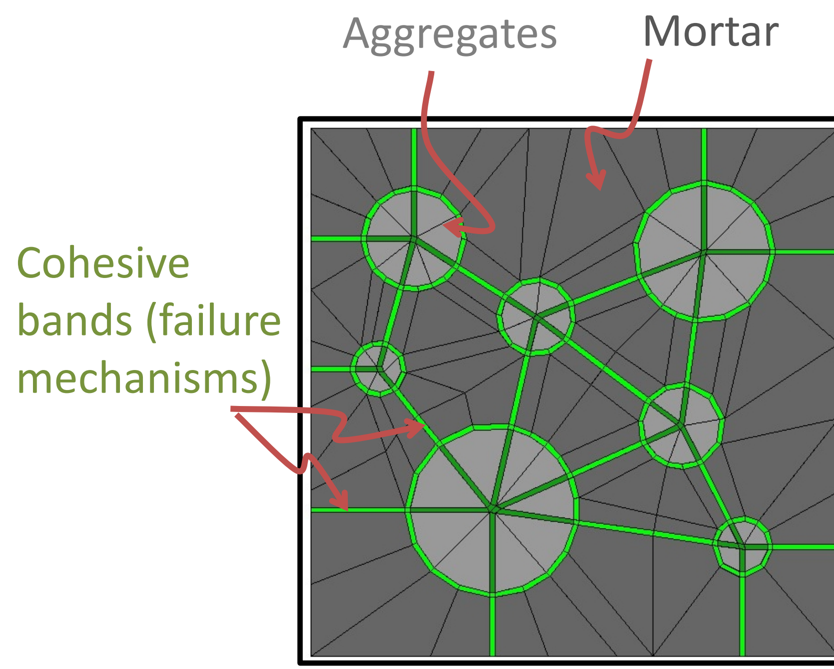










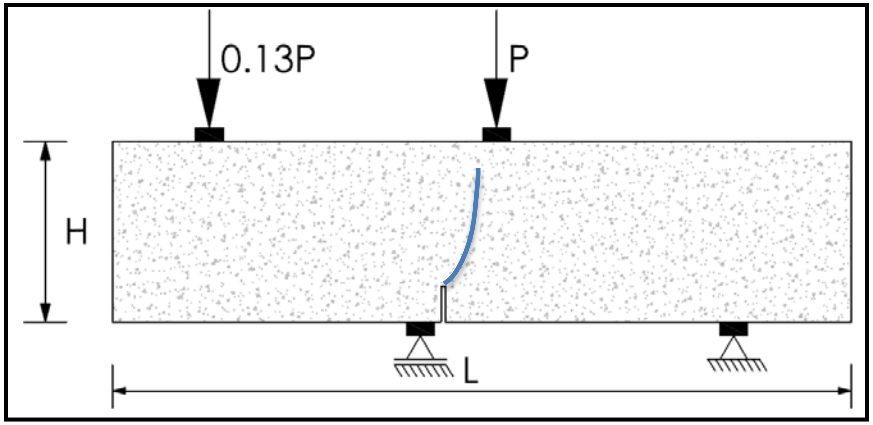


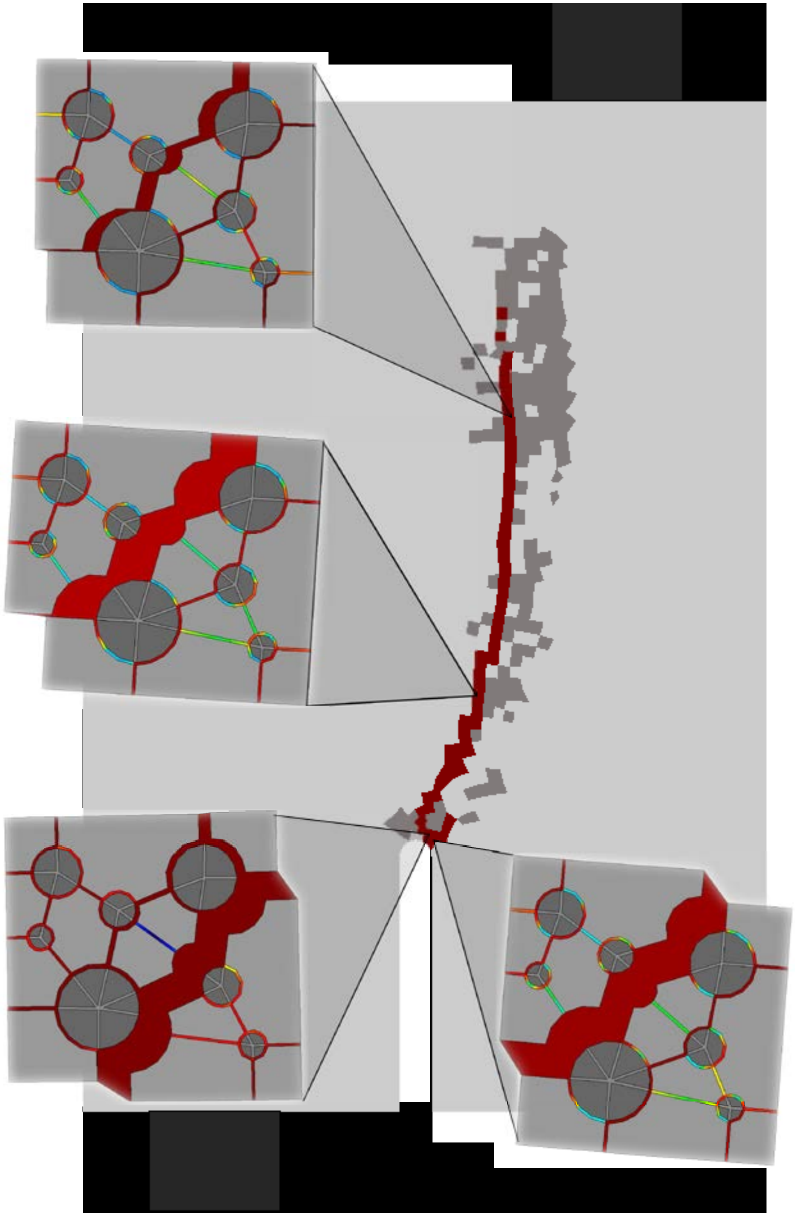
Meso-scale RVE

Ph. D. Thesis: Manuel Caicedo 2015

COMPUTATIONAL MULTISCALE MODELING OF CONCRETE

TWO-SCALE FOUR POINT BENDING TEST



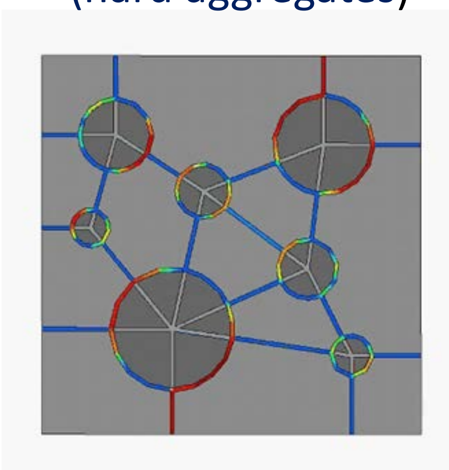


COMPUTATIONAL MULTISCALE MODELING OF CONCRETE

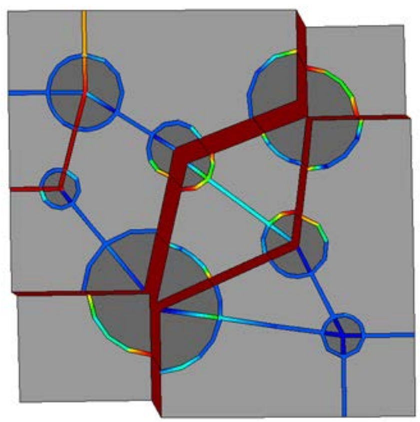
TWO-SCALE FOUR POINT BENDING TEST

Structural response sensitivity to the lower scale failure mechanism

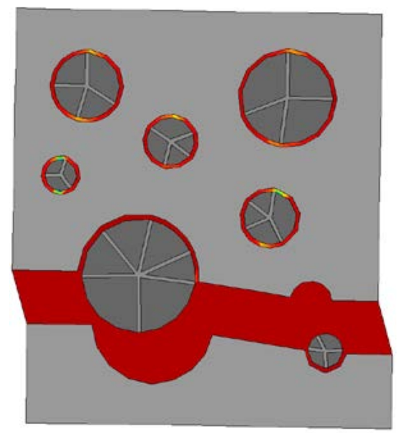
Extra-granular failure
(hard aggregates)



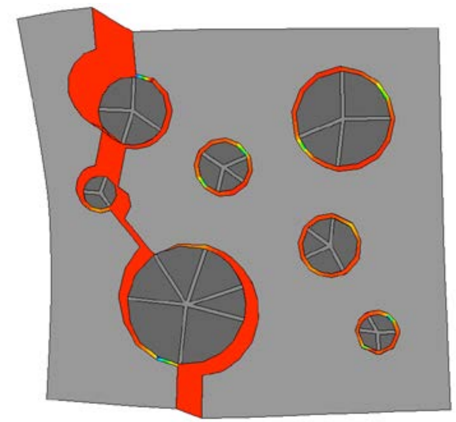
Intra-granular failure
(soft aggregates)



Prescribed horizontal
failure mode



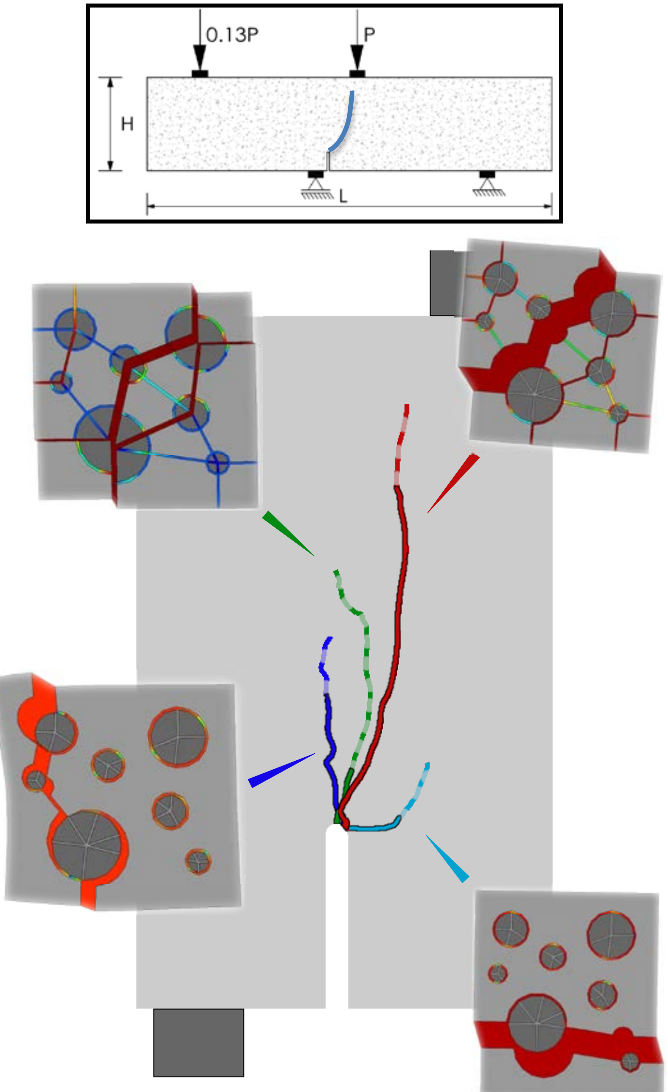
Prescribed vertical
failure mode

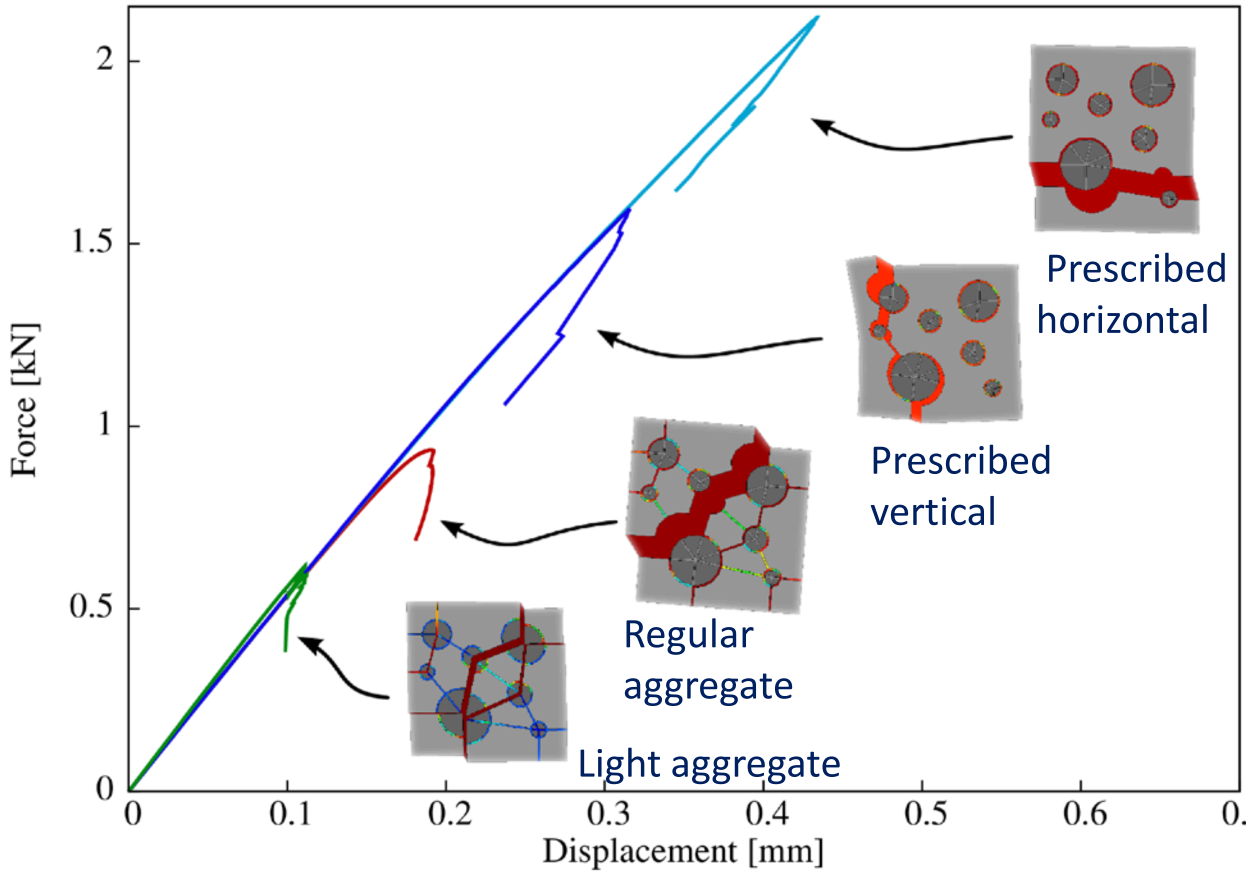


COMPUTATIONAL MULTISCALE MODELING OF CONCRETE

TWO-SCALE FOUR POINT BENDING TEST

Structural response sensitivity to the lower scale failure mechanism





Structural response (force vs. displacement)

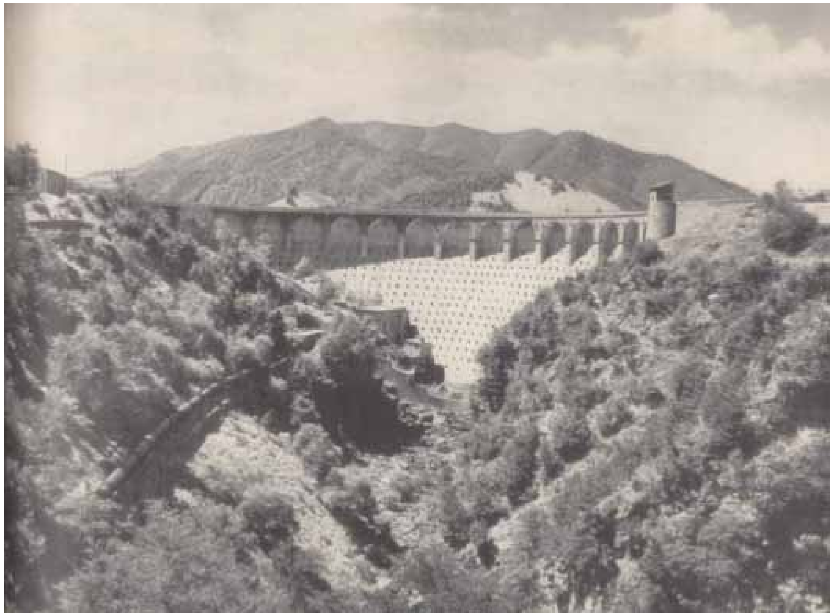
☞ M. Caicedo, E.Roubin 2014

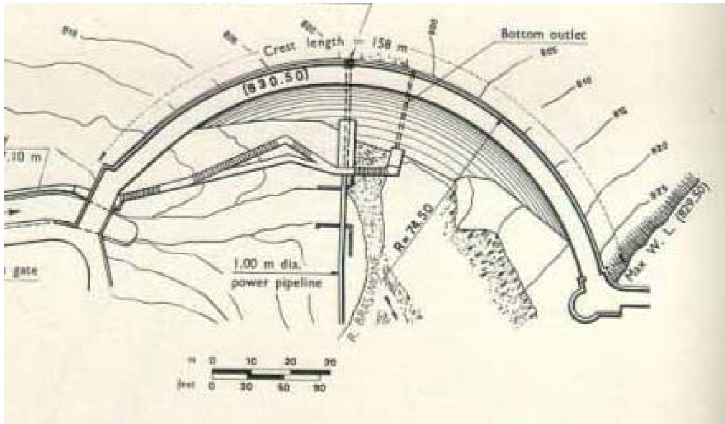
STRUCTURAL COLLAPSE

AND

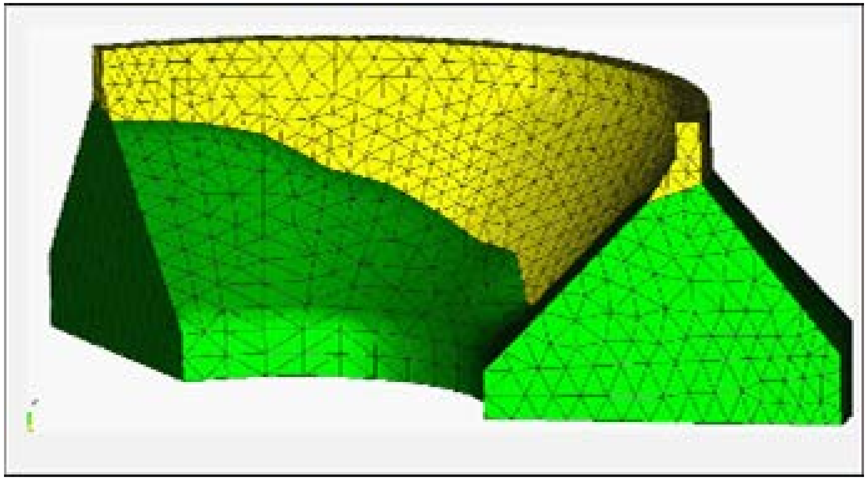
CRACKING OF CONCRETE

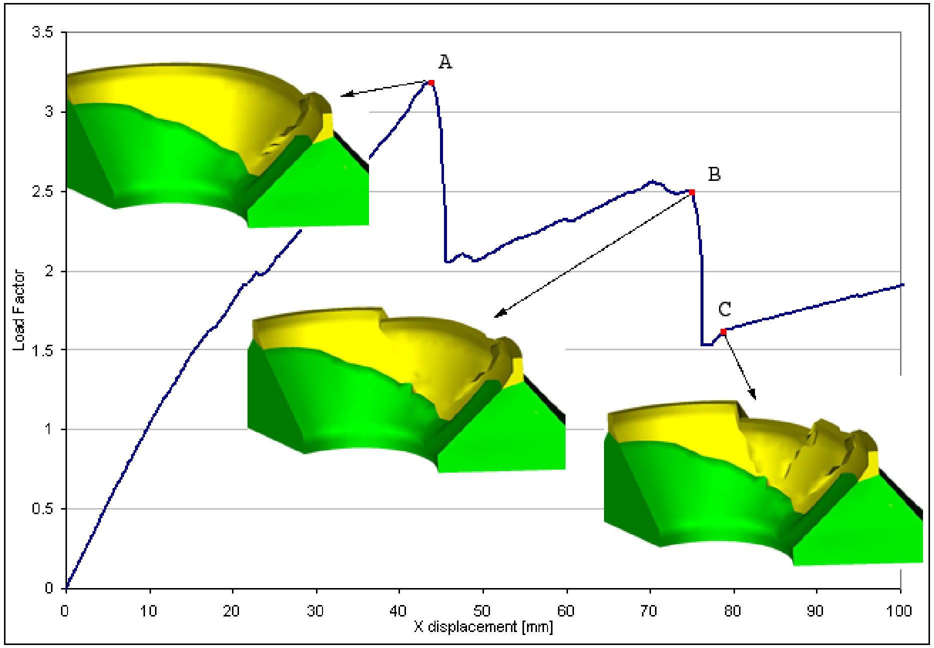
STRUCTURAL COLLAPSE MODELING OF CONCRETE





“Scalere” dam, Italy (1911-1912)

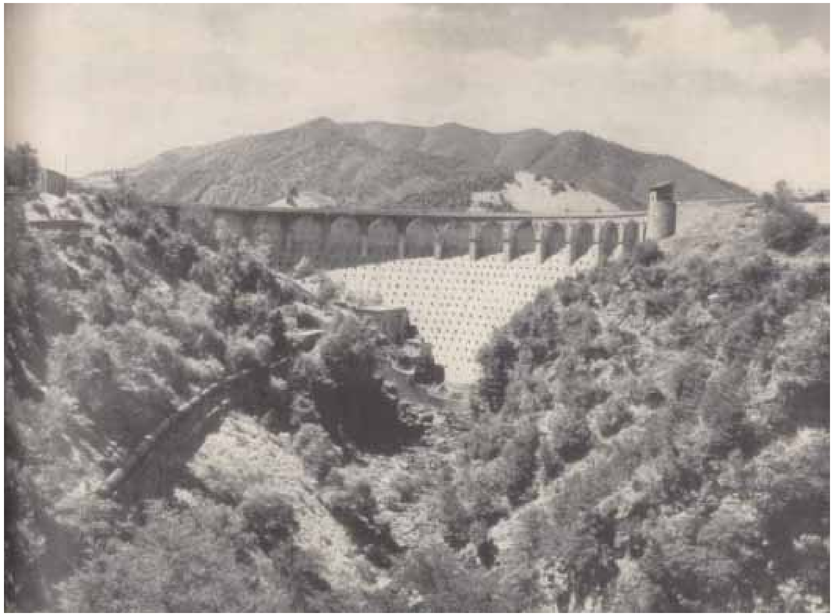


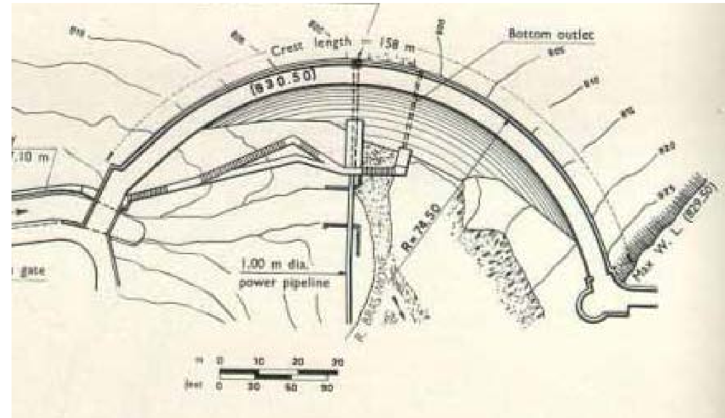


Load-factor vs. crest displacement

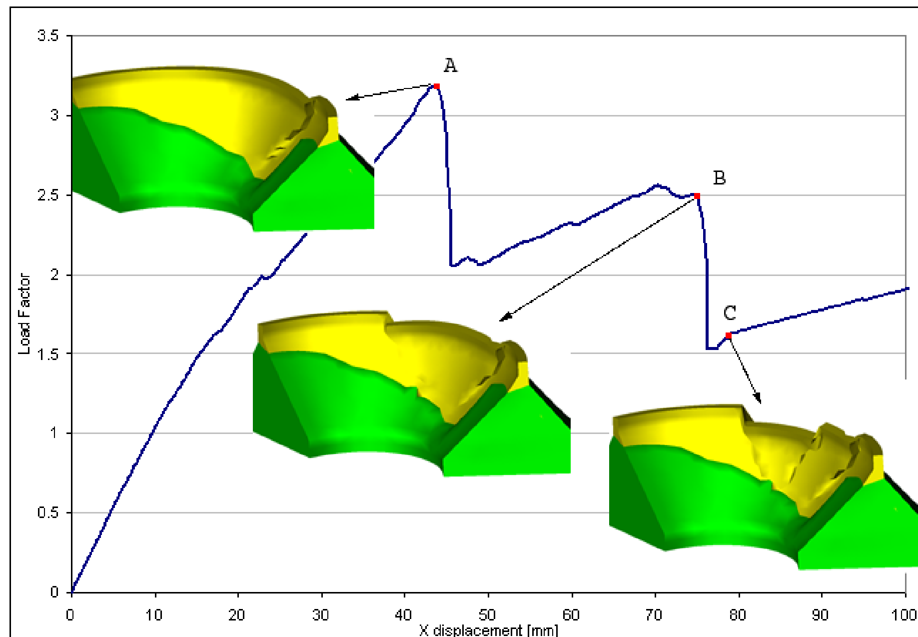
☞ **Ph. D. Thesis**: S. Blanco 2006

STRUCTURAL COLLAPSE MODELING OF CONCRETE





“Scalere” dam, Italy (1911-1912)

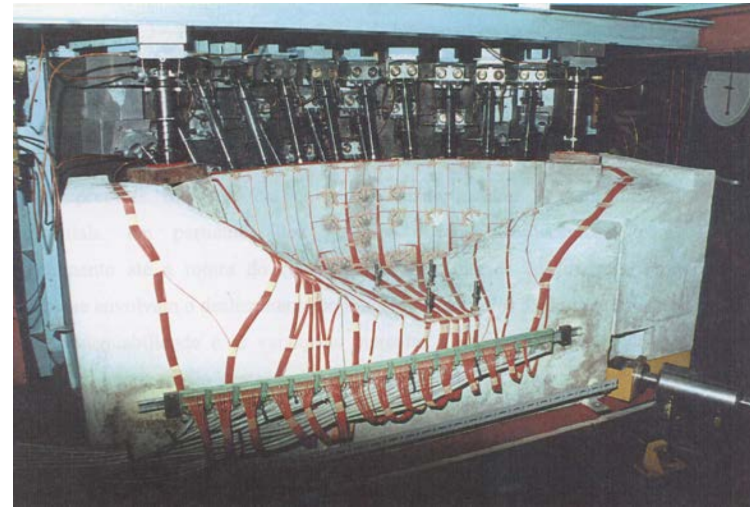


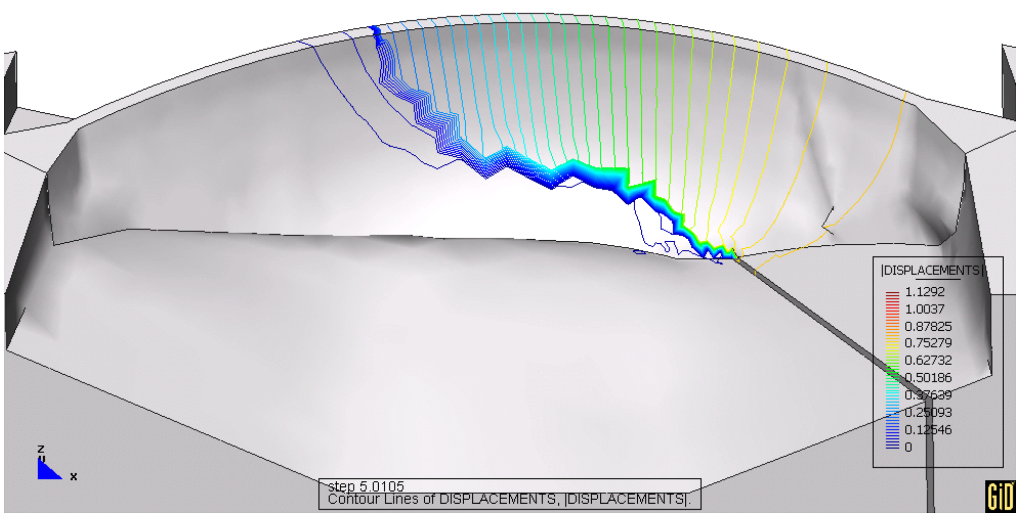
Load-factor vs. crest displacement

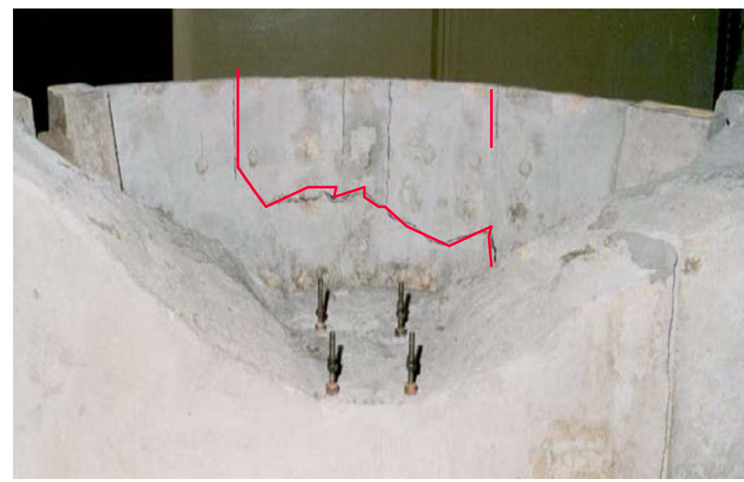
☞ **Ph. D. Thesis**: S. Blanco 2006

STRUCTURAL COLLAPSE MODELING OF CONCRETE

Alqueva dam,(Portugal)





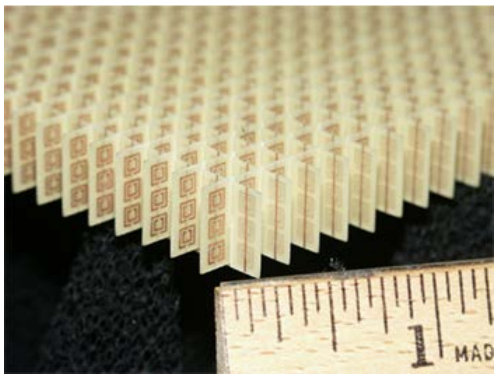


Experimental mock-up
(LNEC, Portugal)

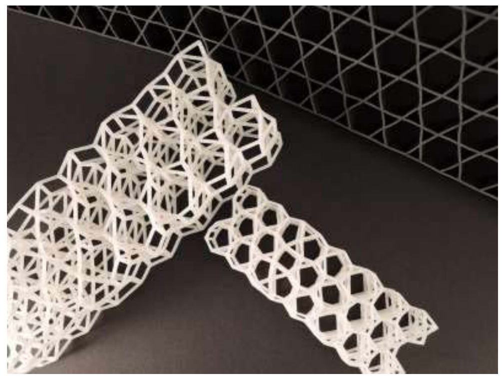
THE FUTURE ....

COMPUTATIONAL MATERIAL DESIGN
Motivation

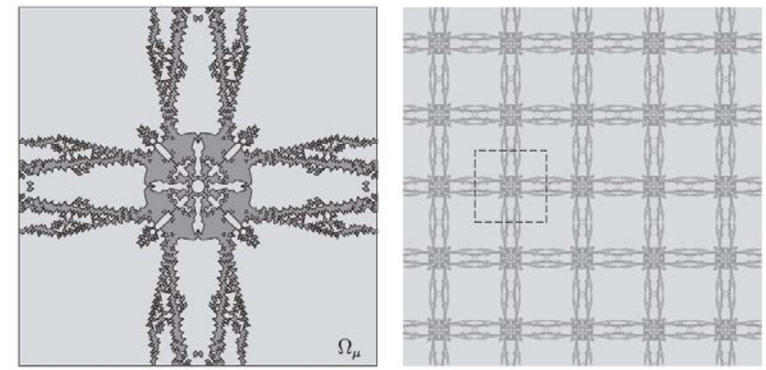
Meta-materials (beyond-materials): materials engineered to have properties that have not yet been found in nature



Negative refractive index
(NASA Glenn Research)



On-demand buckling
(shape-memory materials)



Negative Poisson-ratio
(Giusti 2009)

Reinforced concrete: engineered meta-material !!!
(the first one?)

COMPUTATIONAL MATERIAL DESIGN
COMPDESMAT project

erc European Research Council

ERC - Advanced Grant 2012

Advanced tools for computational design of engineering materials

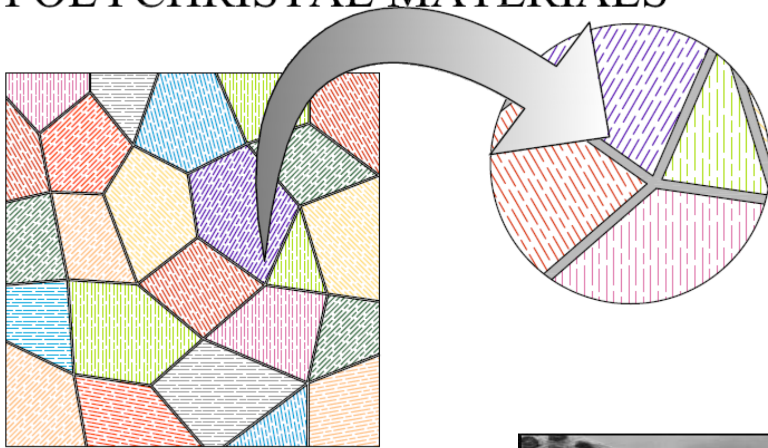
CIMNE

COMPDESMAT

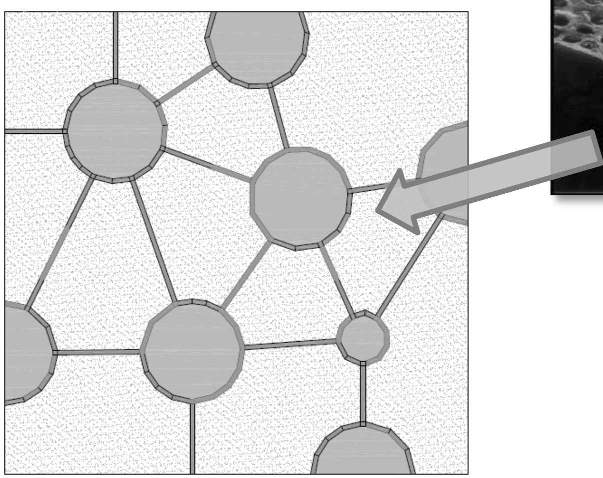
Advanced tools for computational design of engineering materials

COMPUTATIONAL MATERIAL DESIGN
COMPDESMAT project

POLYCHRISTAL MATERIALS

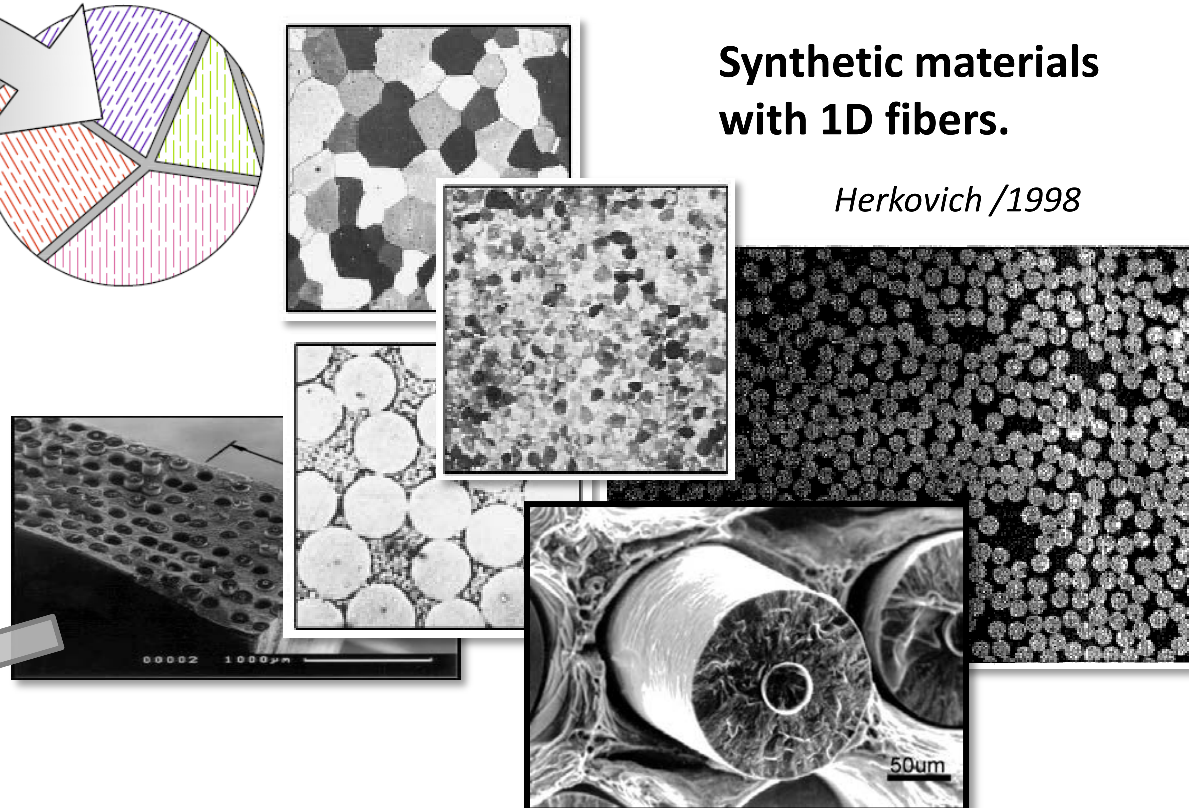


TRASVERSAL FIBERS



Synthetic materials with 1D fibers.

Herkovich /1998



Yates et al.

Miehe /2003

Carbon/Epoxy

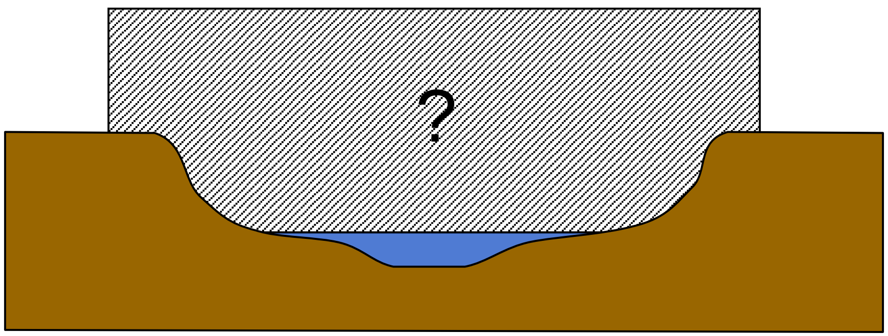
Material design in terms of:

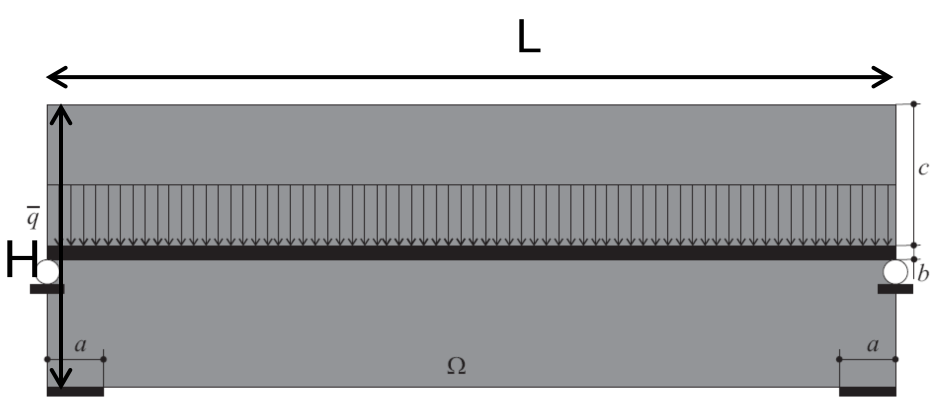
- arrangement
- morphology
- topology (in structural materials)

Structural topological design: minimum compliance (maximum stiffness)

- GOAL: Minimize de structural compliance (maximize stiffness) by optimal design of the structural topology with a given material volume

Bridge design





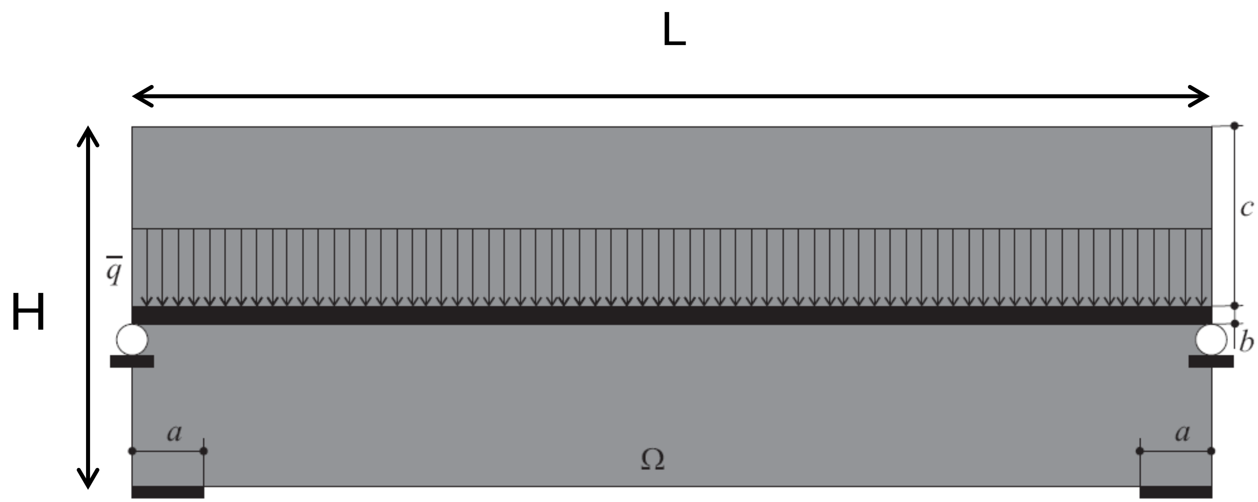
$$\chi(\mathbf{x}) : \mathcal{D} \rightarrow \{0,1\}$$

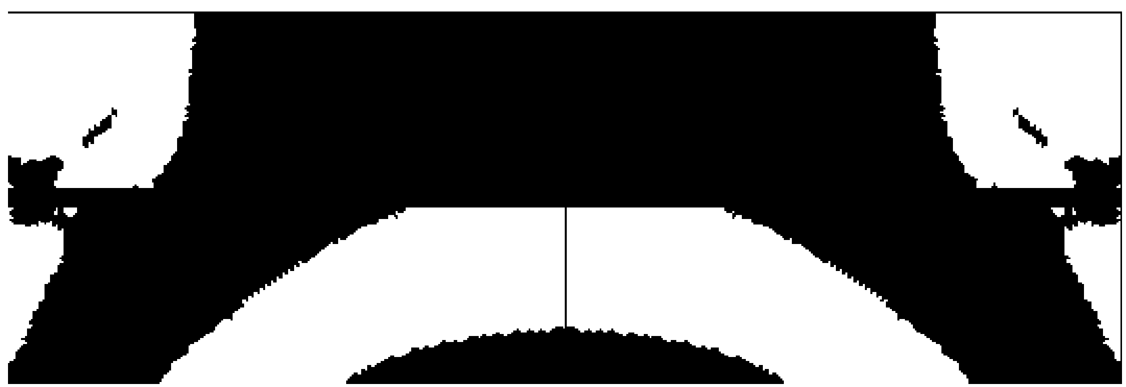
$$\Omega := \{\mathbf{x} \in \mathcal{D} \; ; \; \chi(\mathbf{x}) = 1\}$$

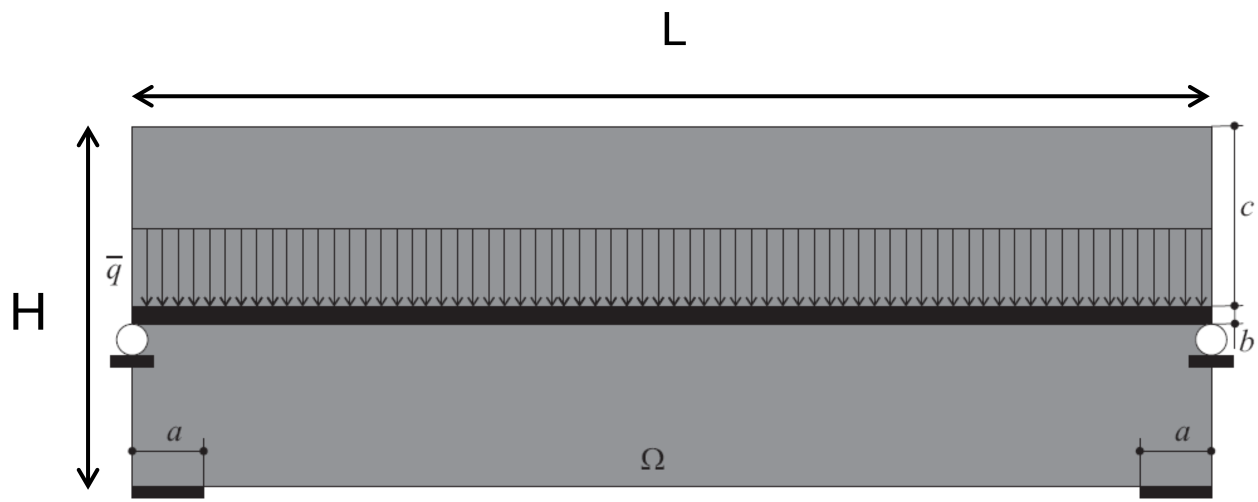
$$\underset{\substack{\chi(\mathbf{x}) \\ \forall \mathbf{x} \in \mathcal{D}}}{\text{minimize}} \quad \underbrace{\int_{\mathcal{D}} \tfrac{1}{2} \boldsymbol{\sigma} \cdot \mathbb{C}^{-1}(\chi_{\mu}) \cdot \boldsymbol{\sigma} \; d\mathcal{D}}_{\text{Structural compliance } (\mathcal{C})}$$

$$s.t. \; |\Omega(\chi)| = V$$

- Volume reduction $\rightarrow$ to 15 %


L
H
$\overline{q}$
c
b
a
a
Ω





L
H
c
$\bar{q}$
b
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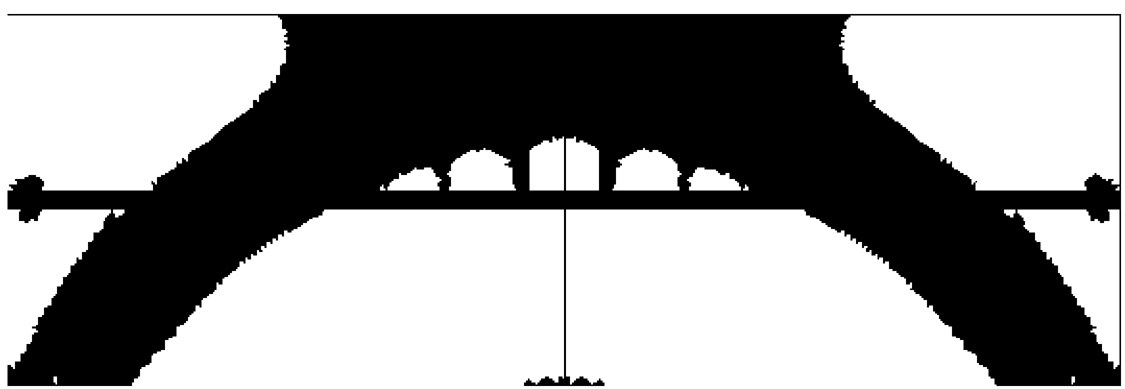
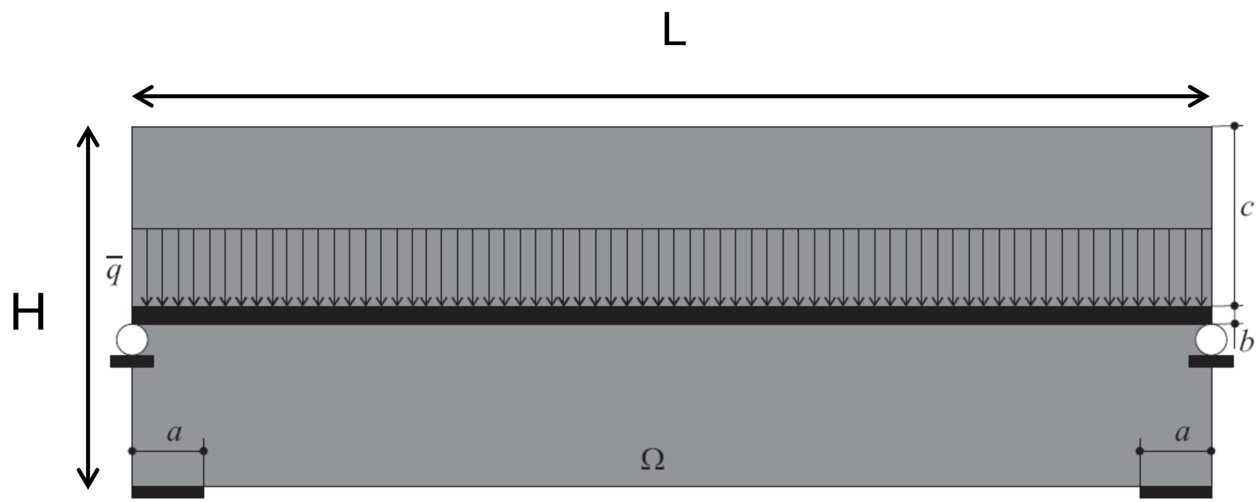
Ω

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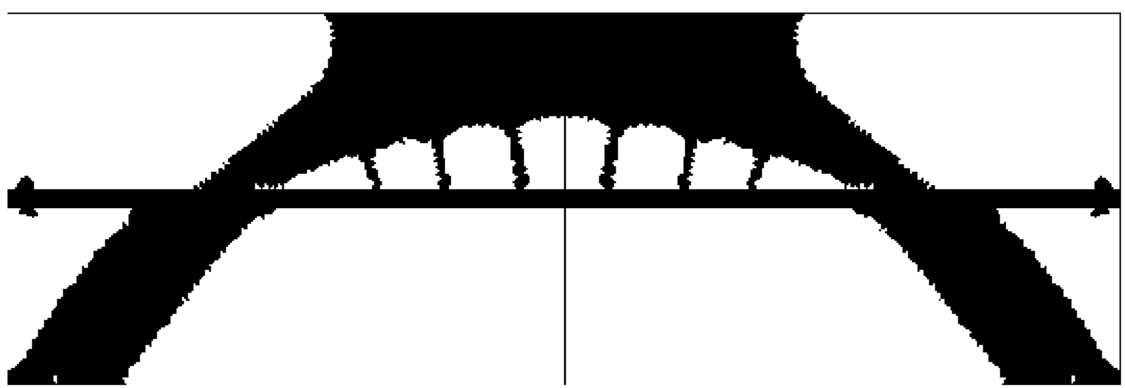

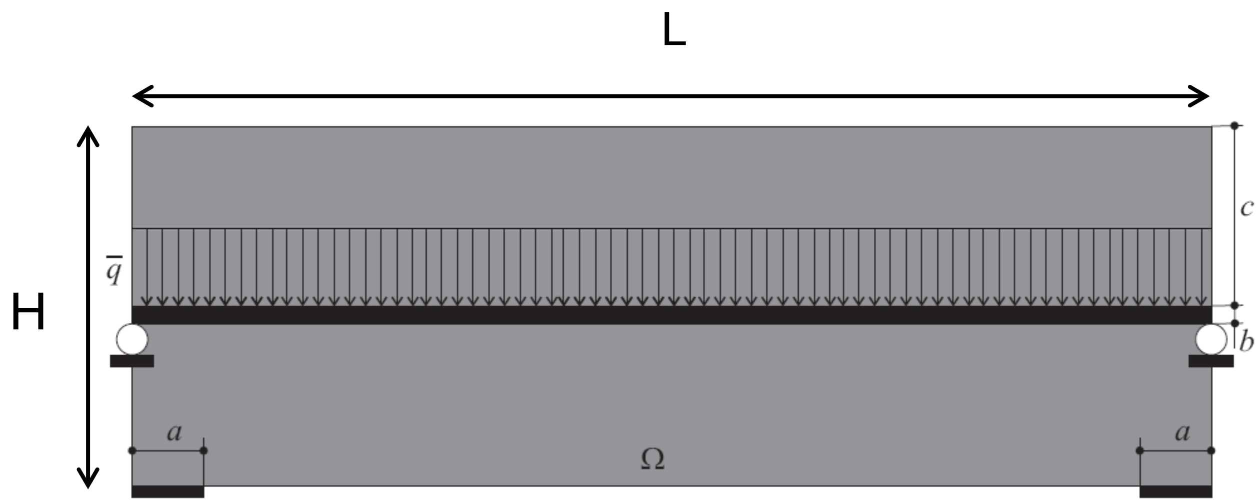
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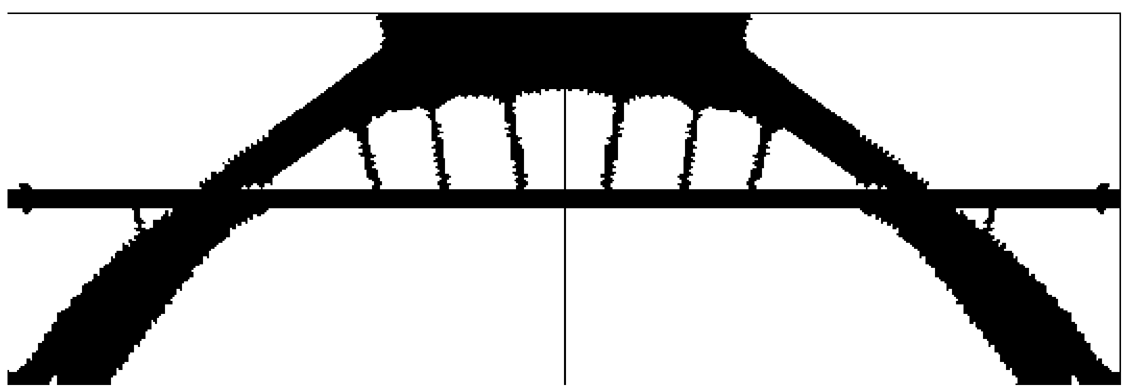

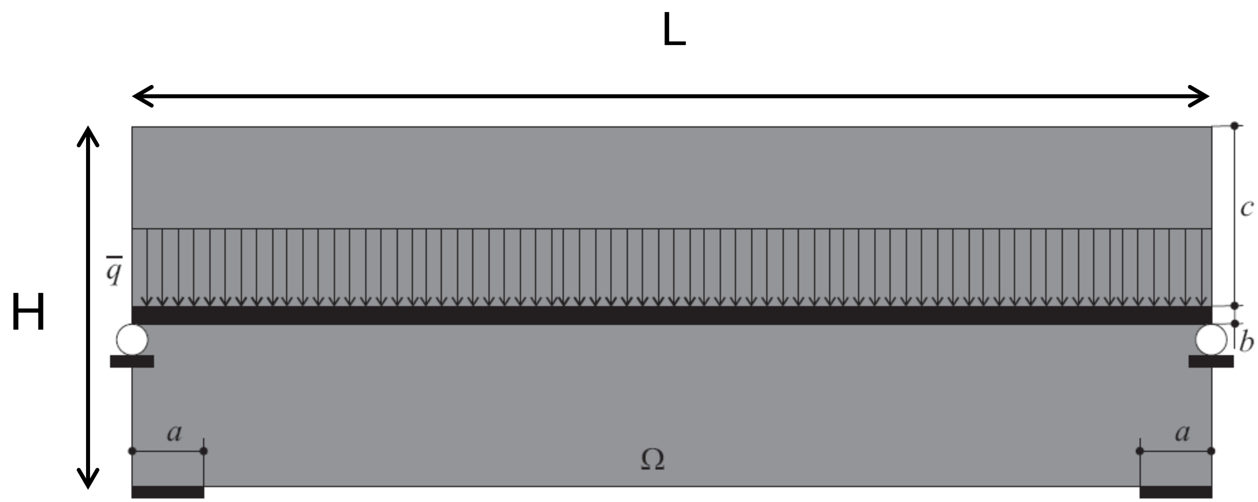
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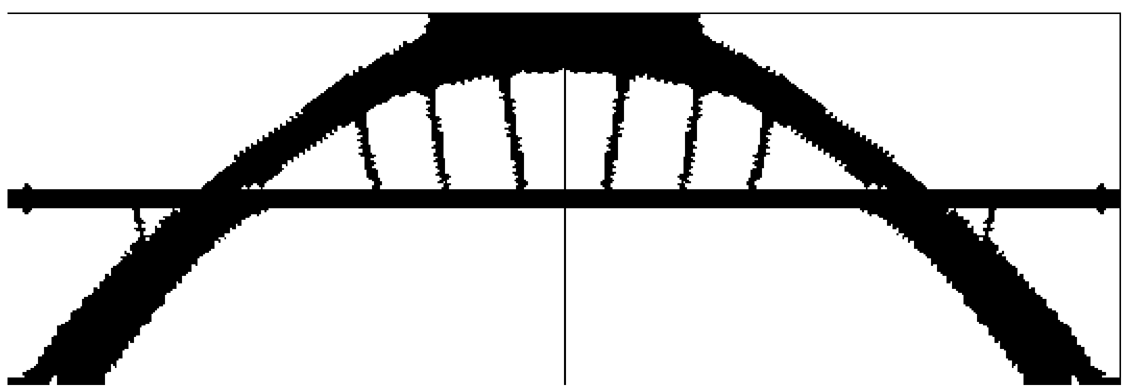
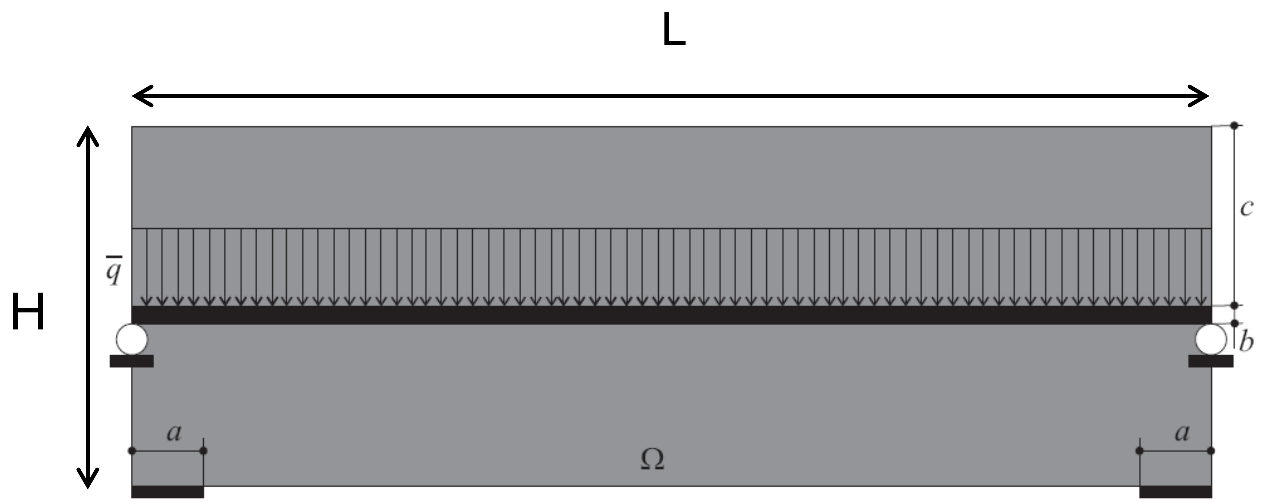
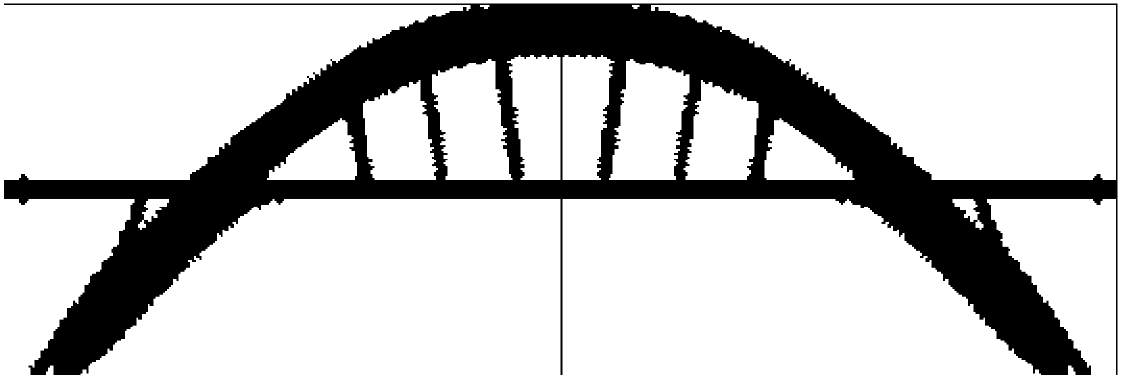
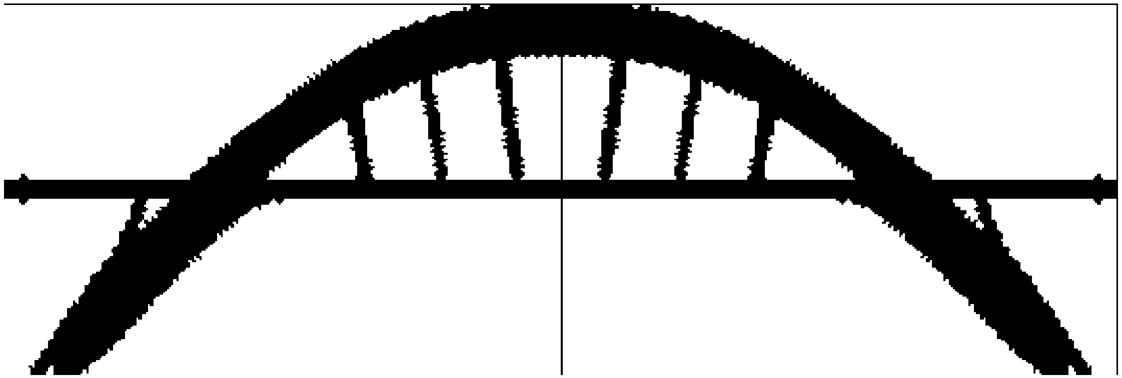






- Volume reduction

→ to 15 %



Material topological design: minimum compliance (maximum stiffness)

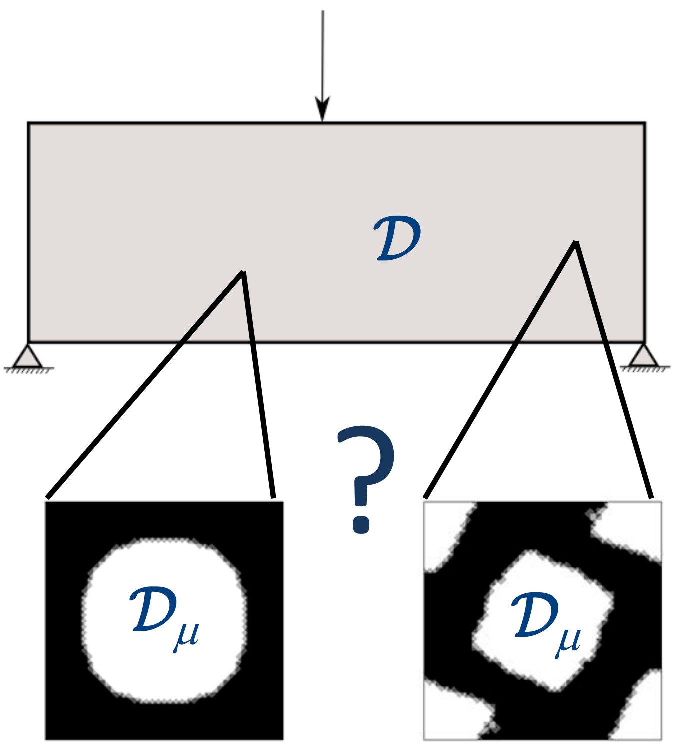
- GOAL: Minimize de structural compliance (maximize stiffness) by optimal design of the material topology for a given material volume reduction

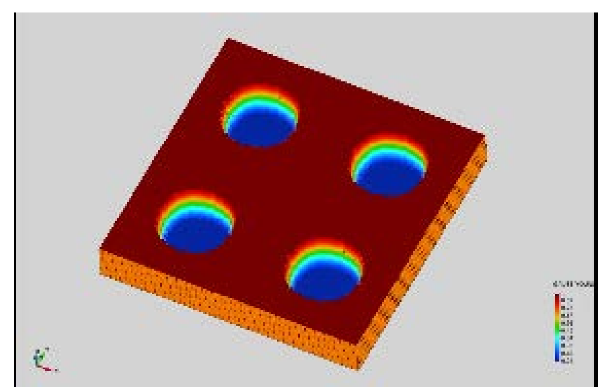
$$\chi_\mu(\boldsymbol{x},\boldsymbol{y}):\mathcal{D}\times\mathcal{D}_\mu \to \{0,1\}$$

$$\Omega_\mu := \{\boldsymbol{y}\in\mathcal{D}_\mu\ ;\ \ \chi_\mu(\boldsymbol{x},\boldsymbol{y})=1\}$$

$$\underset{\substack{\chi_{\mu,\boldsymbol{x}}(\boldsymbol{y})\\ \forall \boldsymbol{x}\in\mathcal{D}}}{\text{minimize}}\ \underbrace{\int_{\mathcal{D}} \tfrac{1}{2}\boldsymbol{\sigma}:\mathbb{C}^{\text{hom}^{-1}}(\chi_\mu):\boldsymbol{\sigma}\ d\mathcal{D}}_{\textit{Structural compliance } (\mathcal{C})}$$

$$s.t.\ \left|\Omega_\mu(\chi_\mu)\right| = V_\mu$$



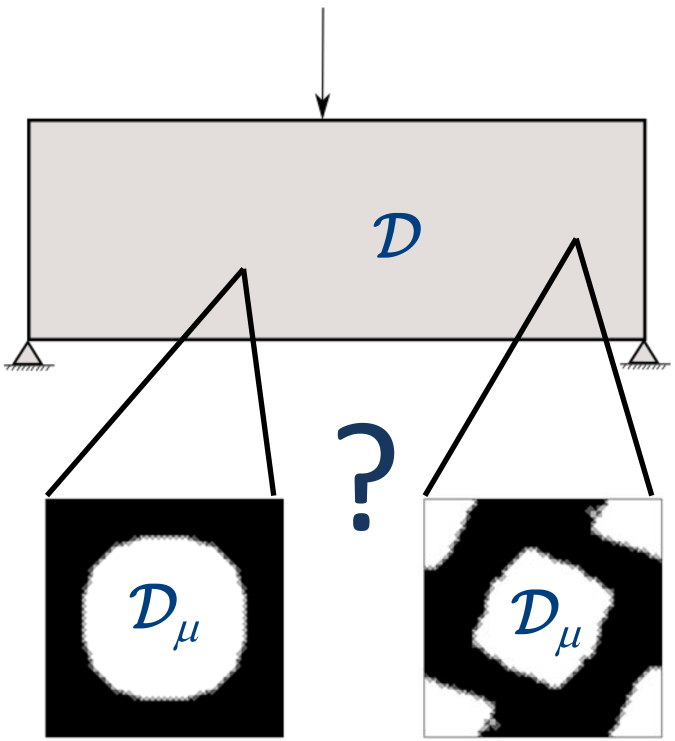


- *Material* volume reduction

  $\to$ to 60 %

Material topological design: minimum compliance (maximum stiffness)

- GOAL: Minimize de structural compliance (maximize stiffness) by optimal design of the material topology for a given material volume reduction



- *Material* volume reduction

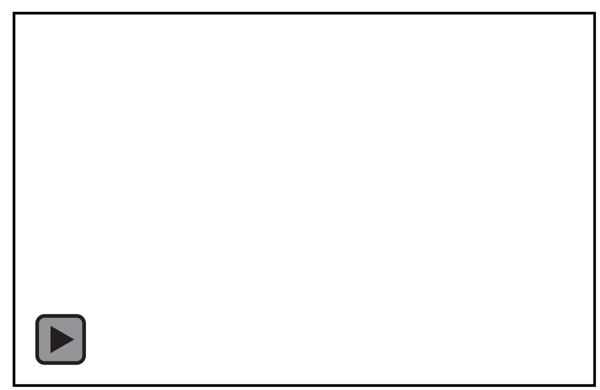
  $\rightarrow$ to 60 %

$$\chi_\mu(\mathbf{x},\mathbf{y}) : \mathcal{D} \times \mathcal{D}_\mu \rightarrow \{0,1\}$$

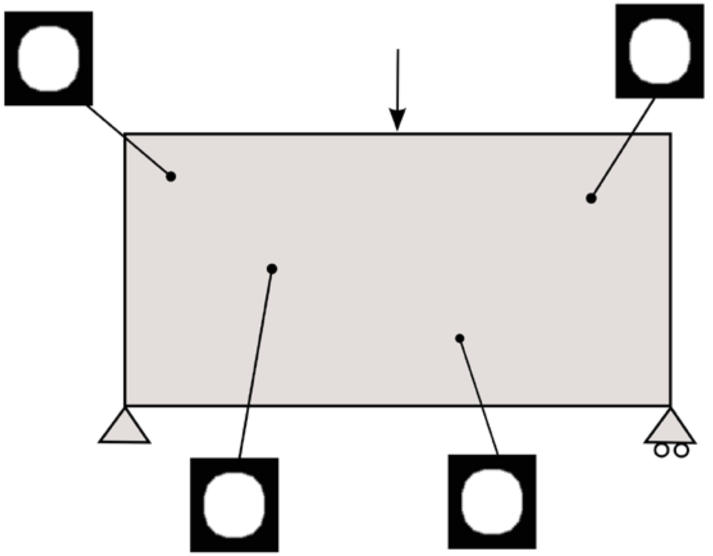
$$\Omega_\mu := \{\mathbf{y} \in \mathcal{D}_\mu \; ; \; \chi_\mu(\mathbf{x},\mathbf{y}) = 1\}$$

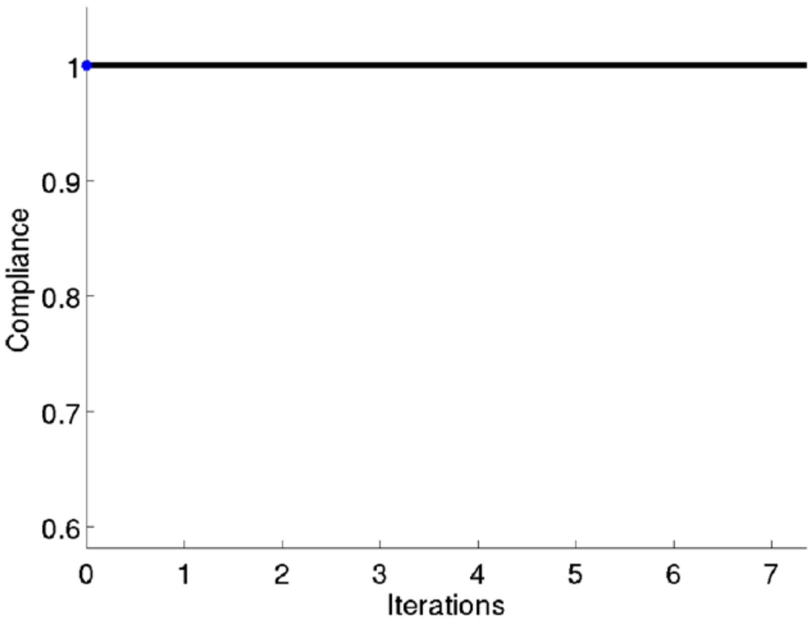
$$\underset{\substack{\chi_{\mu,\mathbf{x}}(\mathbf{y}) \\ \forall \mathbf{x} \in \mathcal{D}}}{\text{minimize}} \quad \underbrace{\int_{\mathcal{D}} \tfrac{1}{2} \boldsymbol{\sigma} : \mathbb{C}^{\text{hom}^{-1}}(\chi_\mu) : \boldsymbol{\sigma} \; d\mathcal{D}}_{\text{Structural compliance } (\mathcal{C})}$$

$$s.t. \; \left|\Omega_\mu(\chi_\mu)\right| = V_\mu$$

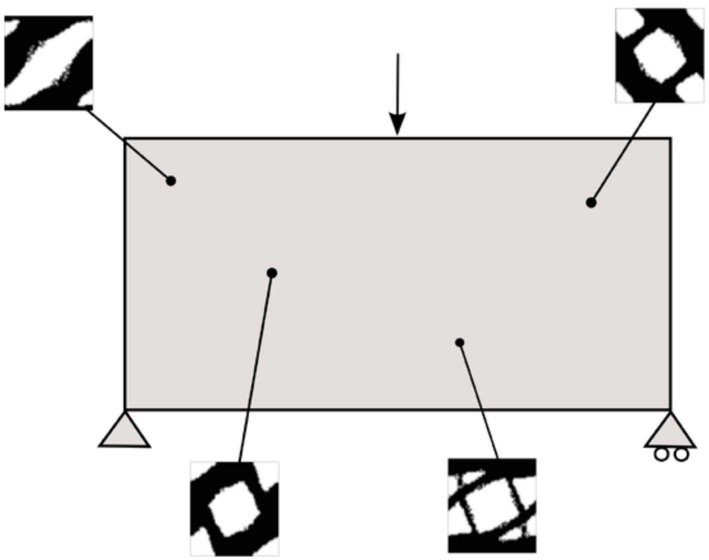


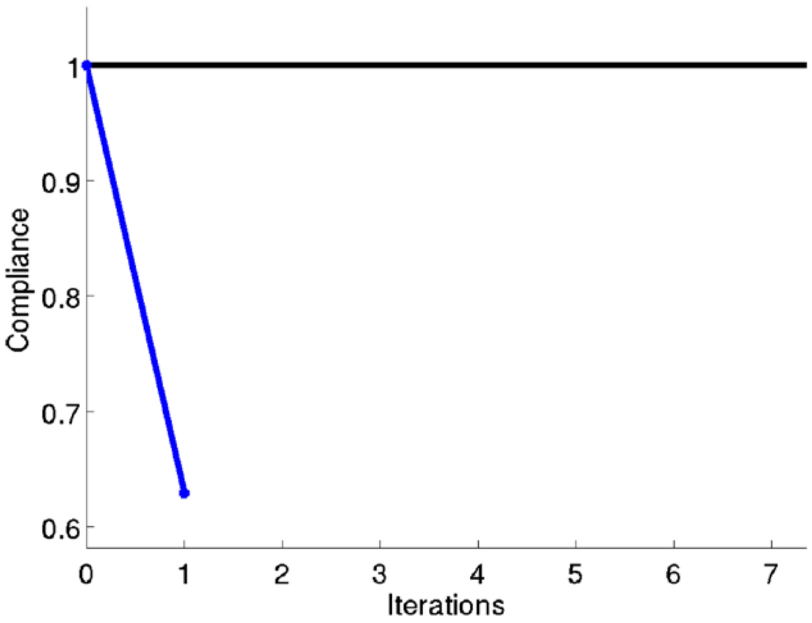
Material topological design:
minimum compliance (maximum stiffness)



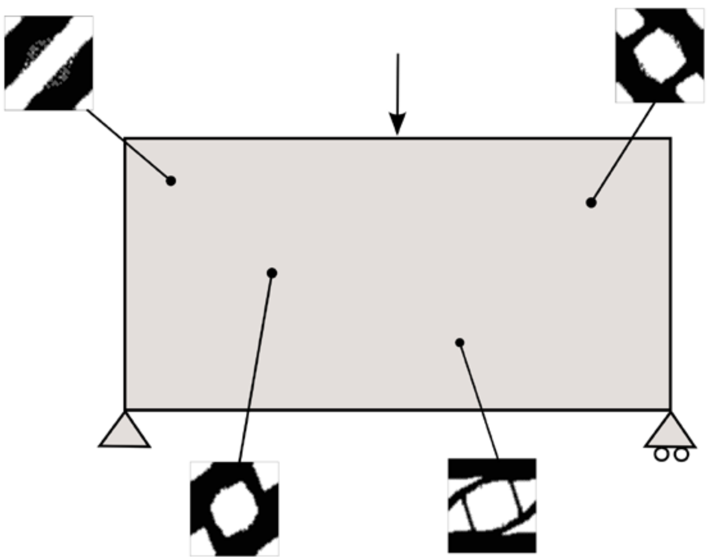


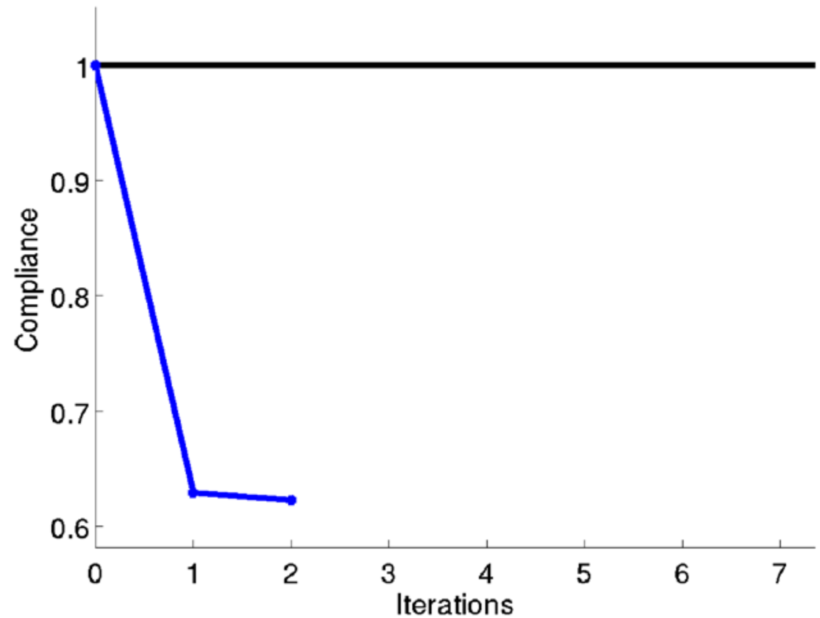
Material topological design: minimum compliance (maximum stiffness)



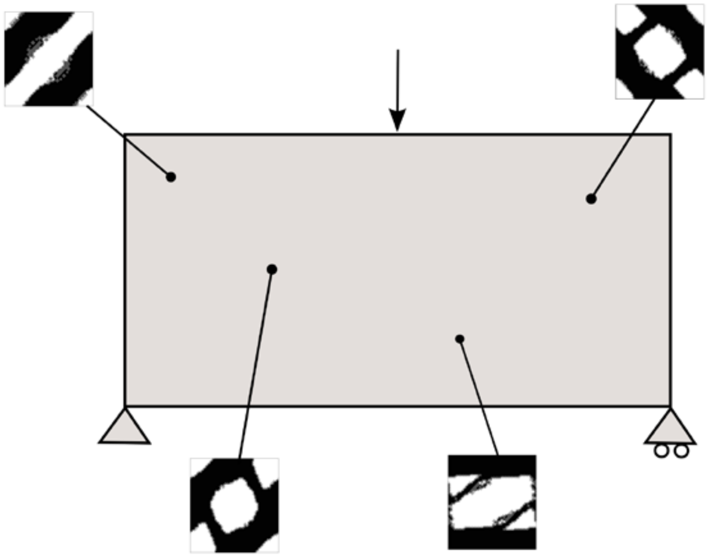


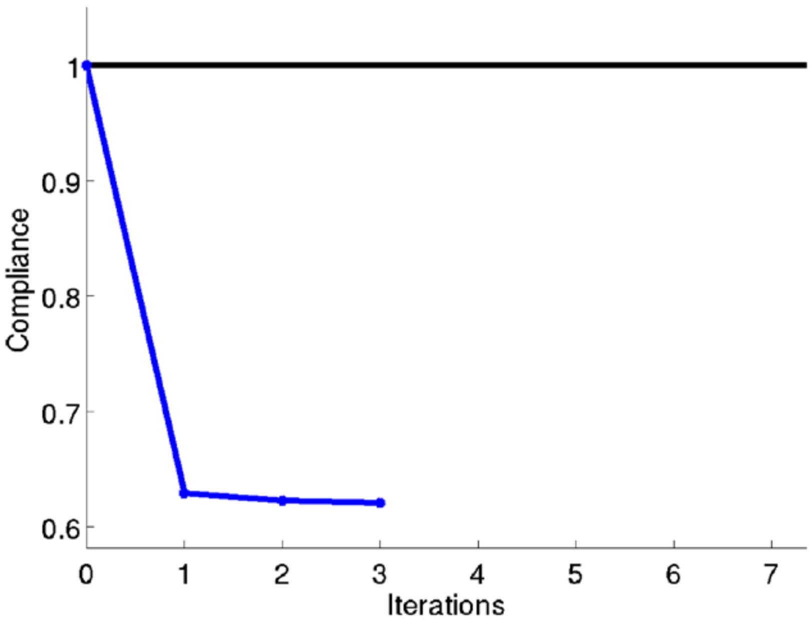
Material topological design: minimum compliance (maximum stiffness)



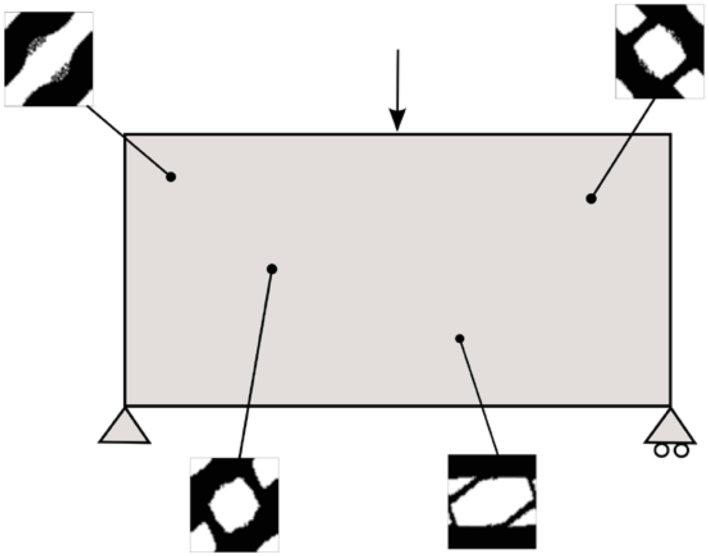


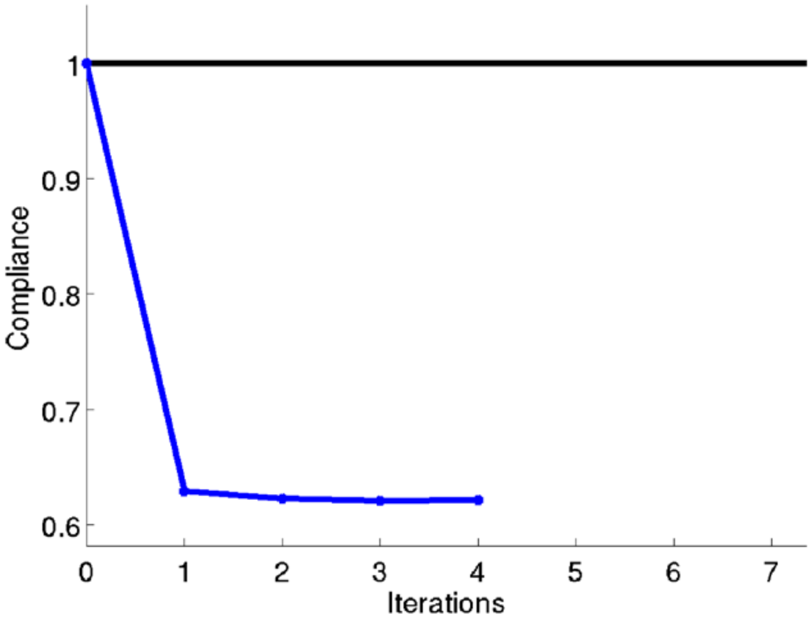
Material topological design: minimum compliance (maximum stiffness)



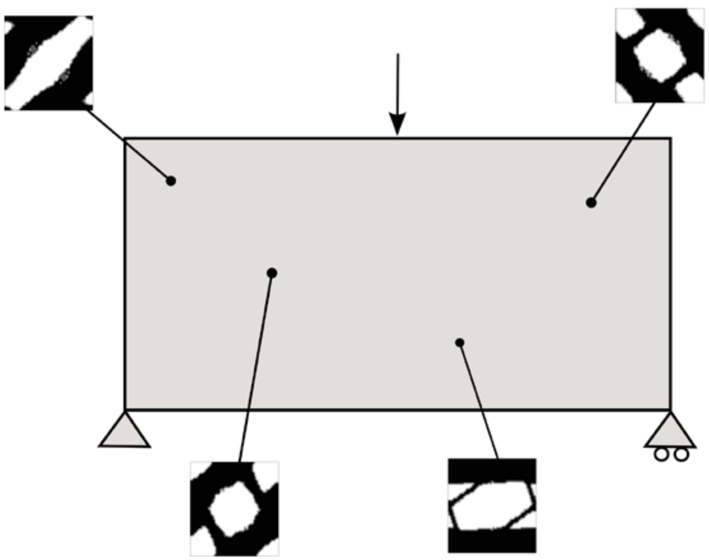


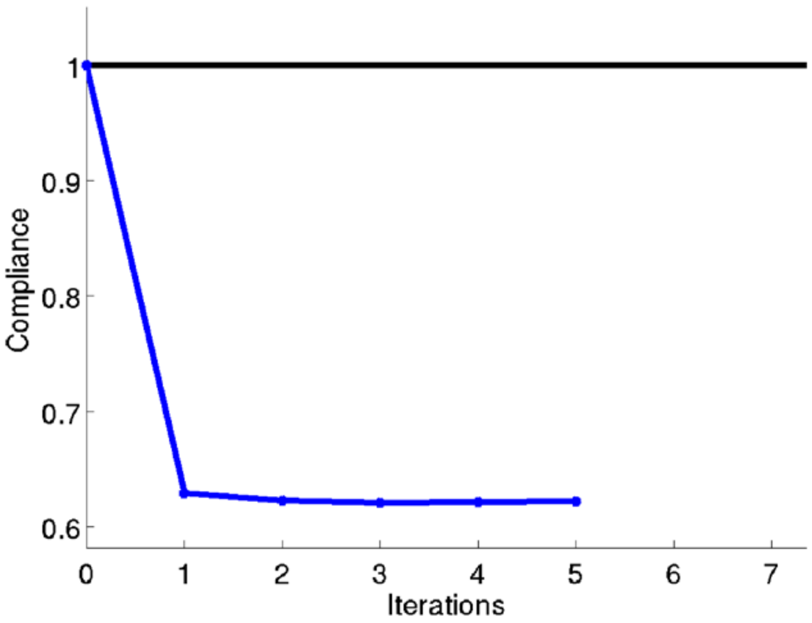
Material topological design: minimum compliance (maximum stiffness)



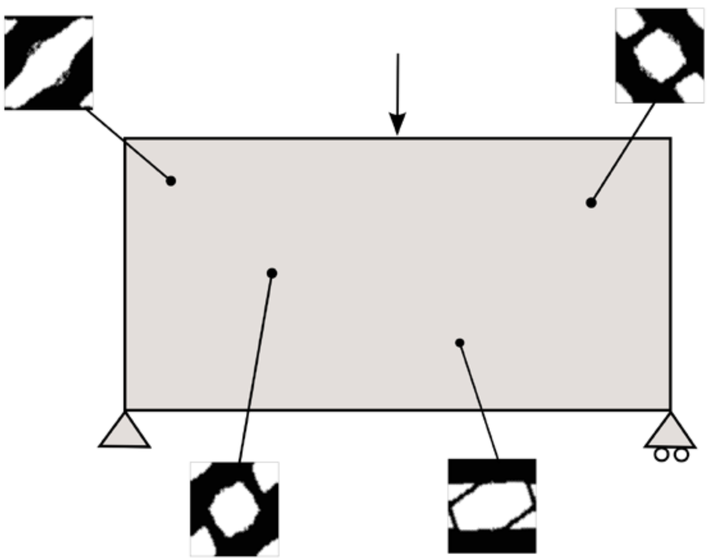


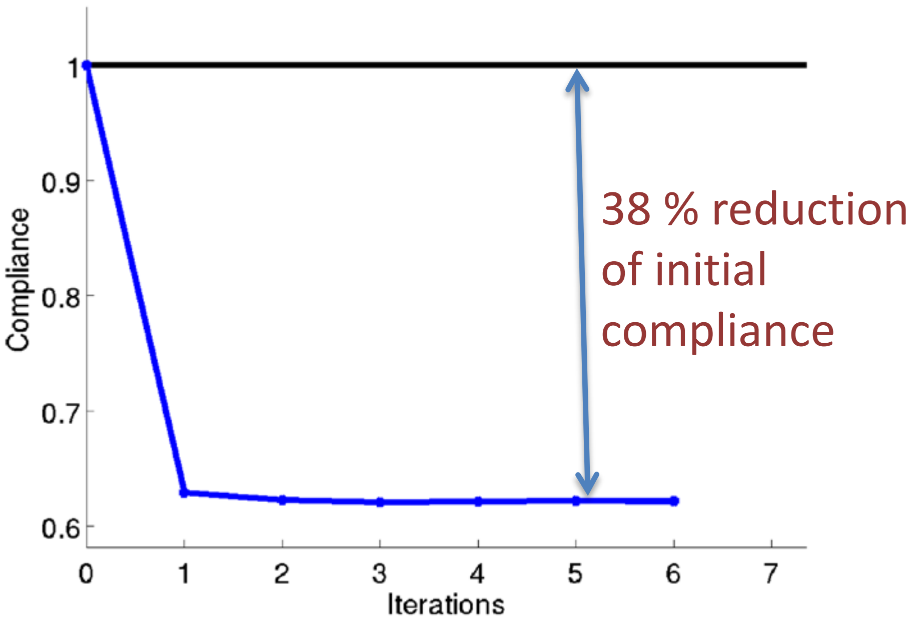
Material topological design: minimum compliance (maximum stiffness)





Material topological design: minimum compliance (maximum stiffness)



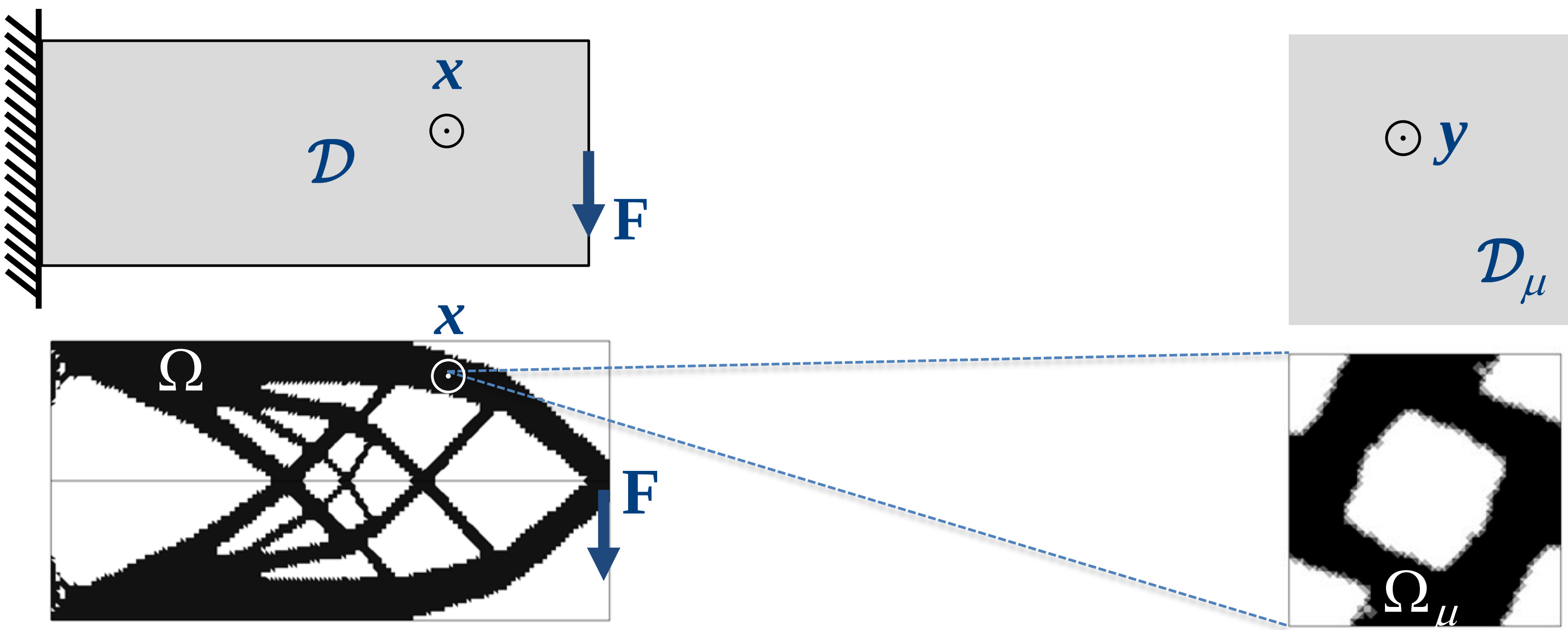


Concurrent (structural & material) topological design

- GOAL: Minimize de structural compliance (maximize stiffness) by optimal design of both the structural and material topology for a given total mass reduction.

$\chi(\boldsymbol{x}): \mathcal{D} \to \{0,1\}$

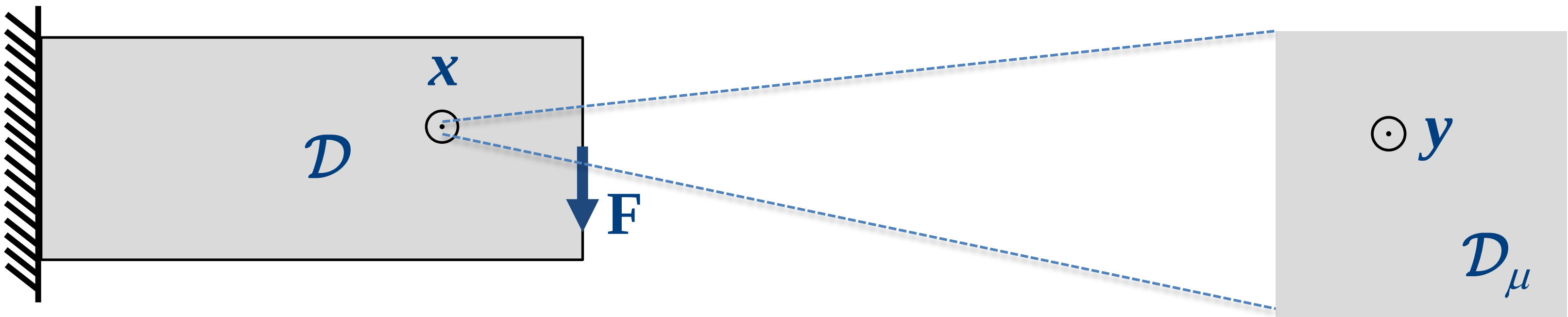
$\chi_{\mu,\boldsymbol{x}}(\boldsymbol{y}): \mathcal{D} \times \mathcal{D}_\mu \to \{0,1\}$



$\Omega := \{\mathbf{x} \in \mathcal{D} \; ; \; \chi(\mathbf{x}) = 1\}$

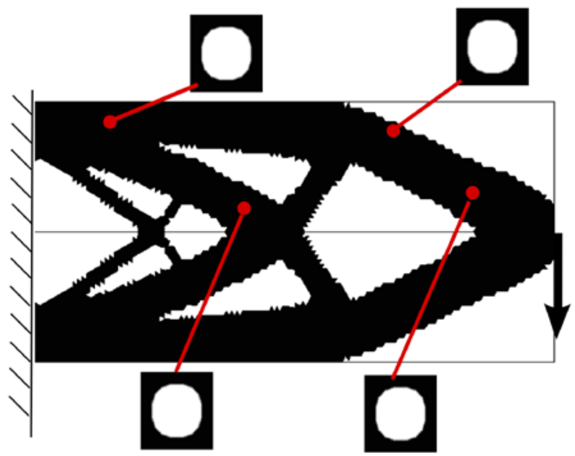
$\Omega_\mu(\boldsymbol{x}) := \{\boldsymbol{y} \in \mathcal{D}_\mu \; ; \; \chi_{\mu,\boldsymbol{x}}(\boldsymbol{y}) = 1\}$

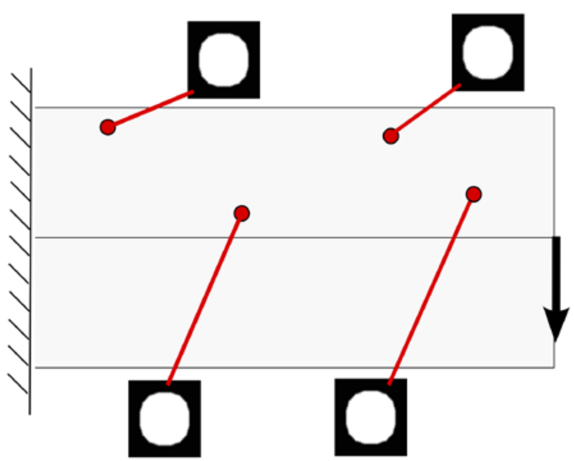
Concurrent (structural & material) topological design

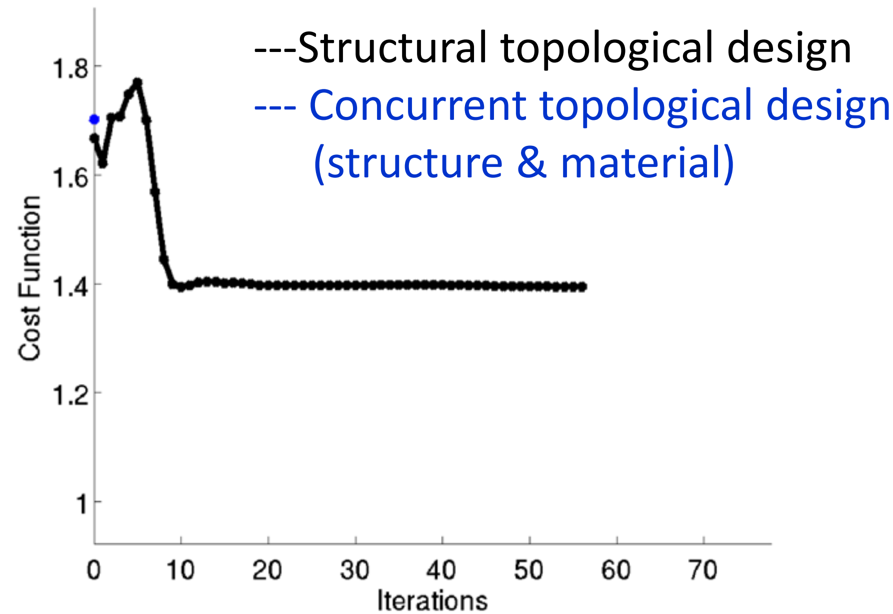


- *Structural* volume reduction (macro - scale) $\rightarrow$ to 60 %
- *Material* volume reduction (micro - scale) $\rightarrow$ to 60 %
- *Total* mass reduction $\rightarrow$ to 36 %

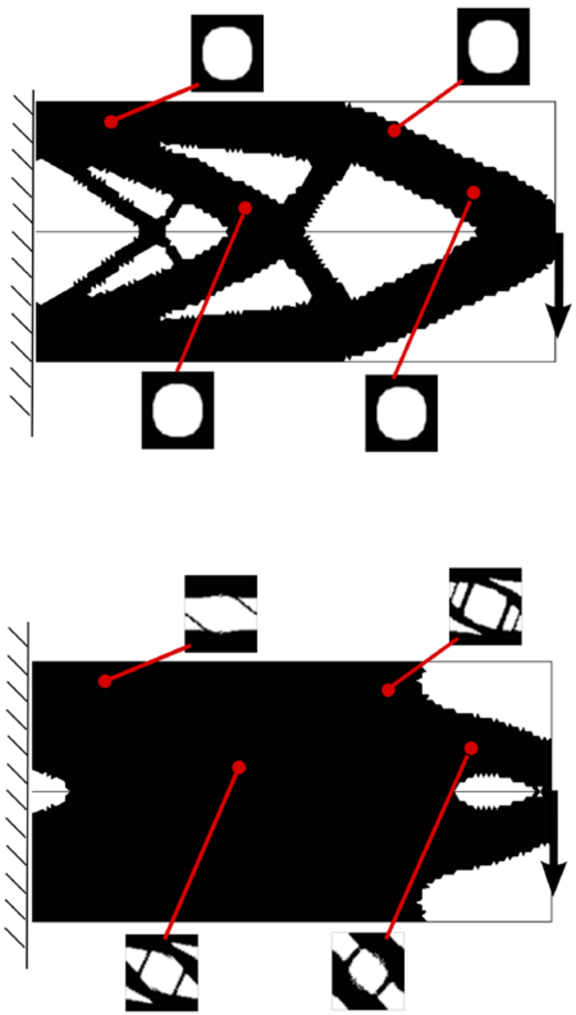
Concurrent (structural & material) topological design

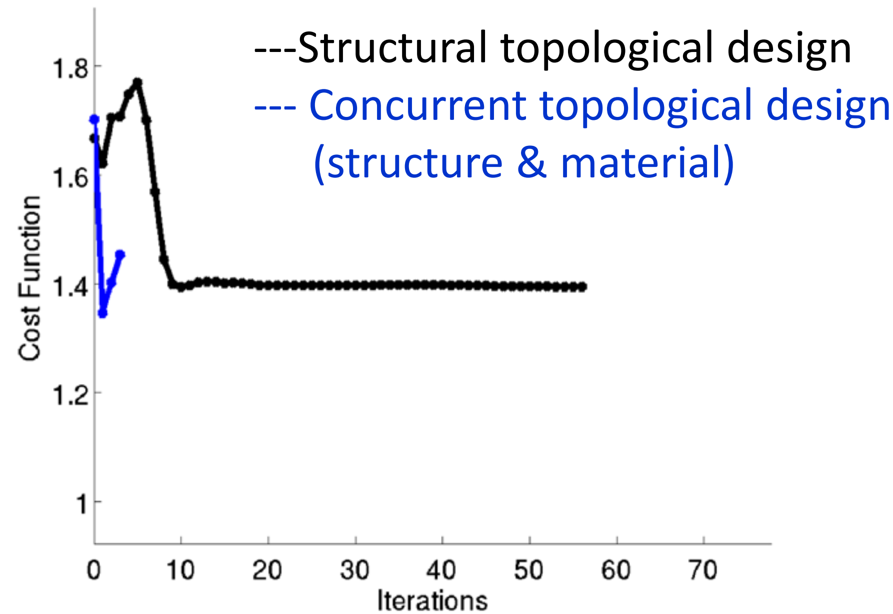




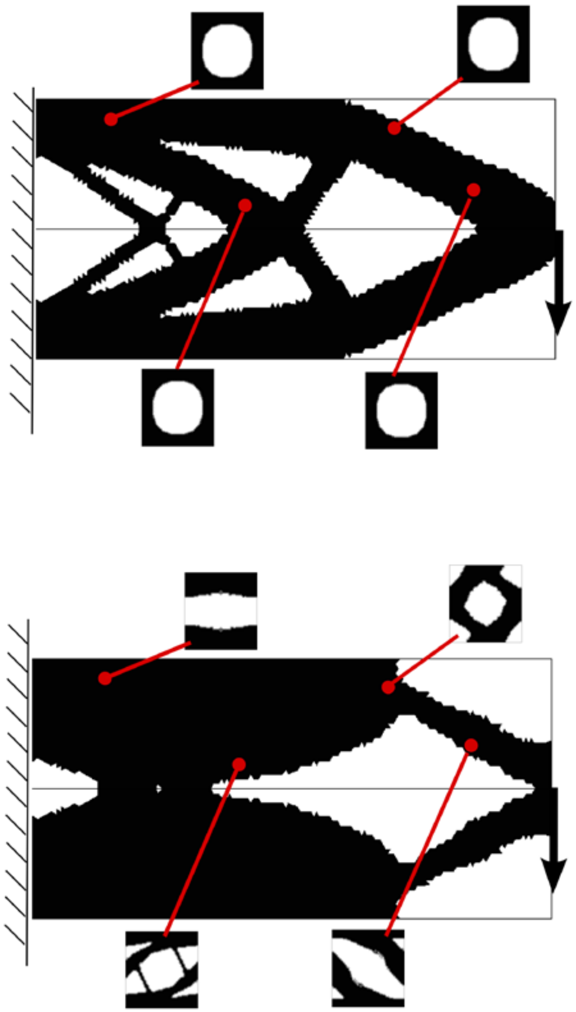


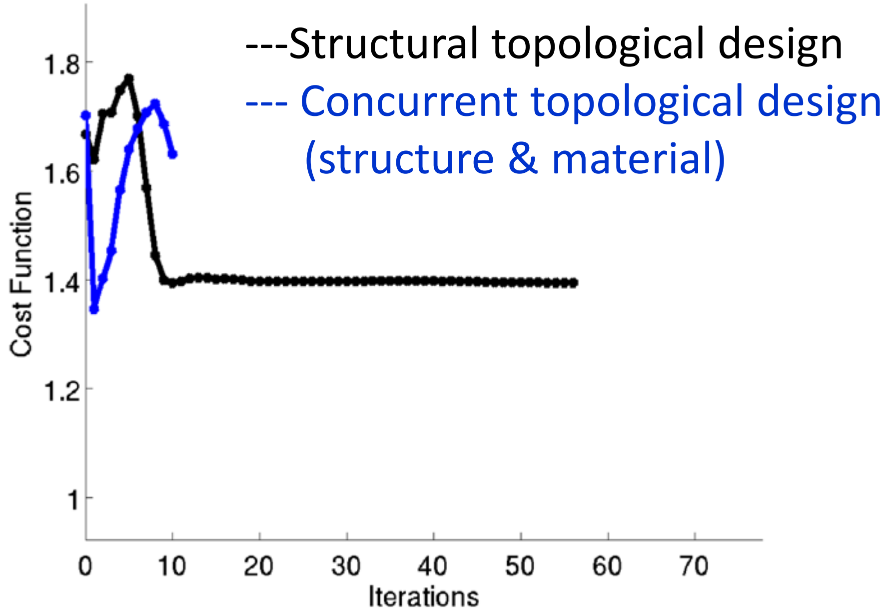
Concurrent (structural & material) topological design



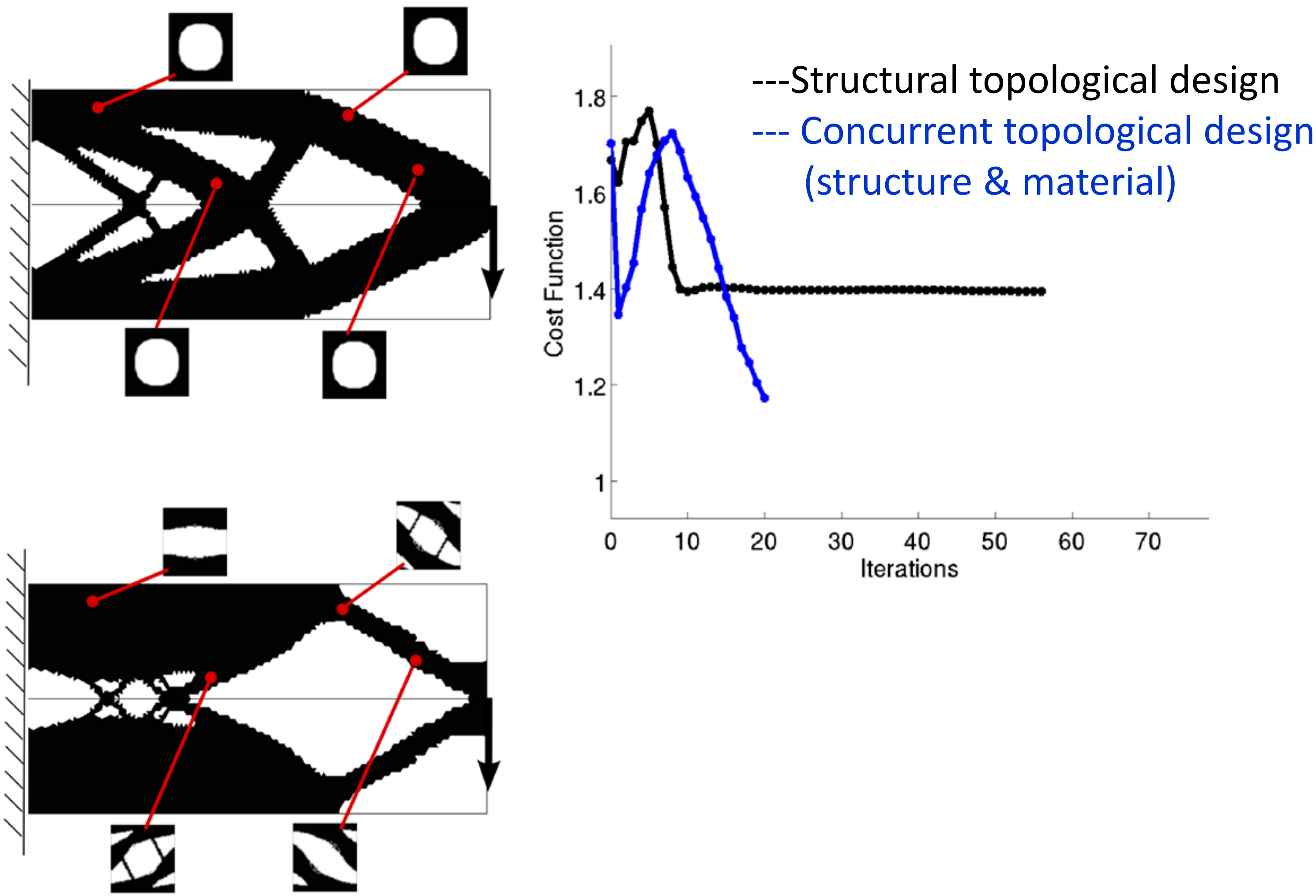


Concurrent (structural & material) topological design

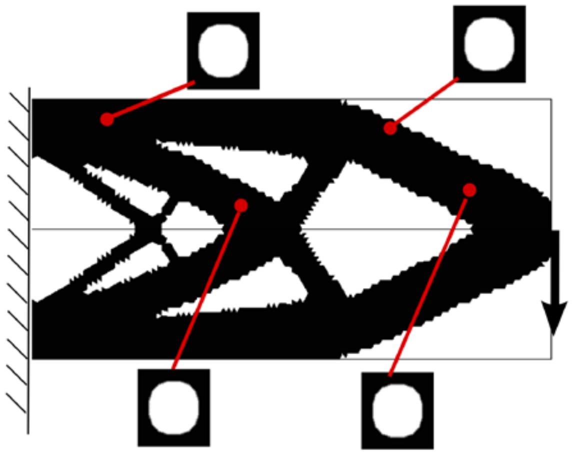


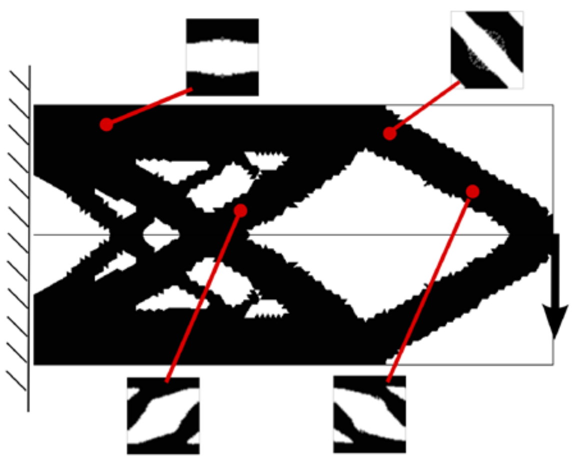


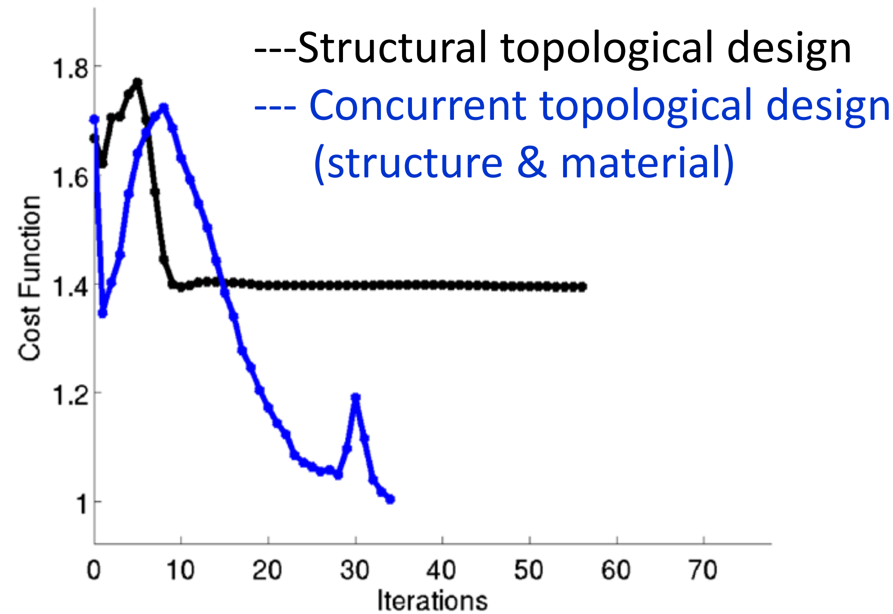
Concurrent (structural & material) topological design



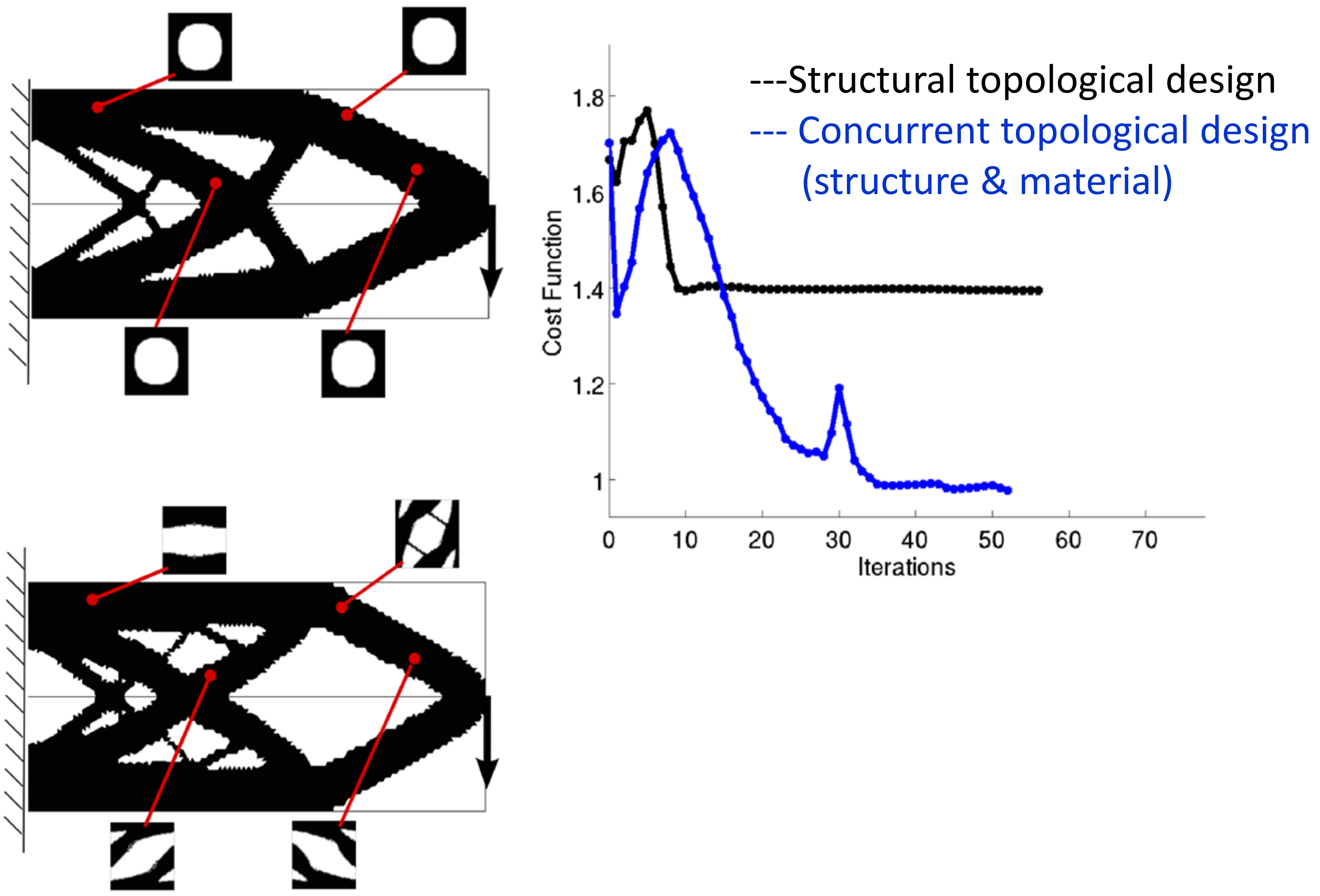
Concurrent (structural & material) topological design



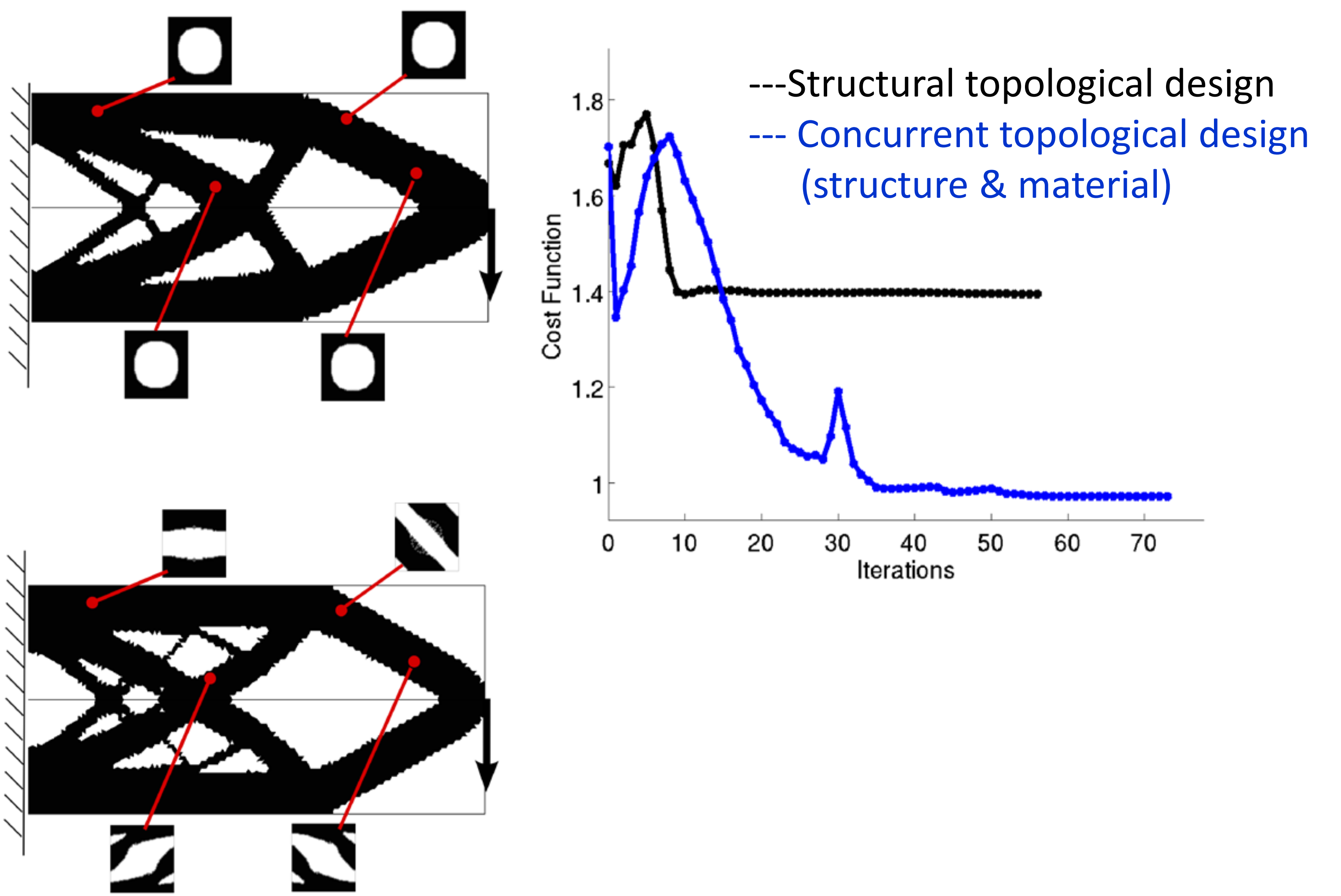




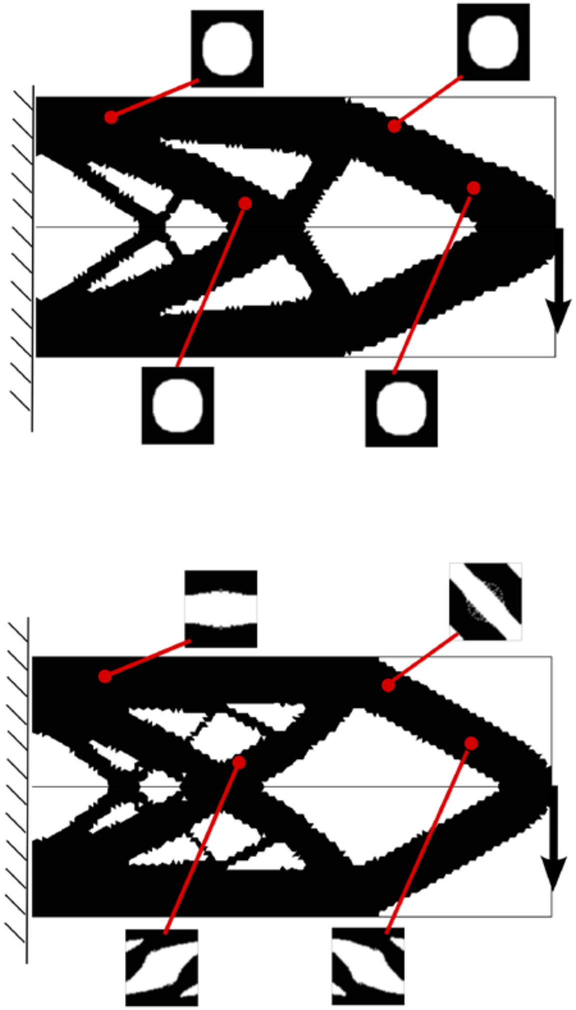
Concurrent (structural & material) topological design

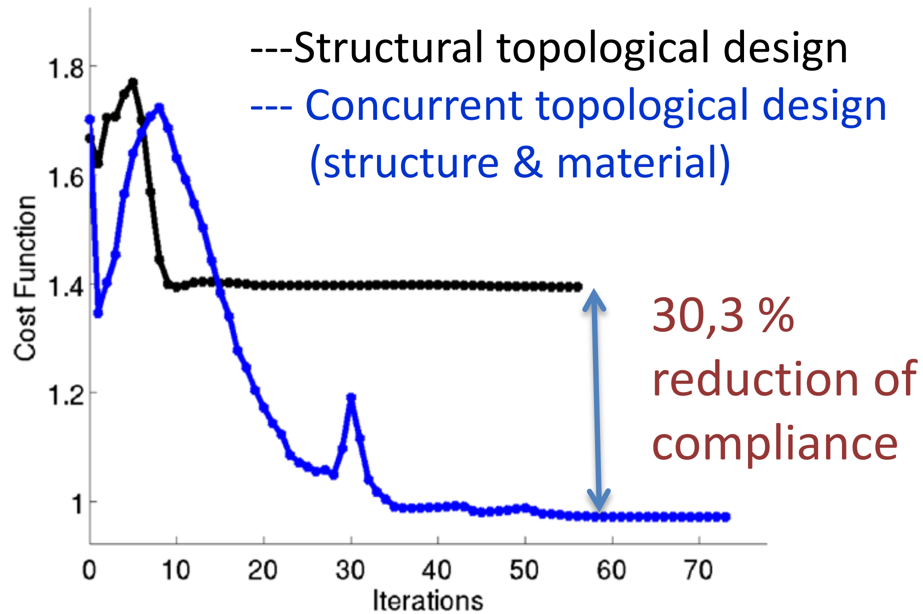


Concurrent (structural & material) topological design



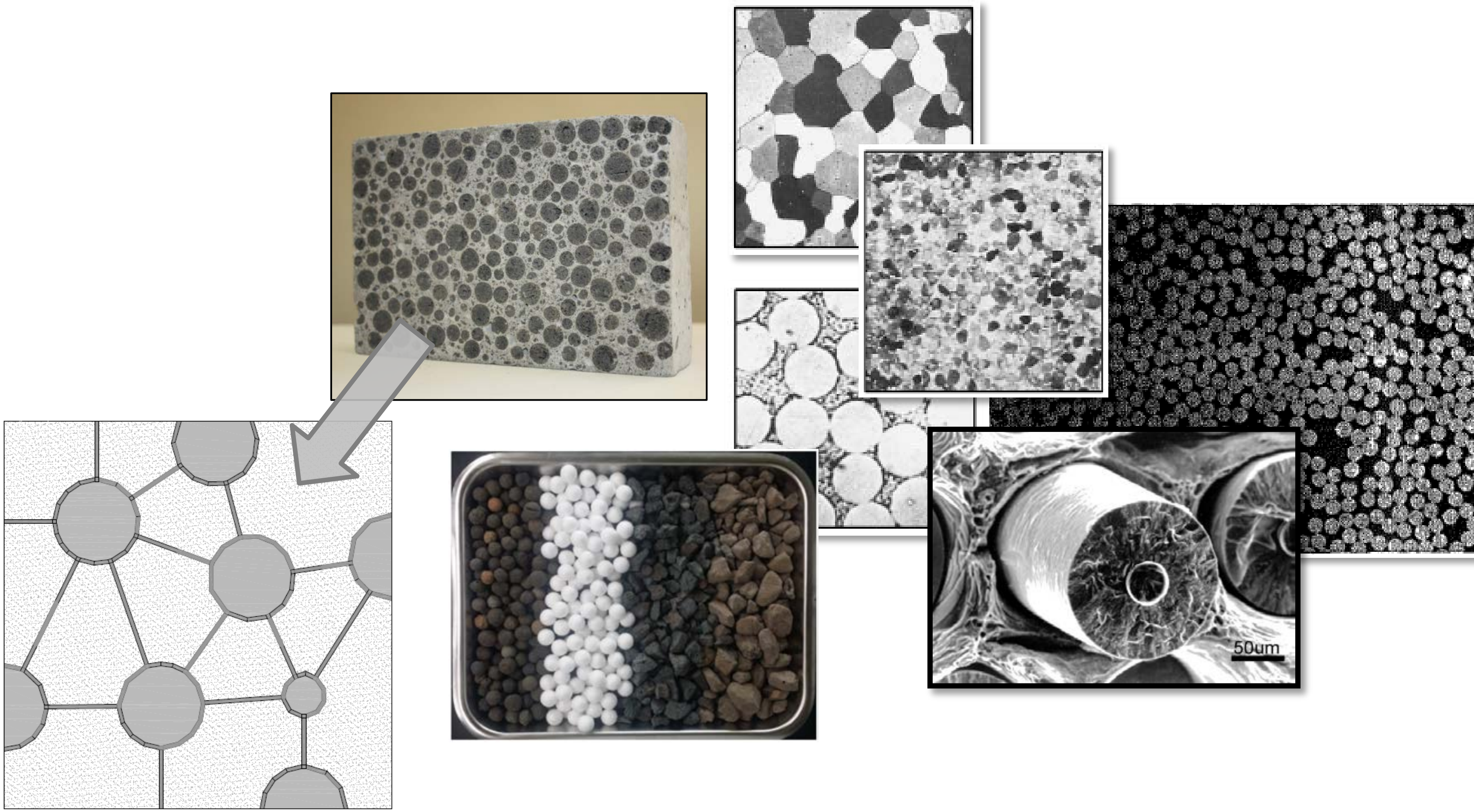
Concurrent (structural & material) topological design





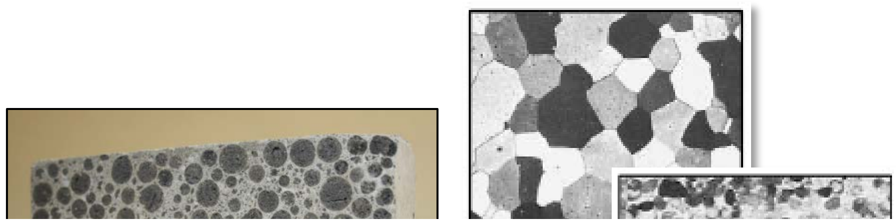
The futur . . .

… a new generation of cementitious meta-materials to be designed through computational material design?

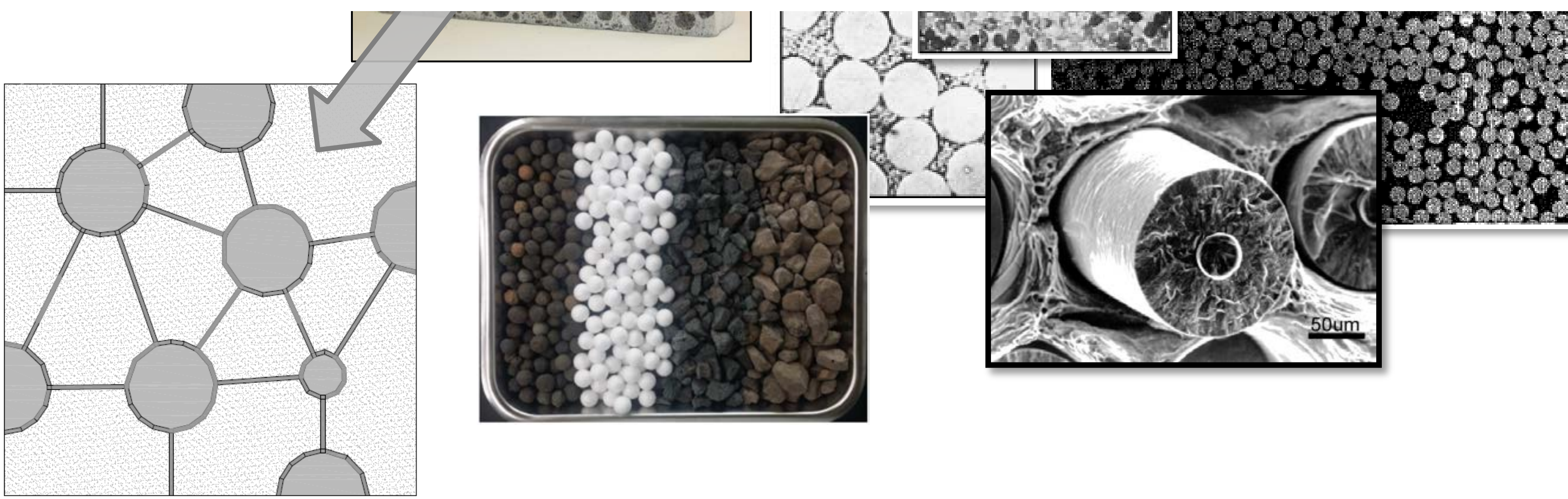


The futur . . .

… a new generation of cementitious meta-materials to be designed through computational material design?



A CHALLENGE FOR THE FUTURE !!!



Support from the European Research Council (ERC) through Advanced Grant ADG_20120216:

ADVANCED TOOLS FOR COMPUTATIONAL DESIGN OF ENGINEERING MATERIALS (COMP-DES-MAT)

is gratefully acknowledged.

Open-source codes and scientific production of this research are available at:

http://cimne.com/compdesmat

European Research Council

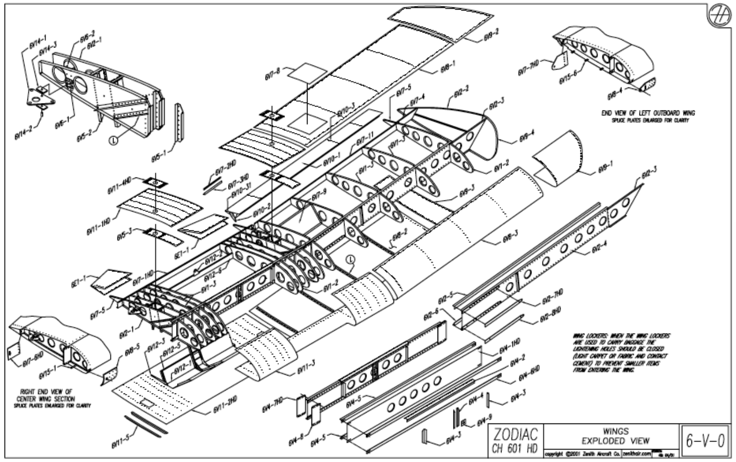
Established by the European Commission

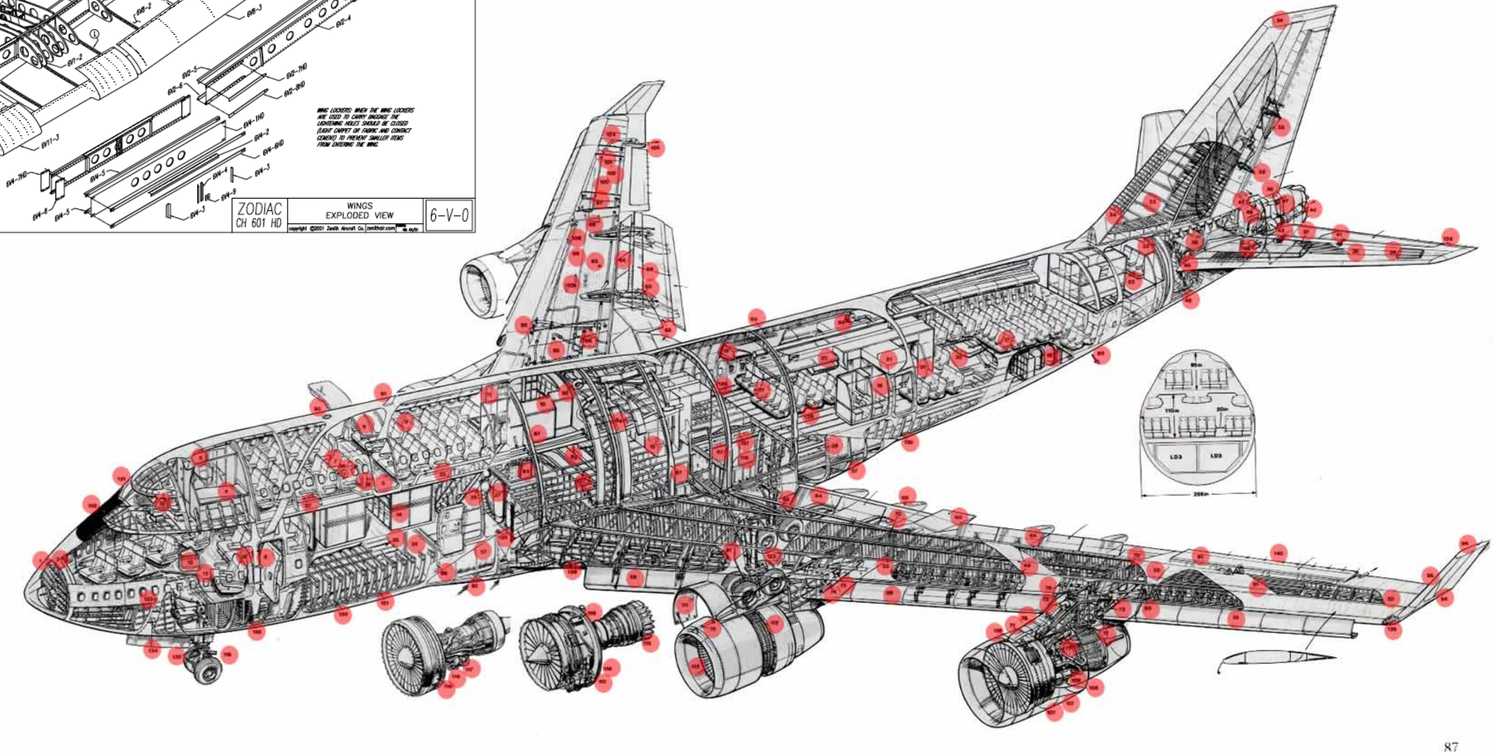
The research leading to these results has received funding from the European Research Council under the European Union's Seventh Framework Programme (FP/2007-2013)/ERC Grant Agreement ADG_20120216

Manufacturability issues

CONTINUUM vs. DISCRETE DESIGN

- Component-based manufacturing





Cantilever beam.

Concurrent (macro/micro-scale) topological design.

- Discrete (by-component) design

Cantilever beam.

Concurrent topological design

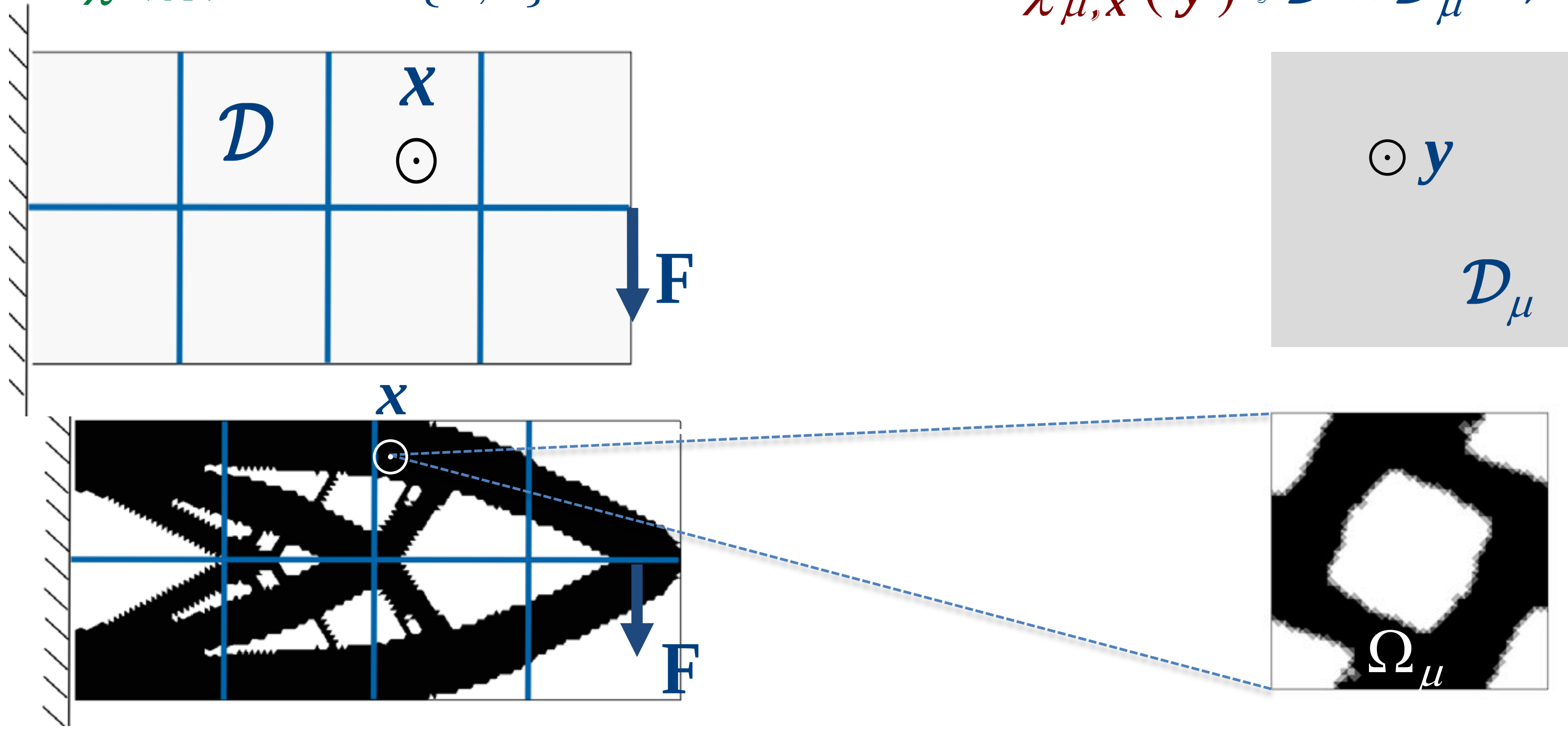
- Discrete (by-component) design

(macro-scale)

$$\chi(\mathbf{x}) : \mathcal{D} \rightarrow \{0,1\}$$

RVE (micro-scale)

$$\chi_{\mu,\mathbf{x}}(\mathbf{y}) : \mathcal{D} \times \mathcal{D}_{\mu} \rightarrow \{0,1\}$$



$$\Omega := \{\mathbf{x} \in \mathcal{D} \; ; \; \chi(\mathbf{x}) = 1\}$$

$$V = \gamma |\mathcal{D}| \; ; \; \gamma = 0.6$$

$$\Omega_{\mu}(\mathbf{x}) := \{\mathbf{y} \in \mathcal{D}_{\mu} \; ; \; \chi_{\mu,\mathbf{x}}(\mathbf{y}) = 1\}$$

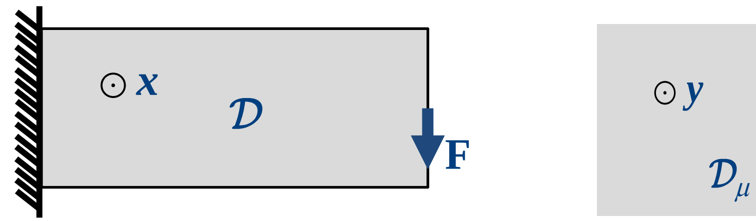
$$V_{\mu} = \gamma_{\mu} |\mathcal{D}_{\mu}| \; ; \; \gamma_{\mu} = 0.6$$

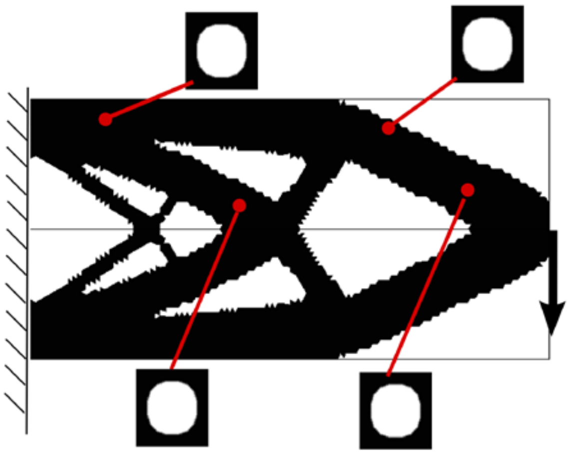
Cantilever beam.

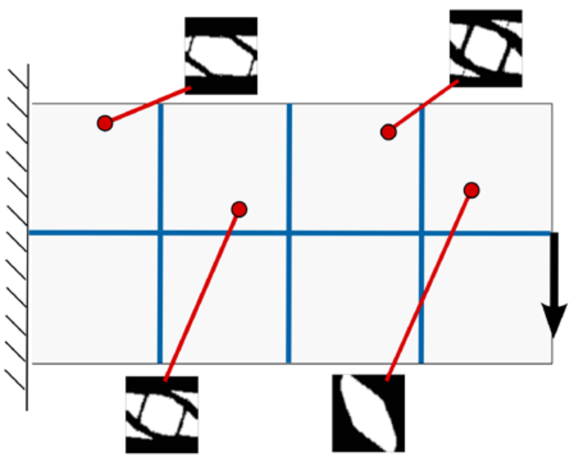
Concurrent topological design

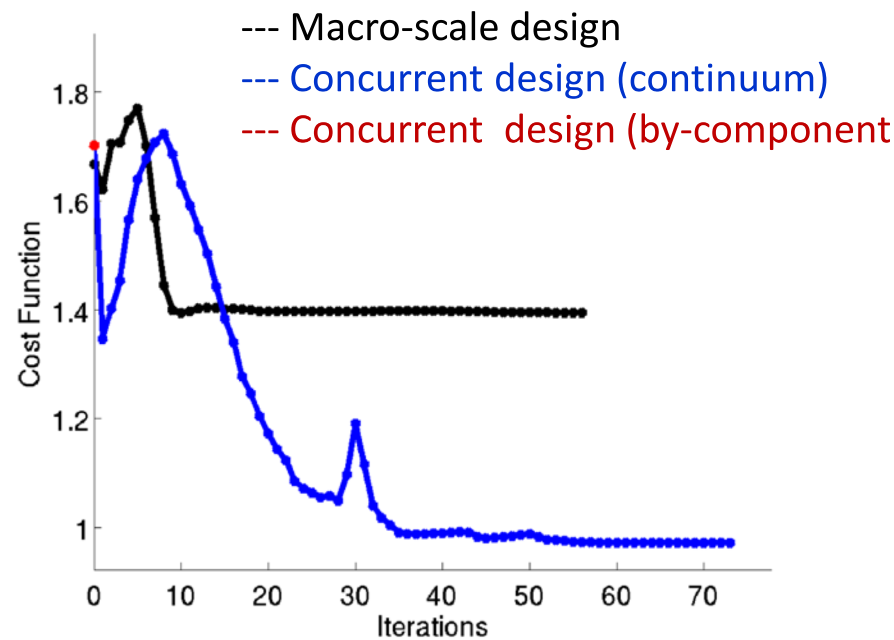
- Discrete (by-component) design

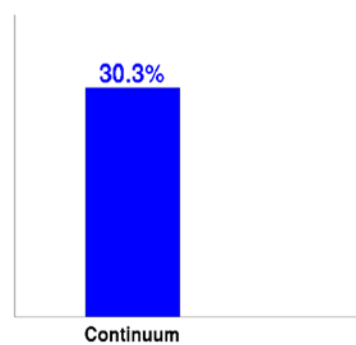
$\chi(x): \mathcal{D} \rightarrow \{0,1\}$ $\chi_{\mu,x}(y): \mathcal{D} \times \mathcal{D}_\mu \rightarrow \{0,1\}$











Cantilever beam.

Concurrent topological design

- Discrete (by-component) design

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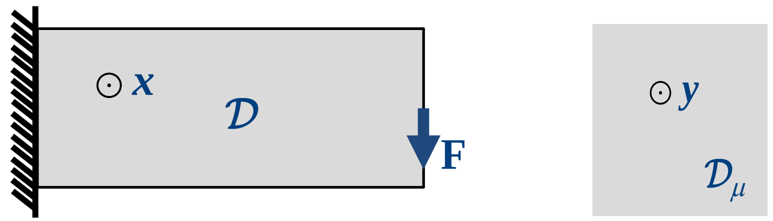

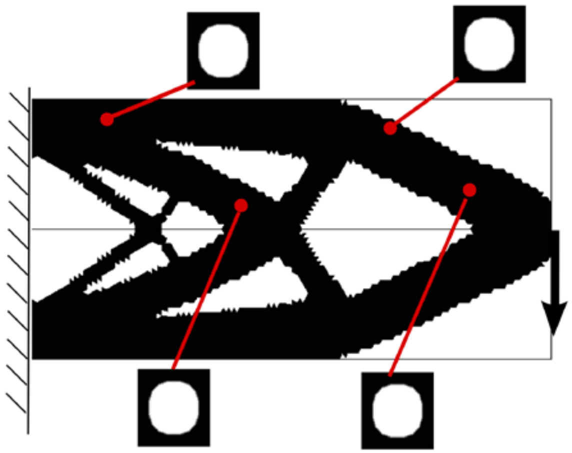

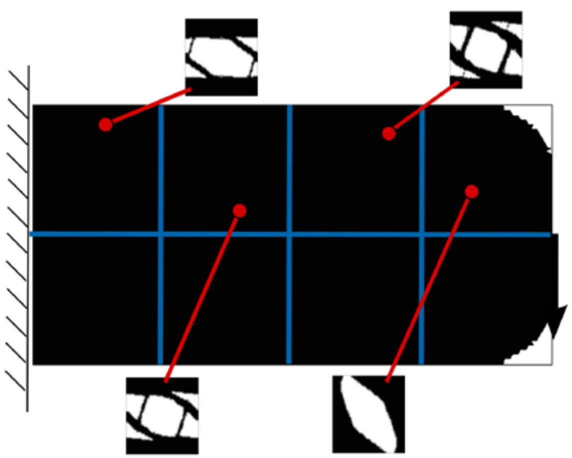

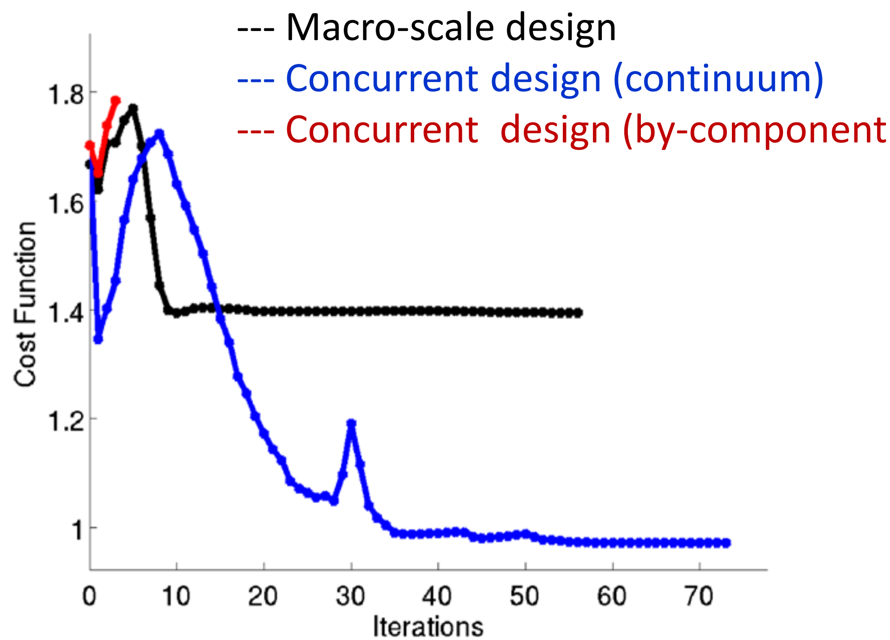

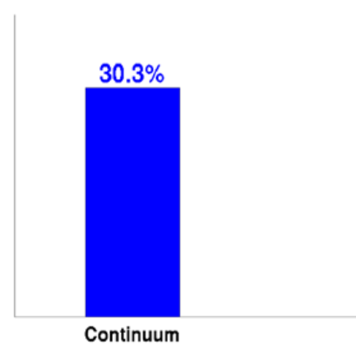

Cantilever beam.

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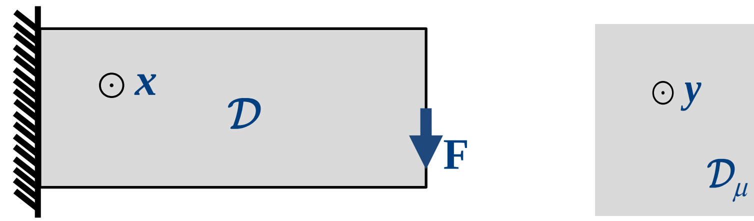

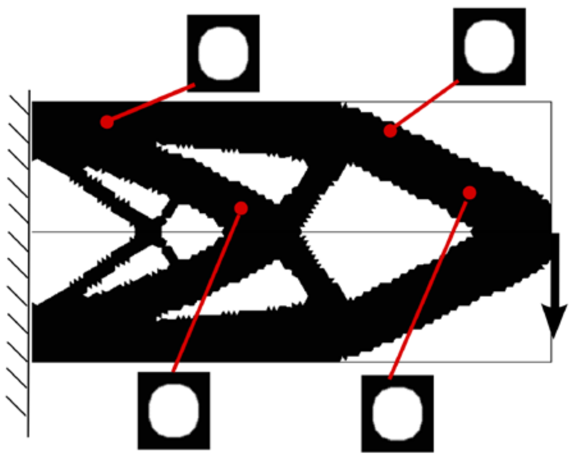

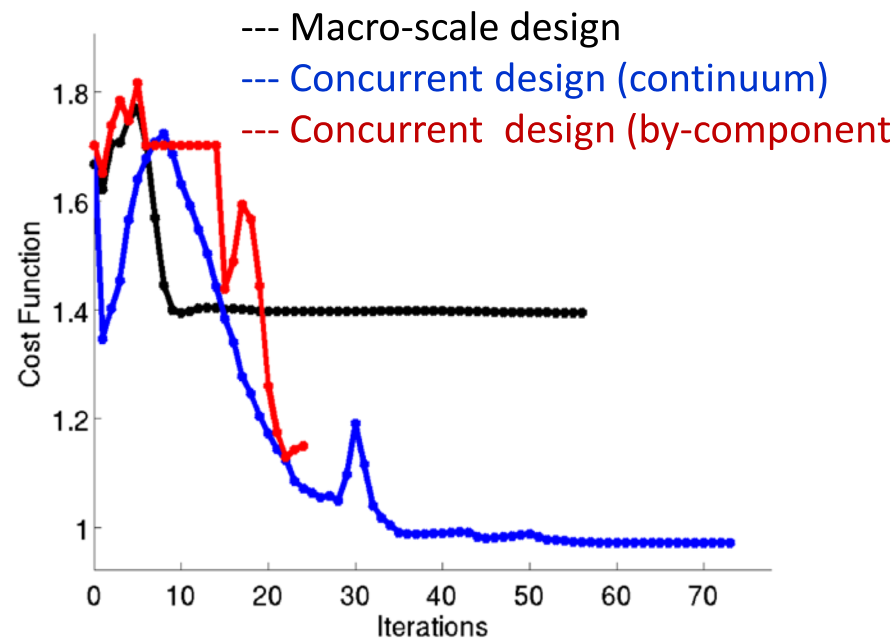

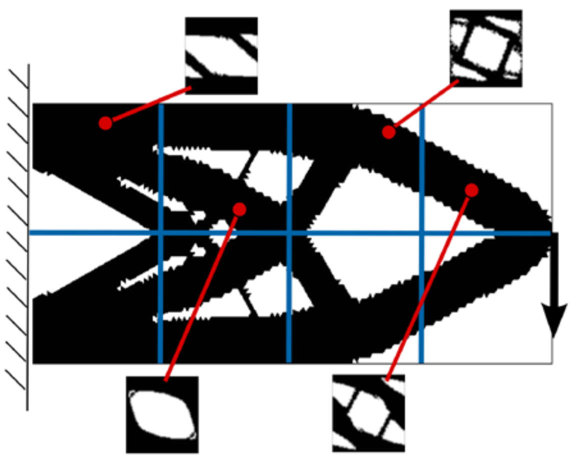

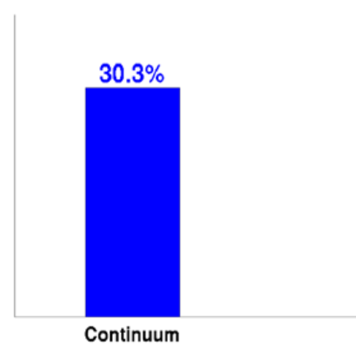

Cantilever beam.

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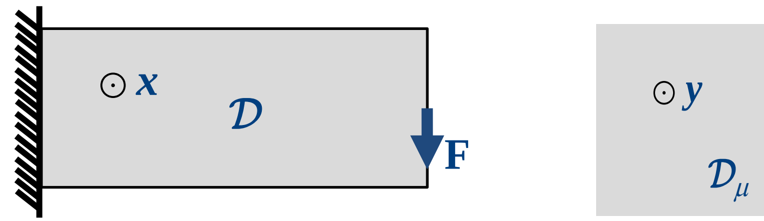

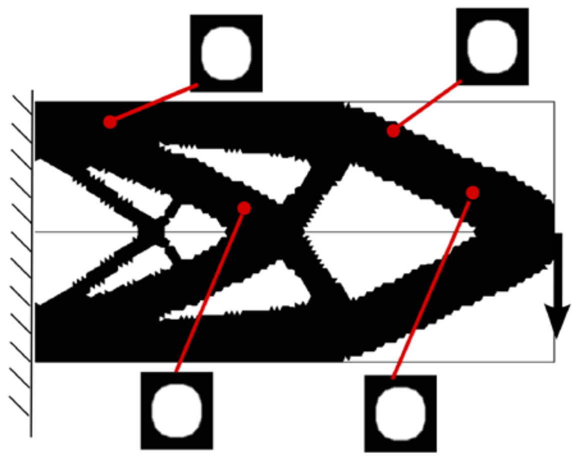

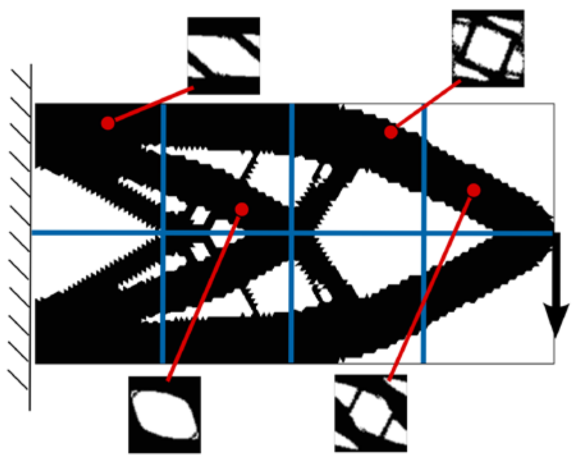

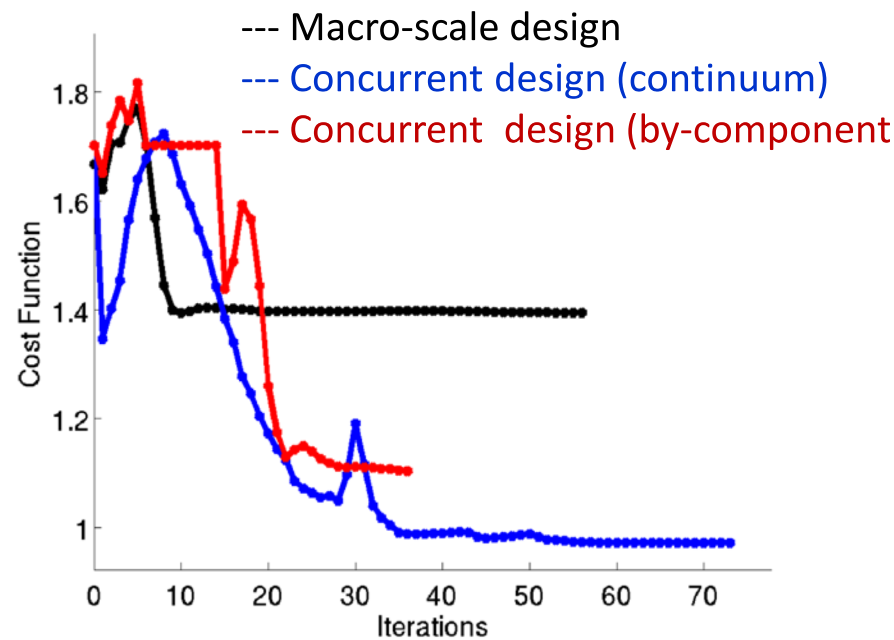

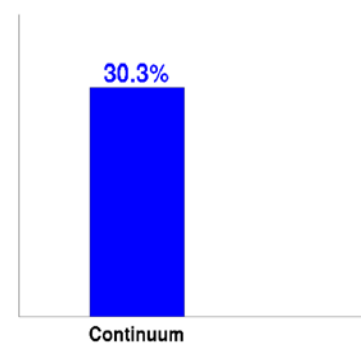

Cantilever beam.

Concurrent topological design

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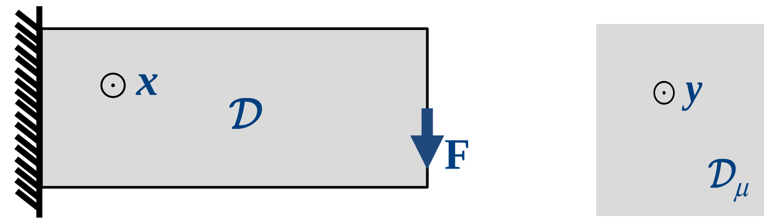

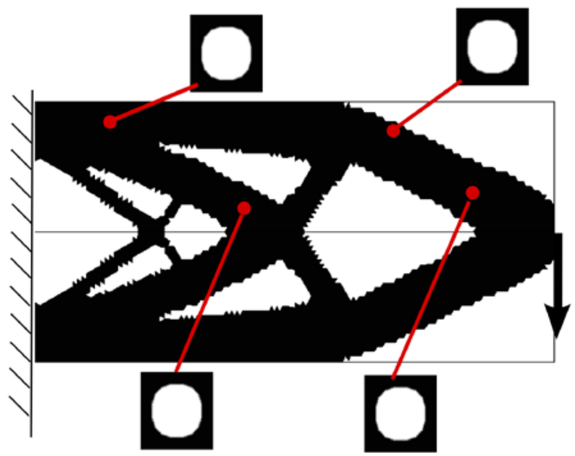

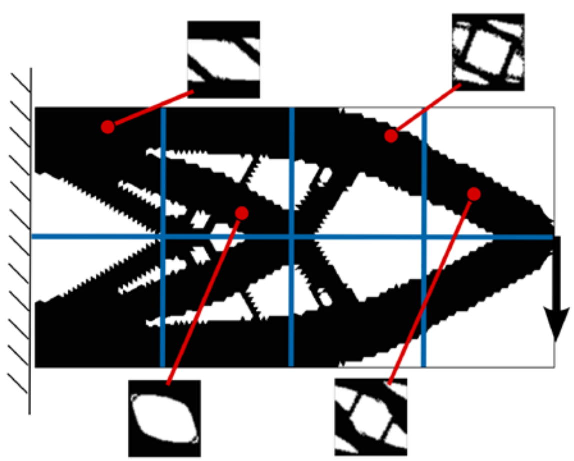

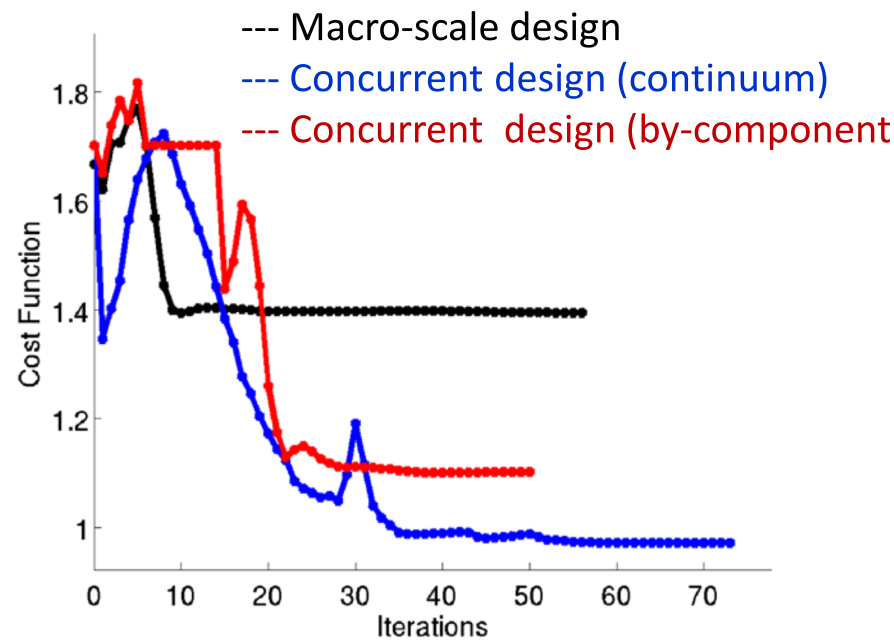

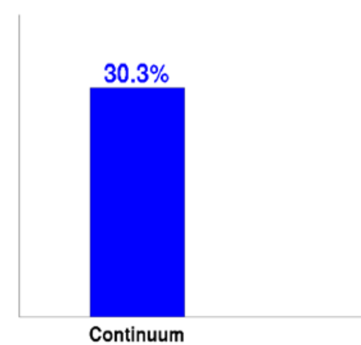

Cantilever beam.

Concurrent topological design

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